APPLICATION OF FUZZY CALCULATION METHODOLOGY TO CHECK THE CORRECTNESS OF FORECAST CALCULATIONS OF FINANCIAL RETURN IN A GLOBAL CONTEXT

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Abstract

Fuzzy set theory is circumscribed to the need to broaden the scope of classical mathematics meaning potentiation possibilities of mathematical modeling of real world systems.
One can appreciate that the nature of reality and our way of thinking and the symbolism of interpersonal communication languages are also sources of uncertainty, imprecision, vagueness, ambiguity. The imperative of handling properties not necessarily "perfect determinable", namely "vaguely defined" (but with a measurable degree of uncertainty) has led to the imposition of fuzzy sets theory, which proved to be the systematic frame suitable to management of ambiguity and imprecision.
Using fuzzy numbers involves some difficulties (related for example to the relaxation of equalities relative to invertible elements) that could be surpassed by using fuzzy numbers characterizing through families of "confidence intervals".

Keywords: fuzzy numbers, Lee inference, confidence intervals.

1. Introduction

Quantitative estimation of economic data can be made as follows: "The price of a product A ...will be between a and c and most likely will be b, where a, b, c are real numbers and a≤ b ≤c ".

Generally a fuzzy number means any application μ: R → [0.1] continuous , convex and so there exists $x_\mu \in R$ that $\mu(x_\mu) = 1$.

Usually is considered the following form (proposed by H. Hellendorn):

$$\mu(x) = \begin{cases} 0, & x \le a; \\ \Pi_1(x), & x \in (a,b); \\ 1, & x \in (b,c); \\ \Pi_2(x), & x \in (c,d); \\ 0, & x \ge d; \end{cases}$$

where Π1 is increasing function , Π2 is a decreasing function, Π1 and Π2 are continuous, $\lim_{\substack{x\to a\\x>a}}\Pi_1(x)=0;\lim_{\substack{x\to d\\x<d}}\Pi_2(x)=0;\lim_{\substack{x\to b\\x<b}}\Pi_1(x)=1;\lim_{\substack{x\to c\\x>c}}\Pi_2(x)=1$, a, b, c, d ∈ R and a ≤ b ≤ c ≤ d.

When $\Pi_1(x)=\frac{x-a}{b-a},\Pi_2(x)=\frac{d-x}{d-c},a\neq b,c\neq d$ trapezoidal fuzzy numbers are obtained (that become, for b = c, triangular fuzzy numbers).

It is clear that a trapezoidal fuzzy number is well defined by a quadruple of real numbers (a, b, c, d) where a≤ b ≤ c ≤d. In the case of the triangular fuzzy number, we have triples (a, b, c) where a ≤ b ≤ c.

Note that a typical number can be interpreted as a fuzzy number (by convention is considered so), namely r ∈ R leads to:

$$\tilde{r}:R\to[0,1],\ \tilde{r}(x)=\begin{cases}1x=1;\\0x\neq 1.\end{cases}$$

Generally, using the extension principle given by L. Zadeh, operations with fuzzy numbers are made by the formula:

– for μ,η:R→[0,1], μ∗η:R→[0,1] is given by:

$(\mu*\eta)(x)=\sup_{x=y*z}\{\min\{\mu(y),\eta(z)\}\}$, where:

"*" can be any of the operations $\oplus,\otimes,\Theta$, ÷.

Are defined:

$$(\tilde{r}\oplus\mu)(x)=\mu(x-r),$$

$$(\tilde{r}\otimes\mu)(x)=\begin{cases}\mu(\frac{x}{r}),r\neq 0;\\ \begin{cases}1,x=0\\0,x\neq 0\end{cases},r=0.\end{cases}$$

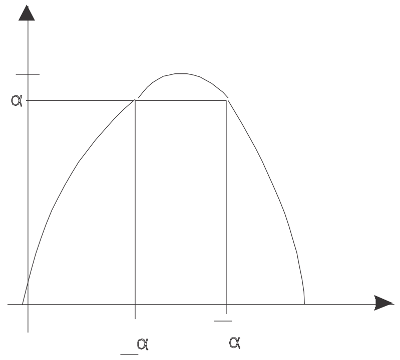
We note here that previous operations return, in case of fuzzy numbers of the form $\tilde{r}$ to the usual arithmetic operations, that is in case of fuzzy numbers of the form $\tilde{r}$, numbers of the same form are obtained.

This property is not fulfilled if we consider the set of trapezoidal fuzzy numbers (triangular). This, along with that, typically, in case of applications trapezoidal or triangular fuzzy numbers are used, required to find new practical ways of defining the operations, so the result is of the same type with the numbers considered. Properties of associativity, commutativeness and the existence of neutral element $(\tilde{0}\ for\ \oplus and\ \tilde{1}\ for\otimes)$ will be preserved, instead equalities for reverse (opposite) of an element will be transformed into equivalents.

Returning to the need to find the best options for defining operations, note first that a (generally) fuzzy number μ: R → [0.1] can be characterized by a family of intervals (called confidence intervals).

$$\mu_\alpha = \left[\underline{x_\alpha}, \overline{x_\alpha}\right], \alpha \in [0,1]$$
$$\underline{x_\alpha} = \inf\{x \mid \mu(x) = \alpha\}, \overline{x_\alpha} = \sup\{x \mid \mu(x) = \alpha\}$$

If, for example, μ: R→[0,1] is represented by:



then $\mu_\alpha = \left[\underline{x_\alpha}, \overline{x_\alpha}\right]$.

2. Submission of data

The first section of the fuzzy calculation methodology applied to a specific economic issue, aims to illustrate in the form of a graph a system of causal relationships between micro, meso and macro variables. All these relations are constituted in a graphical model of global diagnosis and by the use of fuzzy type sizes, gains value of a tool for checking the correctness of the forecasts regarding the evolution of these factors.

Experts opinions related to indetermination between economic factors in the context of forecasts of trends in short and medium term are expressed as [v1; v2] where v1, v2 ∈ { 0, 0.1, ..., 0.9, 1}. Thus for simplification reasons on the one hand, and the materialize of meanings on the other hand, the interval [0, 1] is replaced, in implications, with the set {0, 0.1, ..., 0.9 , 1}.

In this context are allowed the following interpretations:

0 – false;
0.1 – practically false;
0.2 – almost false;
0.3 – quite false;
0.4 – more false than true;
0.5 – no true nor false;
0.6 – more true than false;
0.7 – quite true;
0.8 – almost true;
0.9 – basically true;
1 – true.

Therefore experts estimates relate to assessing the truth of the implications.

Depending on the specific economic application the previous scale of values is used ,in which the words "false" and "true" can be replaced by "infeasible" and "feasible" or "incoherent" and "coherent" etc.

We mention that Lee type inference is used to calculate the values of intermediate nodes, ie if the truth value v (P) of a sentence (economic factor) P is a and the truth value v (P $\to$ Q) is c, then v (Q) = b where b is given by c= $\bar{a} \vee b$, and a, b, c can be triangular fuzzy numbers, the actual calculations being made (for example) through "confidence intervals".

In concrete, if c= [c1, c2], a = [a1, a2], then:

$\bar{a}$ = [1-a2, 1-a1] and b = [c1, c2], if 1-a2 < c1, 1-a1 < c2;
b = [0, c2], if 1-a2 = c1, 1-a1 = c2;
b = 0 in otherwise.

The following four groups of economic factors will be considered:

A1 – Generic trends of sustainable socio-economic development (crisis, recession, stagnation / depression, recovery);
A2 – State Budget (weak, balanced, excess);
A3 – Industrial policy, industrial development priorities namely: sectors, products, markets, technologies, use of specific resources, development of some geographic areas;
A4 – Possibilities estimated (by business) to obtain attractive returns (small, medium, high);
A5 – Fiscal policy (strict or relaxed);
A6 – Change of physical resources allocated to industrial exploitation (small, medium, major);
A7 – Increase of financial resources allocated to financial exploitation (small, medium, high);
A8 – Profit tax rate;
A9 – Volume change in industrial, commercial and financial activity of the enterprise;
A10 – Net profit (result).

B1 – The inflation rate;
B2 – Number and size of firms in the industry;
B3 – The level of costs;
B4 – Consumer income levels;
B5 – Quality of products / consumer preferences;
B6 – Prices of the competitors;
B7 – Barriers to entry / exit to / from branch;
B8 – Competition features;
B9 – The level of the demand to the enterprise;
B10 – The prices of the company;
B11 – Production sold (physical);
B12 – The level of turnover.

C1 – Absolute size of the market (market absorption capacity);
C2 – Market growth rate;
C3 – Regulatory and institutional – economic constraints of the firm behavior;
C4 – Safety (opposite to risk) of the investment;
C5 – Return on investment;
C6 – Investment liquidity;
C7 – Monetary policy;
C8 – Legal, customs, environmental restrictions;
C9 – The social role of the branch;
C10 – The supply of money;
C11 – The interest rate on loans or the cost of borrowed capital;
C12 – Access to borrowed capital;

C13 – Attractiveness of the branch;
C14 – The parameters of the investment in branches;
C15 – The total capital allocated to industrial activity.

D1 – Special tax regulations;
D2 – Level of the dividends;
D3 – Industry strategic forecasts and the strategic position in the enterprise market;
D4 – Depreciation regime;
D5 – Provisions regime;
D6 – The interest of creditors to participate directly in equity and to exercise significant influence over management;
D7 – Future development projects of the enterprise;
D8 – Self-financing;
D9 – Return on capital;
D10 – Conversion of debt;
D11 – New capital contributions of shareholders / associates;
D12 – Incorporation of reserves;
D13 – Equity allocated to the activity.

The generic causal scheme is obtained (fig.no. 1).

Relations "min" or "max" between the influences of economic factor are specified by "$\wedge$" and "$\vee$".

Calculation rules are:

[v1, v2] $\wedge$ [v1', v2'] = [max {v1, v1'}, min {v2, v2'}], for the factors acting simultaneously on an intermediate node (one of the factors moderate the other one);

[v1, v2] $\vee$ [v1', v2'] = [min {v1, v1'}, max {v2, v2'}], when the factors acting simultaneously on an intermediate node catalyze each other (gather).

3. Calculation itself:

Economic trends of the sizes considered in the calculations are:

A1 – General trend of recession materialized in reduction of GDP by 5%;
A2 – The increase of budget deficit with 3.5% of GDP;
A3 – The branch of the industry considered will be a priority of the industrial policy with a probability of 0.6;
A4 – Possibilities estimated (by business) on obtaining attractive returns are low, with a probability of 0.4;
A5 – Tax policy will be restrictive (3% tax increase);
A6 – Increase by 2-5% of physical resources allocated to industrial exploitation;
A7 – Increase by 3-6% of financial resources allocated to industrial activity;
A8 – The profit tax rate will remain 0.16;
A9 – The volume of industrial, commercial and financial activity will increase by 2% at most;
A10 – Net profit will increase by 2% at most.

B1 – The inflation rate will be 10%;
B2 – Number and size of firms in the industry will remain the same with a probability of 0.8;

B3 – The costs will increase by 7%;

B4 – The relative level of consumer income will decrease no more than 15%;

B5 – The quality of the company products and its appreciation by consumers will remain unchanged with a probability of 0.95;

B6 – Competitors prices will increase by 4-8%;

B7 – Barriers to entry / exit to / from industry will be tightened with a probability of 0.7;

B8 – Competition will intensify with a probability of 0.8;

B9 – The level of the demand addressed to the enterprise will fall no more than 3%;

B10 – The company will increase prices by 5% at most;

B11 – Production (physical) sold will remain unchanged with a probability of 0.85;

B12 – The turnover will increase by no more than 5%.

C1 – The absorption capacity of the market will remain unchanged with a probability of 0.9;

C2 – Market growth rate will be decreased with a probability of 0.95;

C3 – Regulatory and institutional – economic constraints of the firm behavior will be tightened with a probability of 0.5;

C4 – Safety of investment will decline with a probability of 0.45;

C5 – Return on investment will increase with a probability of 0.3;

C6 – The liquidity of the investment will decline with a probability of 0.4;

C7 – Monetary policy will be restrictive with a probability of 0.65;

C8 – Legal, customs, environmental restrictions will relax with a probability of 0.4;

C9 – The social role of the branch will remain unchanged with a probability of 0.7;

C10 – Money supply will increase by 15% at most;

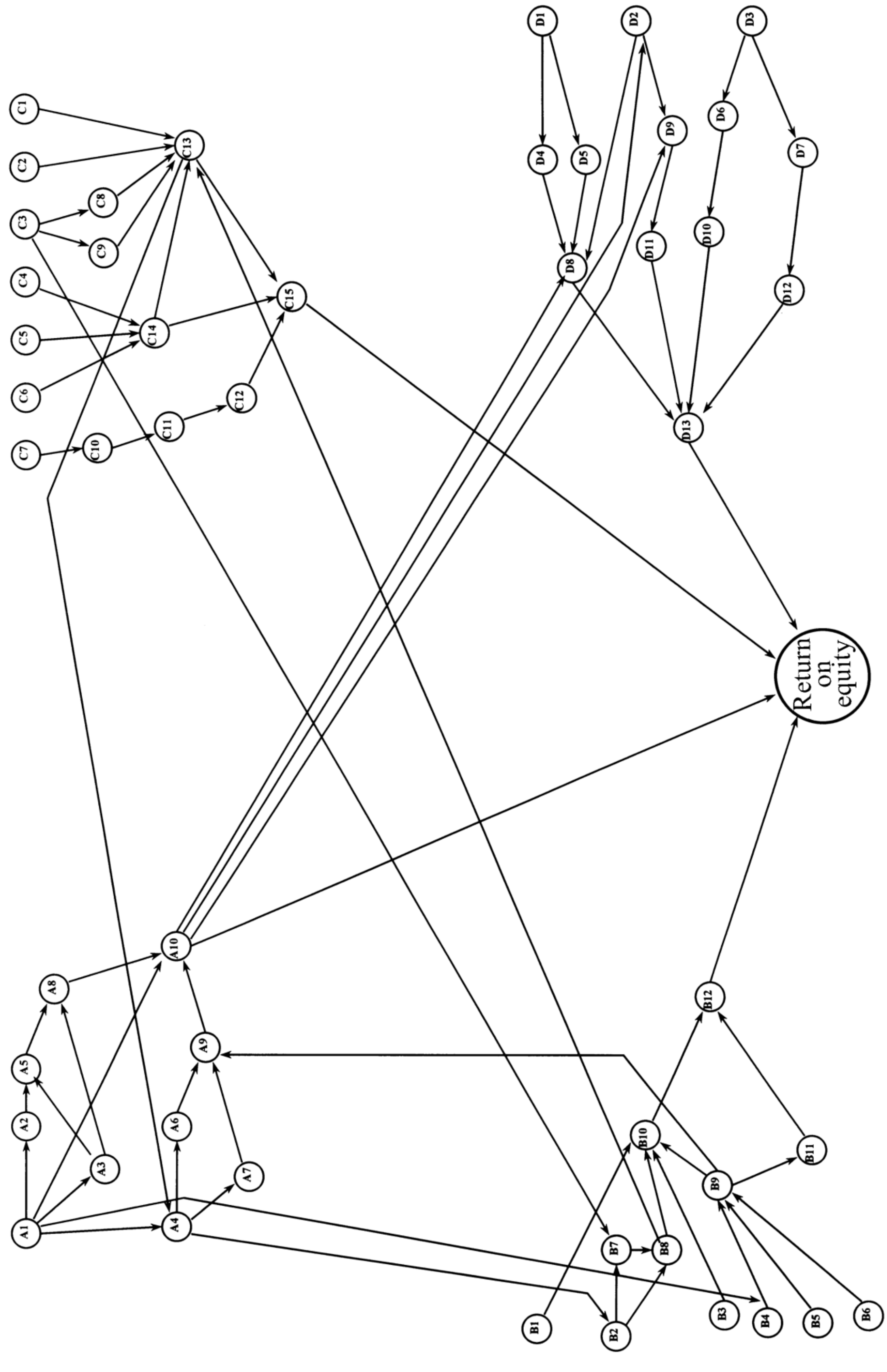


Fig.no.1. Scheme-graph for a global strategic diagnosis

C11 – The interest rate on loans (the cost of borrowed capital) will increase with a probability of 0.3;

C12 – Access to loan capital will be as easy / difficult with a probability of 0.25;

C13 – Industry attractiveness for investors will remain unchanged with a probability of 0.75;

C14 – The parameters of investing in industry will worsen with a probability of 0.2;

C15 – The total capital allocated to the industrial activity will remain unchanged with a probability of 0.4.

D1 – Special tax rules will increase by 6% at most;

D2 – The level of dividends will remain unchanged with a probability of 0.9;

D3 – Industry strategic forecasts and the company's strategic position on the market will improve with a probability of 0.25;

D4 – Depreciation will increase with a probability of 0.6;

D5 -Provisions will increase with the probability of 0.5;

D6 – The interest of creditors to participate directly in equity and exercise significant influence over the management will remain unchanged with a probability of 0.75;

D7 – The volume of future business investment will increase by 10% at most;

D8 – Self-financing capacity of the enterprise will increase with a probability of 0.3;

D9 – Return on capital per share will remain unchanged with a probability of 0.65;

D10 – Conversion of debt will increase by 5% at most;

D11 – New capital contributions will increase with a probability of 0.4;

D12 – Incorporated reserves will increase by 25%;

D13 – Equity allocated to activity increases with a probability of 0.5.

– Consultation of specialists leads to establishment of the following truth values:

V(A1) = [0,7; 0,8]
V(B1) = [0,8; 0,9]
V(B3) = [0,7; 0,9]
V(B5) = [0,4; 0,6]
V(B6) = [0,3; 0,5]
V(C1) = [0,5; 0,7]
V(C2) = [0,4; 0,5]
V(C3) = [0,7; 0,8]
V(C4) = [0,6; 0,8]
V(C5) = [0,7; 0,9]
V(C6) = [0,3; 0,6]
V(C7) = 0,7
V(D1) = [0,4; 0,7]
V(D3) = [0,5; 0,7]

V(A1→A2) = [0,6; 0,8]
V(A1→A3) = [0,3; 0,5]
V(A1→A4) = [0,5; 0,7]
V(A2→A5) = 1
V(A3→A5) = [0,5; 0,7]
V(A3→A8) = [0,5; 0,7]
V(A1→A10) = [0,3; 0,4]
V(A4→A6) = [0,7; 0,8]

V(C1→C13) = [0,2; 0,4]
V(C2→C13) = [0,7; 0,9]
V(C3→C8) = [0,6; 0,8]
V(C3→C9) = [0,7; 0,8]
V(C8→C13) = [0,6; 0,8]
V(C9→C13) = [0,7; 0,8]
V(C4→C14) = 0,8
V(C5→C14) = [0,7; 0,9]

V(A4→A7) = [0,8; 1]
V(A6→A9) = [0,7; 0,8]
V(A7→A9) = [0,8; 0,9]
V(A9→A10) = [0,5; 0,6]
V(B1→B10) = [0,2; 0,5]
V(B2→B7) = [0,5; 0,7]
V(B2→B8) = [0,7; 0,9]
V(B3→B10) = 0,9
V(B4→B9) = 1
V(B5→B9) = [0,8; 1]
V(B6→B9) = [0,7; 0,9]
V(B7→B8) = [0,6; 0,7]
V(B8→B10) = [0,3; 0,5]
V(B9→B10) = [0,5; 0,7]
V(B9→B11) = 0,9
V(B10→B12) = [0,7; 0,9]
V(B11→B12) = [0,9; 1]

V(C6→C14) = [0,2; 0,4]
V(C7→C10) = 0,9
V(C10→C11) = [0,7; 0,8]
V(C11→C12) = [0,8; 0,9]
V(C12→C15) = [0,8; 0,9]
V(C14→C13) = [0,6; 0,8]
V(C13→C15) = [0,7; 0,9]
V(C14→C15) = [0,7; 0,8]
V(D1→D4) = [0,3; 0,6]
V(D1→D5) = [0,4; 0,7]
V(D2→D8) = [0,4; 0,5]
V(D2→D9) = [0,4; 0,5]
V(D3→D6) = [0,4; 0,6]
V(D3→D7) = 0,6
V(D4→D8) = [0,4; 1]
V(D5→D8) = [0,4; 0,5]
V(D8→D13) = 0,3

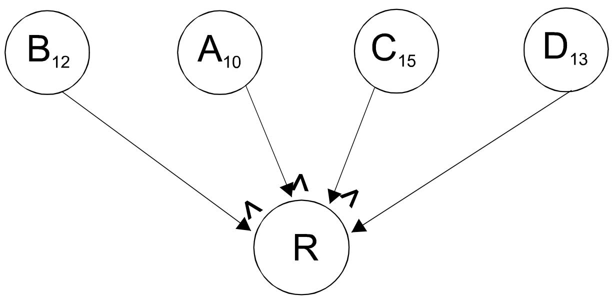
V(D9→D11) = [0,4; 0,5]
V(D11→D13) = [0,3; 0,5]
V(D6→D10) = [0,4; 0,86]
V(D7→D12) = [0,4; 0,5]
V(D10→D13) = [0,4; 0,6]
V(D12→D13) = [0,4; 0,5]
V(A10→D2) = [0,4; 0,5]
V(A10→D8) = 0,6
V(A10→D9) = [0,5; 0,6]
V(B9→A9) = [0,5; 0,7]
V(A9→B2) = [0,5; 0,7]
V(A1→B4) = [0,5; 0,7]
V(C3→B7) = [0,2; 0,4]
V(B8→C13) = [0,6; 0,8]
V(C3→B7) = [0,2; 0,4]

– Using the Lee inference rule the following values of the intermediate nodes are obtained:

V(A2) = [0,6; 0,8]
V(A3) = [0,3; 0,5]
V(A5) = 1 ∨ [0,5; 0,7] = 1
V(A8) = [0,8; 1] ∨ [0,5; 0,7] = [0,8; 1]
V(A4) = [0,5; 0,7]
V(A7) = [0,8; 1]
V(A6) = [0,7; 0,8]
V(A9) = ([0,8; 0,9] ∧ [0,7; 0,8]) ∨ [0,5; 0,7] = [0,7; 0,8] ∨ [0,5; 0,7] = [0,7; 0,8]
V(A10) = [0,3; 0,4] ∨ [0,3; 0,4] ∨ [0,5; 0,6] = [0,5; 0,6]
V(B2) = [0,5; 0,7]
V(B4) = [0,5; 0,7]

V(B7) = [0,2; 0,4] ∨ [0,5; 0,7] = [0,5; 0,7]
V(B8) = [0,6; 0,7] ∧ [0,7; 0,9] = [0,6; 0,7]
V(B10) = [0,2; 0,5] ∨ [0,3; 0,5] ∨ 0,9 ∨ [0,5; 0,7] = 0,9
V(B9) = (1 ∧ [0,8; 1]) ∨ [0,7; 0,9] = [0,8; 1]
V(B11) = 0,9
V(B12) = [0,7; 0,9] ∨ [0,9; 1] = [0,9; 1]
V(C8) = [0,6; 0,8]
V(C9) = [0,7; 0,8]
V(C13) = ϕ ∨ [0,6; 0,8] ∨ [0,7; 0,9] ∨ [0,6; 0,8] ∨ [0,7; 0,8] = [0,7; 0,9]
V(C14) = ϕ ∨ 0,8 ∨ [0,7; 0,8] = [0,7; 0,8]
V(C10) = 0,9
V(C11) = [0,7; 0,8]
V(C12) = [0,8; 0,9]
V(C15) = [0,7; 0,9] ∧ [0,7; 0,8] ∧ [0,8; 0,9] = [0,7; 0,8]
V(D4) = [0; 0,6]
V(D5) = [0,4; 0,7]
V(D2) = [0,4; 0,5]
V(D8) = 0,6 ∨ [0,4; 1] ∨ [0,4; 0,5] ∨ [0,5; 0,6] = [0,6; 1]
V(D9) = [0,5; 0,6] ∨ [0,4; 0,5] = [0,5; 0,6]
V(D11) = [0; 0,5]
V(D6) = [0,4; 0,6]
V(D7) = 0,6
V(D10) = [0,4; 0,6]
V(D12) = [0,4; 0,6] 64
V(D13) = [0; 0,3] ∨ ϕ ∨ [0,4; 0,6] ∨ 0,4; 0,5] = [0,4; 0,6].

Is reached at:



Having V(B12→R) = [0,5; 0,6]
V(A10→R) = [0,4; 0,5]
V(C15→R) = [0,6; 0,7]
V(D13→R) = [0,5; 0,7]

We have: V(R) = [0,5; 0,6] ∧ [0; 0,5] ∧ [0,6; 0,7] ∧ [0,5; 0,7] = [0; 0,5]

Pessimistic alternative was considered on the final node with the ∧ sign between them.

4. Conclusions

In the context considered, the degree of reliability for objective "R" is given by the interval [0, 0.5].

A brief analysis of the scheme-graph for global diagnosis can indicate various ways to increase this degree.

Namely, for example, changing V(A10→R) = [0,6; 0,7] entails:

$$V (R) = [\ 0.5\ ] \wedge 0.6\ [0.6, 0.7] \wedge [0.6, 0.7] \wedge [0.5, 0.7] = [0.5; 0.6]$$

Continuing the reasoning for the factors noted generic by group A, the following alternatives for action are outlined:

– Is chosen the factor from the first neighborhood (A8, A1 or A9) with the greatest influence (value of implication) on A10. In the same way the reverse road in the graph is continued to the original node A1. This course of action is based on the fact that the factor with the implication of greater value is easier to grow.

– The cumulative influences (the sum of values of the implications) on all roads from A1 to A10 are determined. It is chosen for refining the path corresponding to the maximum arithmetic mean obtained on each route taking into account the confidence intervals and the number of arcs.

The previous versions indicate the factor (road) on which action must be taken (by changing the value of truth) so that V (R) is maximal.

A more rigorous solution requires that for each economic factor to specify sentences for all stances in which it can find (e.g. for A1 are four). The values of implications will be different for different situations. The calculation is resumed using the methodology presented. Calculation will also be reconsidered when changes of percentage that occur in sentences take place.

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