The Sequence of two Installations without Initial Deliverance Times

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Abstract. In this paper we shall give a new algorithm instead of Johnson classical in the process of determination the sequence of pieces execution on two installations without initial deliverance times.

Keywords: installation, sequence, piece.

1 Introduction

The sequence operation in production flows appears in the usual practice for the installations waiting time decreasing when a lot of pieces use the same technology line in the same direction.

Let two installations U1 and U2 who process n pieces P1,…,Pn (n · 2) in the same order (first U1 and after U2). We shall consider that U1 and U2 are available from the process beginning and the waiting time to come in execution for U2 does not implies other prices. In addition we shall suppose that the pieces do not have a finish ending date.

Let note with tij the processing time of the j-th piece on the i-th installation.

The problem consists in a determination of the pieces execution beginning order such that the waiting time of the installation U2 to be minimum.

Let the matrix T=(tij) · M2n(R) of the time processing. The classical algorithm of Johnson consists in the following steps:

Step 1 We choose the least element on the first row. This will give us the first piece that will come in execution.

Step 2 We cut the previous column and we choose the least element on the second row. This will give us the last piece that will come in execution.

Step 3 We cut the previous column and we go again at the first step. After this we will obtain the second piece that will come in execution, and after we go again at the second step and we find the penultimate piece and so on.

The algorithm will continue till we shall finish all the pieces.

2 The new method

In the proof of Johnson's algorithm it exists a little but essential error. The author extrapolates a transposition between two consecutive terms to all transpositions. This is the reason that, even if it claim to obtain the optimum, it is not true.

The following method will guide us to the optimum but with a little harder calculus.

Let therefore the table of time processing and a permutation $\sigma=\begin{pmatrix} 1 & 2 & \dots & n \\ i_1 & i_2 & \dots & i_n \end{pmatrix} \in S_n$ – the group of permutations of n elements and an order of pieces, indexed by σ: $P_{i_1}, P_{i_2}, \dots, P_{i_n}$:

Piece/Installation	P_{i_1}	P_{i_2}	…	P_{i_k}	…	P_{i_n}
U_1	$d_{i_1 1}$	$d_{i_2 1}$	…	$d_{i_k 1}$	…	$d_{i_n 1}$
U_2	$d_{i_1 2}$	$d_{i_2 2}$	…	$d_{i_k 2}$	…	$d_{i_n 2}$

We define: $g_1, g_2, \dots, g_n \geq 0$ - the pauses before entrance in execution of pieces $P_{i_1}, P_{i_2}, \dots, P_{i_n}$ on the installation U_2. We have, obviously:

- $g_1 = d_{i_1 1}$ (*from the beginning of the process*)
- $g_2 = \max(d_{i_1 1} + d_{i_2 1} - d_{i_1 2} - g_1, 0)$
- $g_3 = \max(d_{i_1 1} + d_{i_2 1} + d_{i_3 1} - d_{i_1 2} - d_{i_2 2} - g_1 - g_2, 0)$

…

- $g_k = \max\left(\sum_{p=1}^{k} d_{i_p 1} - \sum_{p=1}^{k-1} d_{i_p 2} - \sum_{p=1}^{k-1} g_p, 0\right)$

…

- $g_n = \max\left(\sum_{p=1}^{n} d_{i_p 1} - \sum_{p=1}^{n-1} d_{i_p 2} - \sum_{p=1}^{n-1} g_p, 0\right)$

If we note: $B_{i_1 \dots i_k} = \sum_{p=1}^{k} d_{i_p 1} - \sum_{p=1}^{k-1} d_{i_p 2}$ we have:

- $g_1 = B_{i_1}$
- $g_2 = \max(B_{i_1 i_2} - g_1, 0)$
- $g_3 = \max(B_{i_1 i_2 i_3} - g_1 - g_2, 0)$

…

- $g_k = \max\left(B_{i_1 \dots i_k} - \sum_{p=1}^{k-1} g_p, 0\right)$

…

- $g_n = \max\left(B_{i_1 \dots i_n} - \sum_{p=1}^{n-1} g_p, 0\right)$

The objective function is therefore: $z = \min_{\sigma \in S_n} \left(\sum_{k=1}^{n} g_k\right)$.

We have by iteration:

$$\sum_{p=1}^{k} g_p = g_k + \sum_{p=1}^{k-1} g_p = \max\left(B_{i_1 \dots i_k} - \sum_{p=1}^{k-1} g_p, 0\right) + \sum_{p=1}^{k-1} g_p = \max\left(B_{i_1 \dots i_k}, \sum_{p=1}^{k-1} g_p\right).$$

But: $\sum_{p=1}^{n} g_p$ =max($B_{i_1\ldots i_n}$, $\sum_{p=1}^{n-1} g_p$)=max($B_{i_1\ldots i_n}$,max($B_{i_1\ldots i_{n-1}}$, $\sum_{p=1}^{n-2} g_p$))=

max($B_{i_1\ldots i_n}$, $B_{i_1\ldots i_{n-1}}$, $\sum_{p=1}^{n-2} g_p$)=...=max($B_{i_1\ldots i_n}$, $B_{i_1\ldots i_{n-1}}$,..., B_{i_1}) from where:

z=min ($\sum_{p=1}^{n} g_p$)= $\min_{\sigma\in S_n}$ (max($B_{i_1\ldots i_n}$, $B_{i_1\ldots i_{n-1}}$,..., B_{i_1})).

We have $B_{i_1\ldots i_k} = \sum_{p=1}^{k} d_{i_p 1} - \sum_{p=1}^{k-1} d_{i_p 2} = B_{i_1\ldots i_{k-1}} + d_{i_k 1} - d_{i_{k-1} 2}$, and much generally:

$$B_{i_1\ldots i_k} = \sum_{p=1}^{k} d_{i_p 1} - \sum_{p=1}^{k-1} d_{i_p 2} = \sum_{p=1}^{s} d_{i_p 1} - \sum_{p=1}^{s-1} d_{i_p 2} + \sum_{p=s+1}^{k} d_{i_p 1} - \sum_{p=s}^{k-1} d_{i_p 2} = B_{i_1\ldots i_s} + \sum_{p=s+1}^{k} d_{i_p 1} - \sum_{p=s}^{k-1} d_{i_p 2}$$

For the permutation $\sigma=\begin{pmatrix} 1 & 2 & \ldots & k & \ldots & s & \ldots & n \\ i_1 & i_2 & \ldots & i_k & \ldots & i_s & \ldots & i_n \end{pmatrix}\in S_n$ and z=max($B_{i_1\ldots i_n}$, $B_{i_1\ldots i_{n-1}}$,..., B_{i_1}), let consider a transposition of σ: $\tau=\begin{pmatrix} 1 & 2 & \ldots & k & \ldots & s & \ldots & n \\ i_1 & i_2 & \ldots & i_s & \ldots & i_k & \ldots & i_n \end{pmatrix}\in S_n$.

If we note with bar all the quantities concerning τ we have:

- $t\neq k,s \Rightarrow \bar{d}_{i_t 1} = d_{i_t 1}$ and $\bar{d}_{i_t 2} = d_{i_t 2}$
- $t=k \Rightarrow \bar{d}_{i_k 1} = d_{i_s 1}$ and $\bar{d}_{i_k 2} = d_{i_s 2}$
- $t=s \Rightarrow \bar{d}_{i_s 1} = d_{i_k 1}$ and $\bar{d}_{i_s 2} = d_{i_k 2}$

from where:

- $1\le t<k \Rightarrow \bar{B}_{i_1\ldots i_t} = \sum_{p=1}^{t} \bar{d}_{i_p 1} - \sum_{p=1}^{t-1} \bar{d}_{i_p 2} = \sum_{p=1}^{t} d_{i_p 1} - \sum_{p=1}^{t-1} d_{i_p 2} = B_{i_1\ldots i_t}$
- $t=k \Rightarrow \bar{B}_{i_1\ldots i_k} = \sum_{p=1}^{k} \bar{d}_{i_p 1} - \sum_{p=1}^{k-1} \bar{d}_{i_p 2} = \sum_{p=1}^{k} d_{i_p 1} - \sum_{p=1}^{k-1} d_{i_p 2} + d_{i_s 1} - d_{i_k 1} = B_{i_1\ldots i_k} + d_{i_s 1} - d_{i_k 1}$
- $k<t<s \Rightarrow \bar{B}_{i_1\ldots i_t} = \sum_{p=1}^{t} \bar{d}_{i_p 1} - \sum_{p=1}^{t-1} \bar{d}_{i_p 2} = \sum_{p=1}^{t} d_{i_p 1} - \sum_{p=1}^{t-1} d_{i_p 2}$ + ———————————————- $(d_{i_s 2} - d_{i_k 2}) = B_{i_1\ldots i_t} + d_{i_s 1} - d_{i_k 1} - (d_{i_s 2} - d_{i_k 2})$
- $t=s \Rightarrow \bar{B}_{i_1\ldots i_s} = \sum_{p=1}^{s} \bar{d}_{i_p 1} - \sum_{p=1}^{s-1} \bar{d}_{i_p 2} = \sum_{p=1}^{s} d_{i_p 1} - \sum_{p=1}^{s-1} d_{i_p 2} - (d_{i_s 2} - d_{i_k 2}) = B_{i_1\ldots i_s} - (d_{i_s 2} - d_{i_k 2})$
- $t>s \Rightarrow \bar{B}_{i_1\ldots i_t} = \sum_{p=1}^{t} \bar{d}_{i_p 1} - \sum_{p=1}^{t-1} \bar{d}_{i_p 2} = \sum_{p=1}^{t} d_{i_p 1} - \sum_{p=1}^{t-1} d_{i_p 2} = B_{i_1\ldots i_t}$

Let note now: $\alpha_{sk}=d_{i_s 1} - d_{i_k 1}$ and $\beta_{sk}=d_{i_s 2} - d_{i_k 2}$ for s>k and $\alpha_{sk}=\beta_{sk}=0$ for s≤k. We have now:

$$\bar{B}_{i_1\ldots i_t} = \begin{cases} B_{i_1\ldots i_t} \text{ if } t<k \\ B_{i_1\ldots i_t} + \alpha_{sk} \text{ if } t=k \\ B_{i_1\ldots i_t} + \alpha_{sk} - \beta_{sk} \text{ if } k<t<s \\ B_{i_1\ldots i_t} - \beta_{sk} \text{ if } t=s \\ B_{i_1\ldots i_t} \text{ if } t>s \end{cases}$$

$\overline{z}$=max($\overline{B}_{i_1}$,..., $\overline{B}_{i_1...i_{n-1}}$, $\overline{B}_{i_1...i_n}$)=max(B_{i_1},..., $B_{i_1...i_{k-1}}$, $B_{i_1...i_k}$ +α_{sk}, $B_{i_1...i_{k+1}}$ +α_{sk}-β_{sk},..., $B_{i_1...i_{s-1}}$ +α_{sk}-β_{sk}, $B_{i_1...i_s}$ -β_{sk}, $B_{i_1...i_{s+1}}$,..., $B_{i_1...i_n}$).

We must determine the pair (k,s) of pieces which will be permuted such that, after the computing of $\overline{z}$ to obtain a value less than or equal z.

How this thing leads us at a great number of calculations, we shall act in this way:

For an arbitrary distribution of pieces, corresponding to a permutation $\sigma=\begin{pmatrix} 1 & 2 & ... & k & ... & s & ... & n \\ i_1 & i_2 & ... & i_k & ... & i_s & ... & i_n \end{pmatrix} \in S_n$, we shall determine those piece which permute with the first will lead to the minimization of z.

Suppose now that this thing is for the first piece.

Let therefore P_{i_s} - the searched piece, who will take the place of the first piece P_{i_1}. We have therefore:

$\overline{z}$=max($\overline{B}_{i_1}$,..., $\overline{B}_{i_1...i_{n-1}}$, $\overline{B}_{i_1...i_n}$)=max(B_{i_1} +α_{s1}, $B_{i_1i_2}$ +α_{s1}-β_{s1},..., $B_{i_1...i_{s-1}}$ +α_{s1}-β_{s1}, $B_{i_1...i_s}$ -β_{sk}, $B_{i_1...i_{s+1}}$,..., $B_{i_1...i_n}$).

We shall continue this process till we cannot diminish the value of z.

In this moment, we shall find the permutation with the second piece and so on.

Let conclude:

We build the table where on the rows we have the pieces: P_{i_1},..., P_{i_n} and on columns alone: P_{i_2},..., P_{i_n}.

					...	...	P_{i_s}		...	...	P_{i_n}	
					...	...	d_{i_s1}	d_{i_s2}	...	...	d_{i_n1}	d_{i_n2}
					...	...	α_{s1}	$-\beta_{s1}$			α_{n1}	$-\beta_{n1}$
P_{i_1}	B_{i_1}	d_{i_11}	d_{i_12}	$\overline{B}_{i_1}=d_{i_11}$	...	...	$a_{i_1i_s}=\overline{B}_{i_1}+\alpha_{s1}-\beta_{s1}$		...	...	$a_{i_1i_n}=\overline{B}_{i_1}+\alpha_{n1}-\beta_{n1}$	
	...			...	...	...	...		...	...	...	
P_{i_k}	B_{i_k}	d_{i_k1}	d_{i_k2}	$\overline{B}_{i_k}=\sum_{l=1}^{k}d_{i_l1}-\sum_{l=1}^{k-1}d_{i_l2}$	...	...	$a_{i_ki_s}=\overline{B}_{i_k}+\alpha_{s1}-\beta_{s1}$		...	...	$a_{i_ki_n}=\overline{B}_{i_k}+\alpha_{n1}-\beta_{n1}$	
	...			...	...	...	...		...	...	...	
P_{i_s}	B_{i_s}	d_{i_s1}	d_{i_s2}	$\overline{B}_{i_s}=\sum_{l=1}^{s}d_{i_l1}-\sum_{l=1}^{s-1}d_{i_l2}$	...	...	$a_{i_si_s}=\overline{B}_{i_s}-\beta_{s1}$		...	...	$a_{i_si_n}=\overline{B}_{i_s}-\beta_{n1}$	
	...			...	...	...	...		...	...	...	
P_{i_n}	B_{i_n}	d_{i_n1}	d_{i_n2}	$\overline{B}_{i_n}=\sum_{l=1}^{n}d_{i_l1}-\sum_{l=1}^{n-1}d_{i_l2}$			$a_{i_ni_s}=\overline{B}_{i_s}$				$a_{i_ni_n}=\overline{B}_{i_n}$	
max					...	...	$\max_{t=1,n} a_{i_ti_s}$				$\max_{t=1,n} a_{i_ti_n}$	

Figure 1

We shall choose the piece P_{i_k} for which: z= $\min_{s=2,n}\max_{t=1,n} a_{i_ti_s}$.

The next table will contains the new order of pieces where P_{i_1} will change the place with P_{i_k}.

The process will continue till z= $\min_{s=2,n}\max_{t=1,n} a_{i_ti_s}$ becomes greater than those computed in the preceding table.

This thing suggests the fact that any piece cannot be on the first position without grow the total time. If the value of z remains constant, we can act like in the preceding steps for each pieces order.

In the next table, we shall act analogously, but on the column we shall get only P_{i_3},..., P_{i_n} corresponding to the new permutation.

The process will continue till the last piece.

Example

Piece/Installation	P_1	P_2	P_3	P_4
U_1	15	6	8	9
U_2	19	3	13	7

Johnson's algorithm propose us:

Piece/Installation	P_1	P_3	P_4
U_1	15	8	9
U_2	19	13	7

Piece/Installation	P_1	P_3
U_1	15	8
U_2	19	13

with the final order: P_2,P_3,P_1,P_4, therefore the new table will be, in order of execution:

Piece/Installation	P_2	P_3	P_1	P_4
U_1	6	8	15	9
U_2	3	13	19	7

with times:
$B_2=6$
$B_3=6+8-3=11$
$B_1=6+8+15-3-13=13$
$B_4=6+8+15+9-3-13-19=3$
therefore $z=\max(B_2,B_3,B_1,B_4)=13$.
Our algorithm consists from the following tables:

Table 1

					P_2		P_3		P_4	
					6	3	8	13	9	7
					-9	16	-7	6	-6	12
P_1	B_1	15	19	$\overline{B}_1=15$	6		8		9	
P_2	B_2	6	3	$\overline{B}_2=2$	18		1		8	
P_3	B_3	8	13	$\overline{B}_3=7$	7		13		13	
P_4	B_4	9	7	$\overline{B}_4=3$	3		3		15	
max					18		13		15	

therefore the piece on the first position is P_3.

Table 2

					P_2		P_1		P_4	
					6	3	15	19	9	7
					-2	10	7	-6	1	6
P_3	B_3	8	13	$\overline{B}_3=8$	6		15		9	
P_2	B_2	6	3	$\overline{B}_2=1$	11		2		8	
P_1	B_1	15	19	$\overline{B}_1=13$	13		7		20	
P_4	B_4	9	7	$\overline{B}_4=3$	3		3		9	
max					13		15		20	

The alternative piece on first position can be P_2.

Table 3

					P_3		P_1		P_4	
					8	13	15	19	9	7
					2	10	9	16	3	4
P_2	B_2	6	3	$\overline{B}_2=6$	8		15		9	
P_3	B_3	8	13	$\overline{B}_3=11$	21		36		18	
P_1	B_1	15	19	$\overline{B}_1=13$	13		29		20	
P_4	B_4	9	7	$\overline{B}_4=3$	3		3		7	
max					21		36		20	

therefore the permutation process for the first position is closed.

We go back to the table 1 and continue with the piece on the second position.

Table 4

					P_1		P_4	
					15	19	9	7
					9	-16	3	-4
P_3	B_3	8	13	$\overline{B}_3=8$	8		8	
P_2	B_2	6	3	$\overline{B}_2=1$	10		4	
P_1	B_1	15	19	$\overline{B}_1=13$	-3		12	
P_4	B_4	9	7	$\overline{B}_4=3$	3		-1	
max					10		12	

therefore the piece on second position is P_1.

Table 5

					P_2		P_4	
					6	3	9	7
					-9	16	-6	12
P_3	B_3	8	13	$\overline{B}_3=8$	8		8	
P_1	B_1	15	19	$\overline{B}_2=10$	1		4	
P_2	B_2	6	3	$\overline{B}_1=-3$	13		3	
P_4	B_4	9	7	$\overline{B}_4=3$	3		15	
Max					13		15	

From the table 5 we have that the step is closed.
For the piece on third position:

Table 6

	P_4	
	9	7
	3	-4

P_3	B_3	8	13	$\overline{B}_3=8$	8
P_1	B_1	15	19	$\overline{B}_2=10$	10
P_2	B_2	6	3	$\overline{B}_1=-3$	0
P_4	B_4	9	7	$\overline{B}_4=3$	-1
Max					10

The process is closed. The order will be: P_3,P_1,P_2,P_4 with total time: 10.
If we come again at the table 2 and continue with the piece on the second position we have:

Table 7

					P_1		P_4	
					15	19	9	7
					7	-6	1	6
P_2	B_2	6	3	$\overline{B}_2=6$	6		6	
P_3	B_3	8	13	$\overline{B}_3=11$	18		12	
P_1	B_1	15	19	$\overline{B}_1=13$	7		20	
P_4	B_4	9	7	$\overline{B}_4=3$	3		9	
max					18		20	

Because we obtain a value greater than 13 the process will closed also.

3 References

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