# The Choice of Optimal Decisions in Uncertin Situations

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**Abstract:** The Electre method is a classical algorithm for the decidence of a suitable choice in the process of launching on market of some products. In this paper I shall give a variation of the last part of the algorithm in the direction of simplifying the finally computations.

**Keywords:** Electre method, minimisation, maximisation

**Jel Classification:** C70, C35, C30, C15

In many practical situations, it exist many informations refered to the actions which can be developed, but for each action choice there are many possibilities. The Electre method gives us a way which will make out between different possibilities. Let therefore n choices $V_1$, $V_2$,...,$V_n$ for a decident. We have also m criterions $C_1$,$C_2$,...,$C_m$ who have, each of them, a weighty coefficient (by rule subjective assigned) $k_1$,$k_2$,...,$k_m$. For each pair ($V_i$,$C_j$) we have a numerical value (if it is a qualitative estimation we shall convert in hierarchy numbers).

The algorithm will determine the best choice of action.

**Step I**

We settle, for the beginning the method nature: maximisation or minimisation. We shall add two lines under the table, on which will compute the maximum and the minimum of all numbers on columns.

**Step II**

We shall determine the utilities $U_{ij}$ corresponding to all pairs ($V_i$,$C_j$):

- for the maximisation problem: $U_{ij}=\dfrac{v_{ij}-\min\limits_{k=1,\dots,n} v_{kj}}{\max\limits_{k=1,\dots,n} v_{kj}-\min\limits_{k=1,\dots,n} v_{kj}}$;

- for the minimisation problem: $U_{ij}=\dfrac{\max\limits_{k=1,\dots,n} v_{kj}-v_{ij}}{\max\limits_{k=1,\dots,n} v_{kj}-\min\limits_{k=1,\dots,n} v_{kj}}$.

and after we shall draw the table with these results.

**Step III**

We shall compute the correspondence indicators: $c(V_i,V_j)=\dfrac{\sum\limits_{\substack{p=1,\dots,m\\ U_{ip}\geq U_{jp}}} k_p}{\sum\limits_{r=1}^{m} k_r}$ and

after the disunity indicators: $d(V_i,V_j)=\max\limits_{p=1,\dots,m}(U_{jp}-U_{ip},0)$.

**Step IV**

We shall draw a new table for each pair $(V_i,V_j)$ where on each cell we shall write on the left the correspondence indicator and on the right the disunity indicator.

**Step V**

We shall establish two values p and q (with a significance of complementary probabilities) such that $p,q\in(0,1)$ and $p+q=1$ which will extent the admissible limits for correspondence, respectively disunity. We shall say that $V_i$ is better than $V_j$ if

$$\begin{cases} c(V_i,V_j)\geq p \\ d(V_i,V_j)\leq q \end{cases}$$

Let now the matrice $G=(g_{ij})\in M_n(\mathbf{R})$ such that: $g_{ij}=1$ if $V_i$ is better than $V_j$ and 0 in the other situations. If it exists a row i with all elements equal with 1 it follows that $V_i$ is better than all others therefore it will be preffered. If it not exists such a $V_i$ we shall diminish the value of p (and o course increase q) till we shall obtain the better choice.

In what follows I shall present another way to choice p and q.

The system: $\begin{cases} c(V_i, V_j) \geq p \\ d(V_i, V_j) \leq q \end{cases}$ can be written as: $\begin{cases} c(V_i, V_j) \geq p \\ d(V_i, V_j) \leq 1 - p \end{cases}$ therefore:

$$p \leq \min(c(V_i,V_j), 1-d(V_i,V_j))$$

We shall compute $\min_{j=1,n} c(V_i,V_j)$ and $\min_{j=1,n} (1-d(V_i,V_j))$ from where:

$$p \leq \min(\min_{j=1,n} c(V_i,V_j),\ 1-\max_{j=1,n} d(V_i,V_j))$$

In the last table, we shall add therefore three columns, on which we compute $\min_{j=1,n} c(V_i,V_j)$, $1-\max_{j=1,n} d(V_i,V_j)$ and the minimum of these two values. The better choice will be those who give the maximum value on this last column.

**Example**

Let the following problem (of maximisation):

| **Criterion Choice** | **$C_1$ ($k_1$=0,4)** | **$C_2$ ($k_2$=0,4)** | **$C_3$ ($k_2$=0,2)** |
|---|---|---|---|
| **$V_1$** | 1000 | 0 | 50 |
| **$V_2$** | 800 | 1 | 56 |
| **$V_3$** | 600 | 2 | 60 |
| **$V_4$** | 700 | 1 | 54 |
| **$V_5$** | 500 | 2 | 58 |

| **Criterion Choice** | **$C_1$** | **$C_2$** | **$C_3$** |
|---|---|---|---|
| **$V_1$** | 1000 | 0 | 50 |
| **$V_2$** | 800 | 1 | 56 |
| **$V_3$** | 600 | 2 | 60 |
| **$V_4$** | 700 | 1 | 54 |

| | | | |
|---|---|---|---|
| **$V_5$** | 500 | 2 | 58 |
| **min** | 500 | 0 | 50 |
| **max** | 1000 | 2 | 60 |
| **max-min** | **500** | **2** | **10** |

The utilities table is:

| **Criterion Choice** | **$C_1$ ($k_1$=0,4)** | **$C_2$ ($k_2$=0,4)** | **$C_3$ ($k_3$=0,2)** |
|---|---|---|---|
| **$V_1$** | 1 | 0 | 0 |
| **$V_2$** | 0,6 | 0,5 | 0,6 |
| **$V_3$** | 0,2 | 1 | 1 |
| **$V_4$** | 0,4 | 0,5 | 0,4 |
| **$V_5$** | 0 | 1 | 0,8 |

And the table of correspondence indicators and the disunity indicators:

| | **$V_1$** | | **$V_2$** | | **$V_3$** | | **$V_4$** | | **$V_5$** | |
|---|---|---|---|---|---|---|---|---|---|---|
| **$V_1$** | 1 | 0 | 0,4 | 0,6 | 0,4 | 1 | 0,4 | 0,5 | 0,4 | 1 |
| **$V_2$** | 0,6 | 0,4 | 1 | 0 | 0,4 | 0,5 | 1 | 0 | 0,4 | 0,5 |
| **$V_3$** | 0,6 | 0,8 | 0,6 | 0,4 | 1 | 0 | 0,6 | 0,2 | 1 | 0 |
| **$V_4$** | 0,6 | 0,6 | 0,4 | 0,2 | 0,4 | 0,6 | 1 | 0 | 0,4 | 0,5 |
| **$V_5$** | 0,6 | 1 | 0,6 | 0,6 | 0,4 | 0,2 | 0,6 | 0,4 | 1 | 0 |

If we try for p from 1 back to 0 we shall obtain that, for the first time, we shall have for p=0,4 and q=0,6:

$$G=\begin{pmatrix}1 & 1 & 0 & 1 & 0\\ 1 & 1 & 1 & 1 & 1\\ 0 & 1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1 & 1\\ 0 & 1 & 1 & 1 & 1\end{pmatrix}$$

therefore each of $V_2$ and $V_4$ is better than the others.

I shall suggest at this last step the following:

| | $\min\limits_{j=1,n} c(V_i,V_j)$ | $1-\max\limits_{j=1,n} d(V_i,V_j)$ | min |
|---|---|---|---|
| $\mathbf{V_1}$ | 0,4 | 0 | 0 |
| $\mathbf{V_2}$ | 0,4 | 0,5 | **0,4** |
| $\mathbf{V_3}$ | 0,6 | 0,2 | 0,2 |
| $\mathbf{V_4}$ | 0,4 | 0,4 | **0,4** |
| $\mathbf{V_5}$ | 0,4 | 0 | 0 |

therefore each of $V_2$ and $V_4$ is better than the others, but without consecutive tests.

## References


Ioan C.A. (2008) *Mathematics I*, Galati: Sinteze, Romania.

Stancu-Minasian I.M. (1970). *The study of the simulation problem using the graph theory*, Studies and researches of economical calculus and cibernetics, nr.5.

Teodorescu N. (coord.). (1972). *Methods of operational researchs*, Bucharest: Tehnica.