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The Shapley value for a fuzzy poverty measurement

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Abstract. This article studies the relationship between poverty, inequality and growth. In classical political economic model, we introduce a residual term to maintain the identity of the model. It does not permit us to find the exact contribution of each factor. To derive the results of the decomposition, the Shapley value augmented by the fuzzy approach is used. In order to take its full advantage, it is of interest to calculate the marginal contribution of each factor in the variation of poverty. An application based on individual well-being data from Tunisian households is presented to illustrate use of the proposed concepts.

Keywords: inequality, poverty, growth, Shapley value.

1 Introduction

Persistence of poverty and growing income inequity has continued to be challenging socioeconomic problems in Tunisia. It become the main goal of development efforts, many theories have been constructed to assess the factors that must be at the center of any poverty reducing strategies and explain the correlation between poverty and other factors like inequity and growth. In general, growth could be beneficial in reducing the proportion of the poor, their poverty gap and its severity. Most studies on poverty admit that the welfare of a household is determined by its standard average of living. The literature on income inequality has also allowed us to reminder that diverse authors have highlighted the importance of the income inequality in the genesis of economic growth, which directly affects poverty. Datt and Ravallion (1992) and Kakwani (1993) discuss the impact of income distribution on poverty and they concluded that the poverty have many negative effects on the distribution and on the growth. Ravallion (1997) also found that poverty could move up the growth prospects if inequality is sufficiently high. Barro (1999) using a three-stage least squares estimator which treats the country-specific terms as random, finds that the effect of inequality on growth is negative in poor countries, but positive in rich nations. Ali and Thorbecke (2000) analyzed data from multiple countries in Africa and concluded that poverty was much more dependent on the distribution of income than on the growth. Ravallion (2001) expressed the need for deeper micro empirical work on growth and distributional change to identify specific policies to complement growth-oriented policies, and the evaluation of aggregate impacts and their diversity of impacts. Therefore, it is clear that there is a link between poverty growth and inequality. However, our main problem is what is the exact contribution of inequality and growth to reduce poverty? Therefore, in order to take full advantage of the Shapley value, it is of interest to calculate the marginal contribution of income inequality and growth in the variation of poverty. An application based on individual well-being data from Tunisian households in 2005 and in 2010 is presented to illustrate use of the proposed concepts.

The rest of the paper is organized as follows: Section 2 reviews briefly the method of Shapley. Section

3 presents the model of decomposition of the variation of poverty by integrating inequality and growth through the Shapley value augmented by the fuzzy approach. Section 4 explores the empirical illustration and the most important results. Section 5 deals with the conclusion.

2 The elaboration of the Shapley value

The Shapley value is a solution concept in cooperative game theory. To formalize this situation, we use the notion of a coalitional game. We start out with a set N of n players who have to share a surplus or cost. This sharing will be carried out if players can join to form coalitions of subsets S of N. The force of each coalition is expressed by a characteristic function V. For any coalition S, $V(S)$ measures the share of surplus that S can be obtained without resorting to an agreement with the members of other player's coalitions. For each player i, Shapley proposed a value based on its marginal contribution that is defined as the weighted average of the marginal contributions $(V(S \cup \{i\}) - V(S))$ of a player i in all coalition $S \subset N - \{i\}$. The Shapley value is the sum over all coalition S that contains i. S is the number of elements in a coalition and varies from 1 to n. The Shapley value is defines as follows:

$$Sh(i) = \sum_{s=0}^{n-1} \frac{s!(n-s-1)!}{n!} \sum_{s \subset N-\{i\}} V(S \cup \{i\}) - V(S)) \quad (1)$$

Johan et al. (2002) used this value in the measurement of carbon emissions. In a study of four countries, the Shapley decomposition indicated that the carbon intensity of energy use and the decarbonization of economic growth-variables that are targeted with current climate policy measures have more influence on total emissions than generally proposed in conventional decomposition methods.

Knowing that game theory has played an important role in the study of the link between various economic factors and it is a key concept in the Shapley value. This value attempts to describe a fair way to distribute the gains from cooperation assuming strategic realities. It consists in estimating the marginal effect of different contributing factors in a possible elimination sequence. The operation is repeated for all the given sequences and the average of marginals effects are calculated for each factor. This average measures the contribution of factors that give a true and additive decomposition of the phenomenon in m contribution. Indeed, we use the Shapley value to determinate the marginal contribution of potential explanatory factors of the variation of poverty such as income inequality and growth.

3 The measurement of poverty

The aim of this section is to present the interaction between poverty, inequity and growth and to explore how those factors in turn made efforts to reduce poverty. First, an alternative unidimensional poverty fuzzy measure is proposed. Second, we defined the contribution of growth and the contribution of inequity in the variation of poverty measure proposed. Finally, the sum of those two contributions is supposed as equal to the variation of poverty**.**

3.1 Unidimensional poverty fuzzy measure

To measure poverty, we suggest a fuzzy average of poverty weighted by the inequality index L_{jt} by attribute j, $j=1...m$, defined by specific functions as discussed below. To define this index, we introduce the membership function μ_j from the fuzzy approach. Fuzzy logic is a form of multivalued logic derived from fuzzy set theory, the membership values can range (inclusively) between 0 and 1.The membership function μ_j may be managed by specific functions as discussed below. This membership function is defined by the gap between the median M_e, which is applied to the number of considered units, and the medial M_l, which is applied to the importance of possessed character $(n_i\gamma_i)$.

We suppose that $M_l \in [\gamma_i \ \gamma_{i+}]$ and $M_e \in [\gamma_i \ \gamma_{i+}]$.

The medial and median are expressed respectively as follows:

$$M_l = \gamma_i + \frac{\gamma_{i+1} - \gamma_i}{f(n_{i+1}\gamma_{i+1}) - f(n_i\gamma_i)}\left[0.5 - f(n_i\gamma_i)\right]$$

$$M_e = \gamma_i + \frac{\gamma_{i+1} - \gamma_i}{f(n_{i+1}\gamma_{i+1}) - f(n_i\gamma_i)}\left[0.5 - f(\gamma_i)\right]$$

$f(n_i\gamma_i)$ and $f(\gamma_i)$ indicate respectively the percentage of payroll and employee.

The membership function μ_j measures the degree of the inequality by the attribute socioeconomic j.

$$\mu_j = \frac{M_l - M_e}{M_l} \quad 0 \le \mu_j \le 1 \tag{2}$$

If $M_l = M_e$ then $\mu_j = 0 \Rightarrow$ concentration null

If $M_l \succ M_e$ then $\mu_j \succ 0 \Rightarrow$ presence of concentration

If $M_l \succ\succ\succ M_e$ then $\mu_j \to 1 \Rightarrow$ strong concentration

The income inequality index, across individuals, is:

$$L_j = \mu_j \frac{\sigma_j}{\sigma} \tag{3}$$

σ_j and σ indicate respectively standard deviation by socioeconomic attribute j=1…m, and total standard deviation.

The unidimensional poverty fuzzy function is defined by the fuzzy average individual poverty $P(\gamma_{it}, L_{jt})$ as the following depending on the income γ_{it} and the inequality index L_{it}.

$$P(\gamma_{it}, L_{jt}) = \frac{1}{n} \frac{\sum_{i=1}^{n} L_{jt} (\frac{z - \gamma_{it}}{z})^{\alpha} I(\gamma_{it} \le z)}{\sum_{j=1}^{n} L_{jt}} \quad (4)$$

α is a parameter indicating the sensitivity of the index to the distribution among the poor. The higher α is the more sensitive of the index; it is to the poorest persons in the economy. For $\alpha = 0$, $P(\gamma_{it}, L_{jt})$ is the headcount. For $\alpha = 1$, it represents the poverty gap. In addition, for $\alpha = 2$ it represents the severity of poverty.

3.2 The decomposition of poverty through the Shapley value

The variation of poverty can be decomposed into a component of inequity and a component of growth. The growth factors is defines by the variation of income, $G = (\gamma_{i2} \div \gamma_{i1}) - 1$ and the redistribution factor is defined by the difference between the inequality index at time $t(t = 1,2)$, $R = L_{j2} - L_{j1}$. The decomposition problem consists here to identify the contribution of growth G and the contribution of redistribution R in the variation of poverty ΔP. Moreover, those contributions are calculated through the Shapley value. We have two possible sequences since we have just two factors in the decomposition refer to inequity and growth. We defined the sequences A and B as follows:

Sequence A: $\varphi_A = \{G, R\}$

Sequence B: $\varphi_B = \{R, G\}$

The variation of poverty is defined as follows:

$$\Delta P = P(\gamma_2, \mu_{j2}) - P(\gamma_1, \mu_{j2}) = P(\gamma_1(1+G), L_{j1} + R) - P(\gamma_1, L_{j1}) = F(G, R) \quad (5)$$

The contribution of growth is:

$$C_G^S = \frac{1}{2}\left[\Delta_G F(S(G, \varphi_A)) + \Delta_G F(S(G, \varphi_B)\right]$$

The contribution of growth is calculated through the Shapley value and can be decomposed into two components. The first component relative to the sequence *A*, it is the marginal effect when we add the factors G to the set S. It is given by the value:

$$F(S(G, \varphi_A) \cup \{G\}) - F(S(G, \varphi_A)) = F(G, R) - F(R)$$

The second component is relative to the sequence *B*. It is given by the value:

$$F(S(G, \varphi_B) \cup \{G\}) - F(S(G, \varphi_B)) = F(G)$$

Finally, the contribution of growth is expressed as follows:

$$C_G^S = \frac{1}{2}\left[F(G,R) - F(R) + F(G)\right] \quad (6)$$

If we replace the equation (6) in the equation (5), we obtain:

$$C_G^S = \frac{1}{2}\left[P(\gamma_2, L_{j2}) - P(\gamma_1, L_{j2}) + P(\gamma_2, L_{j1}) - P(\gamma_1, L_{j1})\right] \quad (7)$$

The contribution of growth under the rule of Shapley is the average of two elements. The first element is the variation of the measurement of poverty if inequality is fixed and equal to that in the final period. The second element is the variation of the measurement of poverty if inequality is fixed and equal to that in the initial period.

We consider the same sequences *A* and *B* defined above, the contribution of inequality will be defined similarly as the formula of the contribution of growth.

$$C_R^S = \frac{1}{2}\left[F(G,R) - F(G) + F(R)\right] \quad (8)$$

If we replace the equation (8) in the equation (5), we obtain:

$$C_R^S = \frac{1}{2}\left[P(\gamma_2, L_{j2}) - P(\gamma_2, L_{j1}) + P(\gamma_1, L_{j2}) - P(\gamma_1, L_{j1}\right] \quad (9)$$

The contribution of inequality under the rule of Shapley is equal to the average of two elements. The first element is the variation of measurement of poverty if the income is fixed and equal to that in the final period. The second element is the variation of the measurement of poverty if the income is fixed and equal to that in the initial period.

Finally, the variation of poverty is equal to the sum of the contributions of growth and redistribution. It is expressed as follows:

$$\Delta P = C_R^S + C_G^S \quad (10)$$

It does not present an error term or an interaction between factors unlike in the classic decomposition. The Shapley value helped us to identify the mechanisms of transmission to carry out an economic policy aiming at reducing poverty. Policymakers need more information about dynamics of poverty and the causes of this phenomenon. Therefore, a better understanding of the variation of poverty facilitates effective policies and a great efficiency in social assistance programs.

4 Empirical illustration

Data come from the 13392 and 11281 Tunisian household survey conducted by the Tunisian Institute of Statistics respectively of (2005) and (2010). A brief summary the total annual expenditure variable is respectively given in Tables (1) and (2).

Table 1 Summary Statistics of the Total Annual Expenditure Variable (2005)

Minimum	First quantile	Median	Mean	Third quantile	Maximum
25	871	1367	1887	2201	54420

The survey provides demographic characteristics of households by regions: The Greater Tunis, North, Centre and South. They are carried out about the household including food consumption and nutrition, level of household economy, employment, population, housing conditions and literacy.

Table 2 Summary Statistics of the Total Annual Expenditure Variable (2010)

Minimum	First quantile	Median	Mean	Third quantile	Maximum
259	5328	8486	10580	13230	197000

For the detection of the fuzzy non-parametric boundaries of the fuzzy unidimensional poverty states, we use the method devised by Zedini and Belhadj (2014). This method uses a divisive algorithm to calculate approximately the position of the fuzzy sets. In fact, it is of leave to let the data propose the appropriate threshold instead of fixing it in advance. Therefore, the corresponding fuzzy poverty states will be depicted from data and the procedure used for poverty measurement will be based on a data-driven method instead of an axiomatic framework for the sake of more robust and reliable results. Therefore, we consider the threshold as the 65th percentile of the income distribution. We found the following results.

Table 3 The variation of poverty by regions

	C_G^S	C_R^S	ΔP
Great Tunis	0.00835	-0.008	0.00035
North	-0.0057	-0.0012	-0.0069
Center	-0.00125	0.00005	-0.0012
South	-0.00275	0.00185	-0.0009
whole territory	-0.0029	-0.0028	-0.0318

This result shows that both growth and inequity can play a major role in the change of poverty. However, the impact of these factors depends on the level of expenditure that is the indicator of the income in our work. Moreover, the relative effects of these two phenomena may differ quite across regions:

By examining the second line of the table 1, we notice that in the north there is a decrease in the incidence of poverty of 0.69%. Growth has helped us to reduce this incidence with 0.57% and the redistribution has contributed a decrease of 0.12%.On the one hand, the efficient effect of the two components is probably due to an increase in expenditure. On the other hand, in this region, the poor do not have only the access to the needs that they can buy but also to the natural resources that support their nouriture needs and their water needs. In the region of the Great Tunis, the poverty has increase, reaching 0.035%, this raise may be due to the displacement of the poor in the interior regions of Tunisia to this region, also the problem of demographic growth and the lack of natural resources in this region.

By examining the third and the forth line, we notice that there is a respective decrease of 0.12% and 0.09% in the incidence of poverty. The growth has contributed to reducing this impact while inequality has contributed to increasing poverty. Therefore, the inequality can have a positive impact on poverty, and the poverty responds positively to inequality therefore we can talk about policy that can reduce inequality to be in favor of reducing the poverty.

The region of the north has an important variation of poverty about 0.69% compared to the other regions as the south registered only modest poverty reductions, reaching only 0.09%. This may be due to the regional development policy and to the policy of reducing poverty. Growth is a powerful force for reducing poverty, therefore there are many regionswhere income growth may not adequately be translate to poverty reduction.

Growth benefits the poor but at the absence of effective redistribution policies, it might affect negatively on the income distribution. Growth accompanied by progressive distributional change is better than growth alone. The Policy that aims to reduce poverty has often been founded on the issue of the relative impact of growth and inequality on poverty. We note that growth and inequality can be used to reduce poverty and the redistribution can accelerate the reduction of poverty, so inequality is worth particular interest that implies the need for specific policies to reduce the poverty. On the one hand, the political reforms encourage taxation and redistribution and may be viewed as strategic decisions, but the taxation causes some distortion on employment and will encourages the poor to be indifferent to work. On the other hand, if we tax the rich a lot it encourages them to go abroad or refrain from investing. Whether there is a margin for taxing capital, we must go slowly for fear of practicing a tax optimization policy. Therefore, we can reduce inequality through the creation of employment, encouraging investors to invest and to ameliorate employment in Tunisia and following an appropriate fiscal policy. In reality, taxes coming from rich are not always sufficient and efficient but they are indispensable from the political and social point of view. Therefore, the main solution is the use of a redistributive policy to promote the redistribution of wealth in favor of the poorest, to curb inequality of opportunity, to facilitate access to employment and to a quality education for the most disadvantaged.

Growth plays a crucial role too in reducing poverty, if the growth is important, the decline in poverty will be observed. The results show that the fight against poverty requires not only reducing inequality but also stimulating growth. A strategy to promote growth may be considered as the most appropriate ways to reduce poverty too in Tunisia. To stimulate growth in Tunisia, it is necessary for the governments to invest in infrastructure. It is only with the adequate infrastructure that a country will develop since energy, water, transport will form the structure blocks to growth. Afterward, the entrepreneurs must invest in order to create employment to reduce poverty. The infrastructure projects will provide some local employment and the business will provide employment to facilitate the access to resources. Private sector focused on domestic production and distribution of essentials for profit. The high priority of the government is to promote development of agriculture, textiles, construction and infrastructure, eliminate illiteracy, establish a free press, and provide public assistance via temporary work programs providing essential products and services. The political system must be structured by free of corruption and injustice, education of citizens for the development of various sectors of the economy, investments by multinationals, and government which support for small-scale industries.

5 Conclusion

Poverty, growth and inequality are three subjects of main interet to policy makers and to development economists. Most of them investigated the relationship between these three factors. In general, there exists a negative correlation between poverty and growth. Conventional decomposition techniques have several problems, the contribution assigned to each specific factor does not intuitively a clear sense. Hence, it sometimes introduce some terms such as residual or interaction to ensure the identity of the model.To derive the results of the decomposition and to have a unified theoretical framework,

we use the Shapley value augmented by the fuzzy approach. In order to take full advantage of this value, it is of interest to calculate the marginal contribution of income inequality and growth in the variation of poverty. This application shows that growth and inequity may play a major role in the variation of poverty and these effects differ across regions, it shows that growth and redistribution significantly affected the variation of poverty. As well, it proves that in most regions poverty declined with a negative contribution of growth and positive contribution of redistribution. The growth has contributed to reducing this impact while inequality has contributed to increasing poverty. Therefore, these two phenomena have a positive effect on the reducing of poverty. Consequently, strategies to promote growth and to reduce inequity can be considered as the most appropriate method to reduce poverty in Tunisia.

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