The Modelling of the Volatility of Business cycles in Romania

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Abstract: The latest research highlights the existence of the asymmetry of the volatility of business cycles. In this context, in this paper we firstly aim to test whether the volatility of business cycles in Romania is constant or not and then, according to the identified result we try to model it. For the determination of business cycles of Romania we use the index of the industrial production registered during the period January 2000 – May 2011. The estimation of the business cycles is conducted by means of the Hodrick-Prescot filter. The results obtained confirm that the volatility of business cycles of Romania is not constant and suggest the possibility of taking into account the heteroscedastic models. The estimation of the EGARCH model shows that Romania's business cycles present an asymmetric volatility.

Keywords: asymmetric volatility, EGARCH, Hodrick-Prescot filter.

1 Introduction

In 1862 Clément Juglar publishes the first paper dealing with the cyclical fluctuations of the economic activity entitled Des Crises commerciales et leur retour périodique en France, en Angleterre et aux Etats-Unis. In this study he identifies the regular fluctuations of the business activities with an average length of nine-ten years, which are nowadays called short cycles (Juglar, 1862).

In 1920 J. Kitchin discovers the very short 40 month-long cycles which are enlisted within a Juglar cycle, while in 1925 Kondratiev identifies the long cycles of approximately fifty years in the paper The Major Economic Cycles (Kondratiev, 1925). J. Schumpeter justifies the existence of the five business cycles belonging to Kondratiev by means of the technological shocks determined by: the industrial revolution, the invention of steam engine and the railroads, the steel and the electricity, the oil, the automobiles and the mass production and finally the information and telecommunications.

After the occurrence of these papers, the subsequent studies focused on:

- the analysis of fluctuations: of prices, of business activity or of production and the number of employees. At the end of the XIXth century and the beginning of the XXth century the three economic phenomena had the same evolution. At the end of the 1960s this situation was not valid anymore: the prices were going up while the production and the number of employees remained constant. This period was known under the name of "stagflation".

- the integration in the economic fluctuations of the long-term accumulation process. If the first theories of the cycles took into consideration the regular fluctuations around the long-term horizontal trend, Marx tries to integrate in his paper, The Capital, in the analysis of economic crises, the long-term accumulation process, meaning an increasing trend. Even if the influence of his idea was limited since he didn't leave a complete theory, its consequences still can be found in the papers of Tugan-Baranovsky M. I., Aftalion A. A., Lescure J., Schumpeter J., Goodwin R. M.

- the clipping of the cyclical periodicity, of time series and the identification of a changing points of the steps of cyclical evolution: expansion, crisis, recession and re-launch. The latest studies use the spectral analysis in the cycle determination.

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In 1920, Friedrich Hayek (Nobel Prize laureate in Economics, in 1974 for the analysis of economic changes), creates and becomes the director of the "Austrian Institute of Research of Business Cycles". His interpretations regarding the business cycles are known as The Austrian Theory of Business Cycles.

Due to the discovery of the great importance of business cycles research, in 1920 the National Bureau of Economic Research (NBER) was found, being an American research body. Its researchers developed an observation methodology of the USA cycles. A part of the researchers associated with the NBER were awarded the Nobel Prize in Economics: Jean Tinbergen, Simion Kuznets. The Organisation for Economic Co-Operation and Development (OCDE) is another body which supported the research in the field of business cycles.

Since the latest research regarding the business cycles focus on the study of the volatility of business cycles, the present paper is aiming at approaching the volatility of business cycles in Romania. In order to reach this goal we will go on with a brief presentation of the specialty literature concerning the business cycles and the volatility of business cycles, then the next two chapters will focus on the structured empirical analysis as well as on data, methods and main findings and we will end with conclusions and future research.

2 Literature Review

The present focus on this research theme is justified by the Optimum Currency Area theory (Mundell, 1961) according to which the future as well as the integration of the New Member States of the European Economic Union into the European Monetary Union (EMU), depends on the synchronicity among the business cycles of the member states. If the business cycles are not correlated, then the decisions of the European Central Bank (which are taken in relation to the stage of the business cycle of the countries' economy) are adequate only for those targeted countries which are in the same stage of the business cycle.

A benchmark definition of business cycles belongs to Burns and Mitchell: "Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own." (Burns & Mitchell, 1946)

From this definition, the researchers (Abraham-Frois, 1995) took into consideration several elements underlined by Burns and Mitchell, namely: by means of business cycles we take into consideration the business movement, the business cycles or the cycles of economic activity; in order to highlight the business cycle it is necessary to observe the simultaneous evolution of a certain number of series; the cycles have a recurrent but not strictly regular character; the cycle is structured on four phases of evolution: expansion, recession, contraction, and revival and the business cycles are specific to the capitalist countries.

The Central and East European countries have begun the passage to a capitalist economy after 1990. For these countries the study of business cycles can be undertaken after this year. The first years after 1999 can be considered as being part of the transition period towards the market economy. Since 2000 we can already say that the CEE countries have passed to the market economy.

A recent preoccupation in the field of business cycles focuses on volatility. Certain studies approached the relationship between volatility and economic growth (Fang & Miller, 2009) (Turnovsky & Chattopadhyay, 2003), (Nuță, 2011) others determined the determinant factors of volatility (Furceri &

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Karras, 2007), (Buch & Doepke, 2005) and some others highlighted the asymmetric character of business cycles volatility (Ho & Tsui & Zhang, 2009), (Ho & Tsui & Zhang, 2007).

3 Empirical Analysis

3.1 Data and Methods

The values of the industrial production index for the period January 2000 – May 2011 for Romania are provided by the EUROSTAT database. In order to estimate the business cycles we took the deseasonalized values of the series of industrial production index from the EUROSTAT database. These values were logarithmated and then by means of the Hodrick-Prescott filter we extracted the cyclical component of the series of industrial production index in the period under analysis.

The Hodrick-Prescott filter is very often used in the identification of the long-term trend of a variable. The cyclical component of a time series is obtained as a difference between the original series and its long-term trend obtained by means of the filter. The Hodrick-Prescott filter is given by the following relation:

$$\sum_{t=1}^{t} c_{t}^{2} + \lambda \sum_{t=2}^{t} \left[\left(g_{t} - g_{t-1} \right) - \left(g_{t-1} - g_{t-2} \right) \right]^{2}$$

where: $c_t = lny(t) - lny^*(t)$, $g_t = lny^*(t+1)$, $g_{t-1} = lny^*(t)$, $g_{t-2} = lny^*(t-1)$,

 y^* - the long-term trend of the variable y.

In the case of monthly data, for the estimation of the long-term trend, the most used value of the parameter λ is 14400.

The cyclical component obtained must be stationary. In order to numerically verify the stationarity of a variable there are several tests among which we mention: Dickey-Fuller, Augmented Dickey-Fuller and Philips Perron. The Dickey-Fuller and Augmented Dickey-Fuller tests are parametrical tests while the Philips Perron is a nonparametric one.

The Dickey-Fuller test is used for the autoregressive variables of order 1. If the variable follows an autoregressive model superior to 1 then the Augmented Dickey-Fuller test is used.

The Philips Perron test uses the same equation as that of the Dickey-Fuller test (DF) and computes the tests t to verify the hypotheses presented at the DF test not only under the hypothesis of error independence but also under the hypothesis of potential autocorrelations.

To identify the ARMA process followed by the return rates we may use the autocorrelation functions. For a random variable Y we may define two autocorrelation functions:

- the total autocorrelation function (ACT)
- the partial autocorrelation function (ACP)

In order to test the null hypothesis (implying the absence of the autocorrelation until order k) the

statistics Q of the Ljung-Box is used having the following expression: $Q_k = T(T+2)\sum_{i=1}^k \frac{\hat{\rho}_i^2}{T-i}$.

The tested hypotheses are:

H₀: $\rho_1 = \rho_2 = ... = \rho_k = 0$

$$H_1: \rho_1 \neq \rho_2 \neq \dots \neq \rho_k \neq 0$$

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The null hypothesis supposes the lack of autocorrelation until order k while the alternative hypothesis supposes the existence of the autocorrelation. Supposing the null hypothesis is true, the variable Q follows a law χ_k^2 (with k degrees of freedom).

The recognition of the process followed by a variable by means of the autocorrelation functions is conducted as follows (Berdot, J. P., 2001):

- an AR process AR(p) has an infinite and convergent function towards zero (in case of stationarity) and a truncated function ACP of order p;

- a MA process MA(q) has an ACP infinite and convergent function towards zero (in case of inversibility) and a truncated function of order q;

- an ARMA process (p,q) has the infinite and convergent functions ACT and ACP (in case of stationarity and inversibility), the orders p and q being determined by trials.

If the squares of the values of business cycles are autocorrelated we may say that the business cycles values are dependent and therefore the business cycles may be modelled by means of heteroscedastic models.

The models with conditional variance which vary in time are defined by two equations: the first equation expresses the expectancy of the variable (often presented as an ARMA approach) and the second equation expresses the conditional variance of this variable as being determined by a heteroscedastic process.

The ARCH model (Autoregressive Conditional Heteroskedasticity) was proposed by Engle in 1982 (Engle, 1982). The model takes into consideration the variation of the variable's volatility (the heteroskedasticity), and also a characteristic of the financial variables, "fat tails"

An ARCH(2) model based on an ARMA(2,1) model is expressed y the following equations:

- the ARMA model for Y: $Y_t = a_0 + a_1 Y_{t-1} + a_2 Y_{t-2} + \mathcal{E}_t - m_1 \mathcal{E}_{t-1}$.

$$-h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2$$

In presenting these models, we shall use the following notations:

 Y_t - the analyzed variable

 h_t - the conditional variation of the errors, that is a prediction of the volatility of errors on day t taking into account the knowledge of the phenomenon (knowledge of the previous values of the variable, until the day t-1).

For the variance to be positive or null, the following conditions are required:

 $\alpha_0 > 0, \alpha_1, \alpha_2, \dots, \alpha_p \ge 0.$

The GARCH model (Generalized Autoregressive Conditional Heteroskedasticity) offers the possibility to predict the conditional variance (volatility) starting from the previous errors, but also from the previous predictions of the variance (Boollerslev, 1986). The GARCH model allows volatility to be expressed as an autoregressive process. By means of this model, two characteristics are taken into account: a characteristic of volatility, volatility clustering, and a characteristic of distribution, fat tails. The GARCH(p,q) model may be presented in the form of the following equations:

- the ARMA model for Y: $Y_t = a_0 + a_1 Y_{t-1} + a_2 Y_{t-2} + \varepsilon_t - m_1 \varepsilon_{t-1}$.

$$-h_{t} = \alpha_{0} + \alpha_{1}\varepsilon_{t-1}^{2} + \alpha_{2}\varepsilon_{t-2}^{2} + \dots + \alpha_{p}\varepsilon_{t-p}^{2} + \beta_{1}h_{t-1} + \dots + \beta_{q}h_{t-q}$$

In order for volatility h_i to be positive the following conditions must be met: $\alpha_0 > 0$, $\alpha_i \ge 0$, $\beta_i \ge 0$. Also, the stationarity condition is ensured if $\alpha_i + \beta_i < 1$.

By means of the EGARCH model (exponential GARCH) the asymmetry phenomenon of the impact of news on variables (return) is modeled: a negative shock with the same force as a positive shock leads to a higher increase of volatility (asymmetric volatility).

The EGARCH(1,1) model has the following formulation:

- the ARMA model for Y: $Y_t = a_0 + a_1 Y_{t-1} + \mathcal{E}_t - m_1 \mathcal{E}_{t-1}$

$$- lnh_{t} = \alpha_{0} + \alpha_{1} \left| \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right| + \gamma_{1} \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \delta_{0} lnh_{t-1}$$

The asymmetry effect is highlighted by γ_1 . This estimated parameter must be significant and lower than zero.

The TGARCH(1,1) model has the following formulation(Zakoian,1990):

- the ARMA model for Y: $Y_t = a_0 + a_1 Y_{t-1} + \mathcal{E}_t - m_1 \mathcal{E}_{t-1}$

-
$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma_1 \varepsilon_{t-1}^2 d_{t-1} + \beta_1 h_{t-1}$$

 $d_t = 1$ if $\varepsilon_t < 0$

3.2 Main Findings

After the estimation of business cycles in Romania by means of the above-mentioned methodology we also tested their stationarity. The results are presented in the table below:

Test	Caracteristici	Model cu constantă și trend	Model cu constantă	Model fără constantă și trend
Augmented Dickey	Testul t	-3,925965	-3,913907	-3,941004
Fuller	(Prob. test t)	(0,0025)	(0,0140)	(0,0001)
	Crit. Akaike	-4,909322	-4,894911	-4,923481
	Crit.Schwartz	-4,866281	-4,830349	-4,901960
	Testul t	-3,855213	-3,841955	-3,873442
Philips-Perron	(Prob. test t)	(0,0031)	(0,0172)	(0,0001)
	Crit. Akaike	-4,909322	-4,894911	-4,923481
	Crit.Schwartz	-4,866281	-4,830349	-4,901960

Table 1 The results of the application of the Augmented Dickey-Fuller and Philips Perron tests for the cyclical component of the industrial production index in Romania during January 2000-May 2011

Note: Results generated by means of the Eviews software program

As the Akaike and Schwarz criteria are minimum for the model without intercept and trend, the variable representing the business cycles does not have a unit root and therefore is stationary. Since

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the information criteria are minimum for the same model we do not use other tests to test the stationarity.

Table 2 The correlogram of the cyclical component of the industrial production index in Romania during January 2000-May 2011

Sample: 2000M01 2011M05

Included observations: 136

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
. *****	. ******	1	0.796	0.796	88.050	0.000
*****	. *	2	0.691	0.157	154.94	0.000
. ****	. .	3	0.599	0.030	205.60	0.000
***	* .	4	0.463	-0.152	236.10	0.000
. ***	. .	5	0.373	-0.006	256.01	0.000
. **	* .	6	0.260	-0.100	265.79	0.000
. *	. .	7	0.162	-0.047	269.61	0.000
. *	. .	8	0.084	-0.032	270.65	0.000
. .	* .	9	-0.004	-0.070	270.65	0.000
* .	* .	10	-0.096	-0.112	272.02	0.000
* .	. .	11	-0.153	-0.016	275.55	0.000
** .	* .	12	-0.222	-0.077	283.03	0.000

Note: Results generated by means of the Eviews software program

The probabilities associated with the Ljung-Box test in the table above are smaller than the taken risk of 0.05. Therefore, with a 0.95 probability the hypothesis H0 is rejected while the hypothesis H1 is accepted, the values of the cyclical component of the industrial production index in Romania during January 2000 - May 2011 are autocorrelated. This result proves the possibility of the modelling of the cyclical component by means of the Box&Jenkins methodology.

The analysis of the dependence of the cyclical component values of the industrial production index is also conducted by means of the Ljung-Box test, but when applied to the cyclical component square, it proves that the business cycles volatility is variable and that for modelling other heteroscedastic models should be taken into consideration.

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Table 3 The correlogram of the square of the cyclical component of the industrial production index inRomania during January 2000 – May 2011

Sample: 2000M01 2011M05

Included observations: 136

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
. *****	. *****	1	0.636	0.636	56.259	0.000
***	. *	2	0.478	0.124	88.320	0.000
. ***	. *	3	0.437	0.156	115.26	0.000
. **	* .	4	0.249	-0.180	124.07	0.000
. **	. *	5	0.218	0.093	130.87	0.000
. *	. .	6	0.158	-0.056	134.48	0.000
. *	. .	7	0.096	0.024	135.83	0.000
. *	. .	8	0.090	-0.007	137.00	0.000
. *	. *	9	0.145	0.167	140.11	0.000
. **	. *	10	0.247	0.189	149.24	0.000
. **	. .	11	0.245	0.000	158.22	0.000
. **	. .	12	0.216	-0.057	165.30	0.000

Note: Results generated by means of the Eviews software program

As a consequence, for the modelling of business cycles we will proceed with the following steps:

the identification of the corresponding ARMA model

the estimation of the ARMA model

taking into account the previously estimated ARMA model we will model the volatility by means of the heteroscedastic model

In order to identify the best ARMA model we estimated all the ARMA models with different orders from 0 to 3 while the values of the Akaike, Schwarz and Hannan-Quinn criteria are presented in the table below.

Table 4 The values of the Akaike, 3	Schwarz and H	Iannan-Quinn	criteria for	different A	ARMA 1	nodels of
business cycles in Romania for the	period January	/ 2000 – May 2	2011			

AR/MA		0	1	2	3
Akaike		-3.875814	-4.464359	-4.615383	-4.807874
Schwarz	0	-3.854397	-4.442942	-4.572549	-4.743624
Hannan-Quinn		-3.867111	-4.455656	-4.597976	-4.781765
Akaike		-4.923481	-4.935153	-4.927649	-4.936813
Schwarz	1	-4.901960	-4.892112	-4.863087	-4.850731

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Hannan-Quinn		-4.914736	-4.917663	-4.901413	-4.901832
Akaike		-4.930178	-5.052969	-4.996959	-4.935730
Schwarz	2	-4.886927	-4.988092	-4.910456	-4.827601
Hannan-Quinn		-4.912602	-5.026605	-4.961807	-4.891790
Akaike		-4.913291	-4.938758	-4.927043	-5.001681
Schwarz	3	-4.848095	-4.851831	-4.818383	-4.871289
Hannan-Quinn		-4.886797	-4.903434	-4.882888	-4.948695

Note: Results generated by means of the Eviews software program

The values of the minimum Akaike, Schwarz and Hannan-Quinn criteria which are presented in the above table show us as the best model for the business cycles in Romania for the period under analysis the ARMA (2,1) model. In order to correct the error heteroscedascity we use the heteroscedastic models. The values of the Akaike, Schwarz and Hannan-Quinn criteria for the tested heteroscedastic models are described in the table below:

Table 5 The values of the Akaike, Schwarz and Hannan-Quinn criteria for different heteroscedastic models of the business cycles in Romania during January 2000 - May 2011

Model	ARCH(1)	ARCH(2)	GARCH(1,1)	GARCH(1,2)	GARCH-M(1,1)
Akaike	-4.957769	-4.950658	-5.073704	-4.966018	-4.950355
Schwarz	-4.849641	-4.820904	-4.943950	-4.814638	-4.798975
Hannan-Quinn	-4.913829	-4.897930	-5.020976	-4.904502	-4.888839

Note: Results generated by means of the Eviews software program

Table 6 The values of the Akaike, Schwarz and Hannan-Quinn criteria for different heteroscedastic models of the business cycles in Romania during January 2000 - May 2011 (continuation)

Model	EGARCH 1	EGARCH-M	TGARCH
Akaike	-5.103084	-4.952089	-5.066865
Schwarz	-4.951704	-4.800709	-4.893859
Hannan-Quinn	-5.041568	-4.890573	-4.996561

Note: Results generated by means of the Eviews software program

Since the information criteria are smaller for the ARCH (1) model than for the ARCH (2) model, we choose the estimation of the ARCH (1) model. After the estimation of the GARCH (1,1) and GARCH (1, 2) models, we notice that the smallest information criteria are for the GARCH (1, 1) model, therefore we do not estimate GARCH models with higher orders. When estimating the GARCH (1,1) model we observe that the condition specifying that in the equation of conditioned volatility all parameters must be positive and smaller than zero is not met, therefore this model cannot be taken into consideration. The GARCH-M (1,1) model estimated for the business cycles in Romania is not a better model than the ARCH (1) model.

	ARC	CH(1)	GARC	CH(1,1)	EGARCH(1,1)		TGARCH(1,1)	
y_{t-1}	0,7833	(0,0763)	0,6078	(0,0270)	0,8888	(0,0000)	0,7851	(0,0154)
y_{t-2}	0,0946	(0,7946)	0,2112	(0,3306)	0,0951	(0,0896)	0,1182	(0,6543)
\mathcal{E}_{t-1}	-0,1917	(0,6607)	-0,0095	(0,9717)	-0,220	(0,0011)	-0,1675	0,6080
$lpha_0$	0,003	(0,0000)	0,00001	(0,5349)	-0,8688	(0,0000)	0,00069	(0,0000)
$arepsilon_{t-1}^2$	0,1590	(0,1525)	-0,0715	(0,0000)	-	-	0,0705	(0,3376)
$arepsilon_{t-2}^2$	-	-	-	-	-	-	-	-
$\sigma_{_{t-1}}^2$	-	-	1,0655	(0,0000)	-	-	-0,91317	(0,0000)
$\left rac{\mathcal{E}_{t-1}}{\sigma_{_{t-1}}} ight $	-	-	-	-	-0,05138	(0,0000)	-	-
$rac{\mathcal{E}_{_{t-1}}}{\sigma_{_{t-1}}}$	-	-	-	-	-0,2258	(0,0000)	-	-
$\ln(\sigma_{_{t-1}}^2)$	-	-	-	-	0,8382	(0,0000)	-	-
$\mathcal{E}_{t-1}^2 d_{t-1}$	-	-	-	-	-	-	0,03463	(0,6199)

Table 7 The estimation of heteroscedastic models

Note: Results generated by means of the Eviews software program

EGARCH(1,1) is the heteroscedastic model with the smallest information criteria therefore it is the best model. The estimated value of the parameter γ_1 is smaller than zero therefore it shows us that business cycles in Romania present an asymmetric volatility. The asymmetry of business cycles volatility is higher during the periods of economic downturn than during those of economic growth.

4 Conclusions and Future Research

The business cycles modelling in Romania is a relatively recent research topic determined by the occurrence of business cycles after the transition to the market economy. For the analysis we took into consideration the period January 2000 – May 2011 because we believed that Romania passed to the market economy in 2000.

The obtained results prove that business cycles have a variable volatility in time and that this volatility is also asymmetric: the volatility of business cycles is higher during downturn economic periods than during economic growth times.

Due to the result obtained, I intend to continue the study of this topic and take into account the countries from the Central and East Europe as well in order to be able to make comparisons among the

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results obtained. It would also be interesting that for the same period of analysis I should also approach the European and Monetary European Union countries in order to identify whether there are differences among these countries and those from the Central and Eastern Europe.

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