EE
E u r o E c o n o m i c a
Issue 2(34)/2015 *ISSN: 1582-8859*

Fuzzy multidimensional inequality measurement. Policies to reduce inequality in Tunisia

Lamia HASNAOUI[1], Besma BELHADJ[2]

[1]*University of Tunis El Manar, Tunis, Tunisia, hasnaoui.lamia@yahoo.fr*

[2]*University of Carthage, Tunis, Tunisia, Besma.Kaabi@isg.rnu.tn*

Abstract. This article debates a systematic treatment of the conceptual frame work of the multidimensional fuzzy measurement of inequality. Fuzzy logic is a type of multivalued logic consequential from fuzzy set theory. The introduction of the dimension relative to the human existence like health, energy and housing provides considerable enrichment to our understanding of inequality and its causes. In fact, we propose a multidimensional fuzzy measurement with the membership functions of inequality then we suggested some policies to reduce inequality. An application based on individual well-being data from Tunisian households in 2010 is presented to illustrate the use of the proposed index.

Keywords: income distribution, inequality, Fuzzy approach

1 Introduction

The income inequality is the disparate distribution of household income across the different participants in an economy. It varies between societies, historical periods, economic structures and systems. The phenomenon can refer to cross sectional distribution of income or wealth at any particular period or to the lifetime income and wealth over longer periods of time. There are a variety of numerical indices for measuring economic inequality. Economic inequality refers to how economic metrics are distributed among individuals in a group, among groups in a population, or among countries or regions. Economists usually think of three metrics of economic disparity: wealth inequality, income inequality, and consumption. The issue of economic inequality can implicate notions of equity, equality of outcome, and equality of opportunity.

Indeed, the causes of income inequality can vary considerably by region, gender, education and social status. Economists are divided as to whether income equality is ultimately positive or negative and what are the implications of such disparity. In fact, income distribution has always been a central concern of economic theory and economic policy. Therefore, many researches on the measurement of this concept are based on Gini index. Differences in national income equality are measured by the national Gini coefficient. The Gini coefficient is a number between 0 and 1, where 0 corresponds with perfect equality (where everyone has the similar income) and 1 corresponds with absolute inequality (where one person has all the income, and everyone else has zero income). Indeed, economists have developed new Gini decomposition.

Bhattacharga and Mahalabonis (1967) introduce a Gini decomposition in order to study regional disparities. Eugene et al (2002) defined and studied the beta-nornal distribution highlighting great flexibility in modeling not only symmetric heavy-tailed distribution, but also skewed and bimodal distribution. Zenga (2007) proposed an alternative curve based on the ratios of lower and higher means. Akinsete et al (2008) introduced the beta-pareto distribution. Barreto-Souza et al (2009) proposed and studied the proprieties of the beta-generalized exponential distribution. Pescimet et al (2010) introduce the beta-generalized half-normal distribution. Moreover, inequality in access to resources, and in particular in education, may inhibit reductions in child mortality and fertility rates

and prevents the expansion in education of the next generation. These important development policy goals are closely linked to the educational attainment levels of the female population (Summers, 1994; Murthi et al., 1995). To the extent that these linkages exist, gender bias in education may thus prevent progress in the improvement in the wellbeing of the people in a considerable number of developing countries. Noglo (2014) aims to measure and analyze the inequalities in the distribution of non-monetary wealth in Togo. The decomposition of the Gini index in Shapley-value showed evidence that the within-groups inequality is preponderant and policies must give priority to it. This paper debated concerning a systematic treatment of the conceptual frame work of the measurement of inequality. An alternative approaches are evaluated based on the fuzzy approach. Fuzzy logic is a type of multivalued logic consequential from fuzzy set theory to deal with probable reasoning rather than the precise one. In binary logic, in contrast to fuzzy logic, the variables can have a membership value equal to 0 or 1. In fuzzy set theory with fuzzy logic the set membership values can range inclusively between 0 and 1. Indeed, the degree of truth of a statement can range between 0 and 1 and is not constrained to the two true values (true (1), false (0)) the same as in classic predicate logic (Novák et al., 1990). And when linguistic variables are used, those degrees may well be managed by definite functions as discussed below. Zadeh (1965) has proposed fuzzy sets theory, as a consequence of this theory the fuzzy logic has emerged. Wilkinson (1963) has introduced the concept without using the term and therefore preceded fuzzy set theory. However, our main problem is what is the level of inequality in Tunisia and what is the appropriate politic to reduce it? Therefore, in order to take full advantage of this approach, it is of interest to calculate the rate of inequality by attribute and across regions. An application based on individual well-being data from Tunisian households in 2010 is presented to illustrate use of the proposed concepts. The rest of the paper is organized as follows: Section 2 presents the distribution of income per quantile. Section 3 presents the multidimensional inequality measure augmented by the fuzzy approach. Section 4 explores the empirical illustration, the most important results and policies to reduce inequality. Section 6 deals with the conclusion.

2 Distribution of income per quantile

In this section, we measure the monetary inequalities by using the distribution curve that shows how the consumption (or income) is divided in the population and across regions. The division of the distribution in quantile shows how the share of income are divided through of the same size of population groups, ranging from the poorest to the richest and present how this distribution evolves between regions. Due to lack of data, the illustration is limited to a measure using data from Tunisia in 2010. The information used is supplied from the household survey data come from the 11281 Tunisian household survey conducted by the Tunisian Institute of Statistics (2010). A brief summary the total annual expenditure variable is given in Tables (1).

Table 1. Summary statistics of the total annual expenditure variable (2010)

Minimum	First quantile	Median	Mean	Third quantile	Maximum
259	5328	8486	10580	13230	197000

The survey provides demographic characteristics of households. In order to take into account diverse geographical and socioeconomic characteristics of regions in Tunisia, we split the country into 7 different homogenous regions. Tunisia is usually subdivided into four naturals regions: The Greater Tunis, North East, North West, Centre East, Centre West, South East and South West. These investigations are carried out about the household including food consumption and nutrition, level of household economy, employment, population, housing conditions and literacy.

In Tunisia, we have two classes (figure1) therefore we noticed the absence of the middle class. The definition of the middle class indicate that the first criterion used to define its contours is the level of income or expenditure in deed we have other criteria such as socio-professional category, the level of

actual consumption of durable goods, the size of the family, residential space, employment status, heritage and perception of belonging to the middle class.

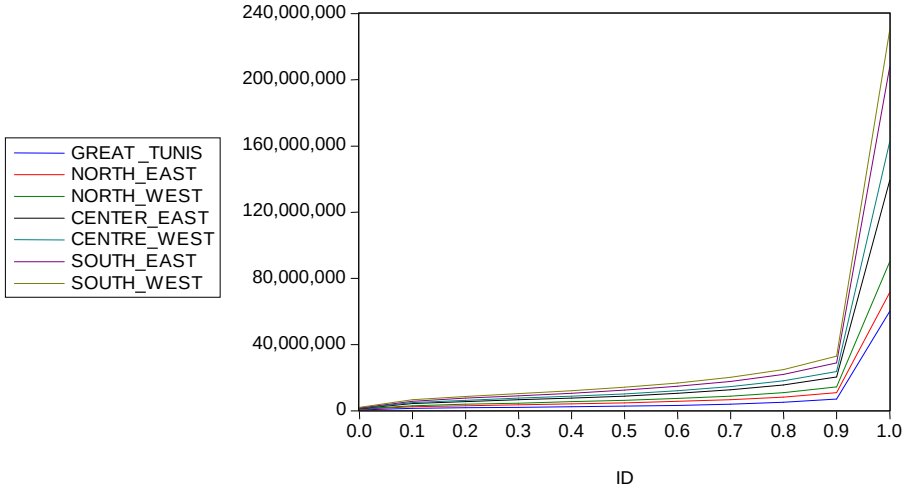

Figure1.Distribution of income in 2010

Figure 1 shows that the percentage of middle class in Tunisia including poor class is reached about 90 percent of the population. This class split into three classes the middle class with law spender capacity can easily switch in poverty it is about 12 percent, the intermediate middle class and the superior middle class it is about 80 percent. We did not notice a high level of inequity across regions. The government must work hard to bring development back to center stage. Our main concern in this quest is the urgent need to decrease the levels of inequalities. Development policies may help our country to develop the middle class. Our country need to design whole-of-government strategies to reduce inequalities. Any of these strategies should rest on three most important pillars: The first challenge is to create new productive and rewarding jobs. Second, we have to investment in education and skills development to promote employment and employability. Finally, reforming tax and benefit policies can also progress the distribution of income. Indeed, the choice of this welfare indicator depends on other factors. Inequality is a multidimensional phenomenon. They should in principle account different aspects of economic and social life, such as food, housing, clothing or the possibility of pursuing a profession, the state of health and education, for example. When admits inequalities integrate social and demographic components as life expectancy or literacy, this has undoubtedly affect the measurement of inequality, particularly in the context of regional comparisons, where coverage health or education, for example, can differ from one region to another. Indeed, it is interest to consider the level of inequality in a multidimensional form. We present in the section below an alternative approaches are evaluated based on the fuzzy approach. In order to take full advantage of it, we propose an index augmented by this approach.

3 Multidimensional inequity fuzzy measure

To measure inequity, we suggest the multidimensional inequality index I_{ki} by attribute k, $k = 1...K$, defined by specific functions as discussed below. To define this index, we introduce the membership function μ_{ki} from the fuzzy approach. This function can be inclusively range between 0 and 1.The

membership function μ_{ki} may be managed by specific functions as discussed below. This membership function is defined by the gap between the median M_{ki}^{e}, which is applied to the number of considered units, and the medial M_{ki}^{l}, which is applied to the importance of possessed character $(n_i\gamma_i)$.

We suppose that $M_{ki}^{l} \in [\gamma_{ki} \quad \gamma_{ki+1}[$ and $M_{ki}^{e} \in [\gamma_{ki} \quad \gamma_{ki+1}[$

The medial and median are expressed respectively as follows:

$$M_{ki}^{l} = \gamma_{ki} + \frac{\gamma_{ki+1} - \gamma_{ki}}{f(n_{ki+1}\gamma_{ki+1}) - f(n_{ki}\gamma_{ki})}[0.5 - f(n_{ki}\gamma_{ki})]$$

$$M_{ki}^{e} = \gamma_{ki} + \frac{\gamma_{ki+1} - \gamma_{ki}}{f(n_{ki+1}\gamma_{ki+1}) - f(n_{ki}\gamma_{ki})}[0.5 - f(\gamma_{ki})]$$

$f(n_{ki}\gamma_{ki})$ and $f(\gamma_{ki})$ indicate respectively the percentage of payroll and employee.

The membership function μ_{ki} measures the degree of the inequality by the socioeconomic attribute k.

$$\mu_{ki} = \frac{M_{ki}^{l} - M_{ki}^{e}}{M_{ki}^{e}} \quad 0 \le \mu_j \le 1 \tag{1}$$

If $M_{ki}^{l} = M_{ki}^{e}$ then $\mu_{ki} = 0 \quad \Rightarrow$ concentration null

If $M_{ki}^{l} > M_{ki}^{e}$ then $\mu_{ki} > 0 \quad \Rightarrow$ presence of concentration

If $M_{ki}^{l} >> M_{ki}^{e}$ then $\mu_{ki} \to 1 \quad \Rightarrow$ strong concentration

The income inequality index, across individuals, is:

$$I_{ki} = \frac{M_{ki}^{l} - M_{ki}^{e}}{M_{ki}^{e}} \times \sum_{j=1}^{k \neq j} \frac{V(\gamma_{ki})}{\mathrm{cov}(\gamma_{ki}, \gamma_{ji})} \tag{2}$$

This measure of inequality is ranged from 0 refers to the perfect equality to 1, which refers to perfect inequality. While this coefficient has many desirable properties mean independence, population size independence, symmetry, and Pigou-Dalton Transfer sensitivity it cannot easily be decomposed to show the sources of inequality. This inequity index is not entirely satisfactory. To see this, consider the axioms that make a good measure of income inequality, namely:

Axiom 1: Mean independence

This means that if all incomes were doubled, the measure would not change. The index satisfies this.

$I_{ki}(2y) = I_{ki}(y)$

Axiom 2: Population size independence

If the population were to change, the measure of inequality should not change, ceteris paribus. The index satisfies this too.

Axiom 3: Symmetry

If you and I swap incomes, there should be no change in the measure of inequality. The index satisfies this. For example, if x' is obtained from y' by permutation of income, therefore $I_{ki}(y) = I_{ki}(x)$

Axiom 4: Pigou-Dalton Transfer sensitivity

Under this criterion, the transfer of income from rich to poor reduces measured inequality. The index satisfies this too. We suppose a distribution of income x' obtained from a distribution y' where we make a handoff progressive income. X is obtained from y by a single progressive transfer if there is i and r such as $x_i \prec y_i \prec y_r \prec x_r, y_i - x_i = x_r - y_r, y_s = x_s,$ for all $s \neq i, r \Rightarrow I_{ki}(x) \prec I_{ki}(y)$

The index satisfies this too.

Axiom 5: Decomposability

This means that inequality may be broken down by population groups or income sources or in other dimensions. The index is decomposable or additive across groups. That is, the total index of society is equal to the sum of the coefficients of its subgroups.

Axiom 6: Statistical testability

One should be able to test for the significance of changes in the index over time. This is less of a problem than it used to be because confidence intervals can typically be generated using bootstrap techniques.

Axiom 7: the relative invariance property

The relative invariance property is the homogeneity of degree zero of a function. $I_{ki}(\lambda y) = I_{ki}(y), \lambda \in \Re_+^*$. This property allows as overcoming the different nature units.

4 Empirical illustration

In this section, the data used is the same as in the section 2. The survey also provides demographic characteristics of households. In order to take into account diverse geographical and socioeconomic characteristics of regions in Tunisia, in this section, we divided also the country into 7 different homogenous regions to create use of the characteristics of different regions in Tunis; we split the households according to their location with respect to 7 different regions.

Table 2. The measurement of inequity by regions

	Great Tunis	North East	North West	Center East	Centre West	South East	South West
Consumption	0.5853	0.52	0.511	0.6726	0.5863	0.56	0.609
Alimentation	0.336	0.038	0.049	0.0318	0.065	0.0359	0.1557
Alcohol and tobacco	0.0019	0.0066	0.0033	0.003	0.0062	0.0017	0.0027
Dress	0.0049	0.0241	0.0291	0.017	0.0171	0.00141	0.028
Housing and energy	0.1925	0.1231	0.2204	0.2423	0.212	0.2476	0.1534
Furniture	0.0132	0.0182	0.0127	0.0152	0.0246	0.0093	0.0174
Health and	0.0232	0.063	0.0235	0.038	0.037	0.0316	0.013

	Great Tunis	North East	North West	Center East	Centre West	South East	South West
care							
Transport	0.06311	0.06	0.0282	0.113	0.306	0.0494	0.131
Telecom	0.0074	0.0057	0.0063	0.0197	0.0049	0.0047	0.0025
Entertainment	0.00181	0.0036	0.0019	0.0052	0.0013	0.0011	0.0016
Education	0.0018	0.0024	0.0013	0.0014	0.0035	0.0006	0.0007
Hotel	0.00303	0.0016	0.0006	0.0014	0.002	0.0024	0.00167
Other expenditure	0	0.00066	0.0049	0.0079	0.0033	0.0069	0.00072

The region of North West (table 2, column 3) has relatively low levels of consumption reached about 51.1 percent compared to the other regions. And the region of Centre East (table 2, column 4) presented the highest inequity index reached about 67.26 percent. The inequality of the condition of life refers to the inequality of access to goods and services that satisfy the needs of household like foods, health, education. Access to education has always been one of the historical claims of the regions. A number of economists have argued that inequality leads to economic instability. One mechanism by which this happens is that the rich consume a smaller proportion of their income than the poor. Meanwhile, as inequality grows, individuals facing low or declining relative incomes maintain their consumption through borrowing.

The region of South West (table 2, column 7) has relatively low levels of inequality index of education reached about 0.07percent compared to the other regions. And the region of Centre West (table 2, column 5) presented the highest index reached about 0.35percent. We remark that we don't have high difference in the rate of inequality but the governments have to investment in education and skills development to promote employment and employability. The Tunisian politic of education was regarded as the prototype of success, we must take attention to the problem of mass illiteracy. The principal reform in Tunisia will be devoted to developing and restructuring higher education and driven by technological skills and needs. In fact, the government effort must increase in rural primary education not only have large economic returns, but also contribute to equity in rural and interior areas to facilitate the access to best education. The number of pilot school increase over this decade. Indeed, we can analyses the education like investment as describe in human capital theory. Schultz (1961), Becker (1975) and Mincer (1958) showed that the most efficient technology is adopted more rapidly by the richly endowed economies for highly skilled workers. According to this authors, the level of education increase the economic growth rate, by accelerating the assimilation of technical progress. In fact, if we invest in education, we realize the wage gain over the long term. While the national perspective, this investment increase the economic growth rate and reduce inequality.

The region of South East (table 2, column 6) presents the highest level of inequality index of energy reached about 24.76 percent. Indeed, the region of North East (table 2, column 2) presents the low level it is about 12.31 percent of inequality of housing and energy. Moreover, it presents the highest level of inequality of health and care, reached about 6.3 percent. In fact, the region of South West (table 2, column 7) presents the low level, reached about 1.3 percent. The region of the Center East (table 2, column 5) has an important index of inequality of transport about 30.6percent compared to the other regions as the North West (table 2, column 1) registered only modest level, reached only 2.82percent. We notice that the level of the index of inequality is important but the main interesting is the quality of this services energy, health and care and transport so we can talk about the degree of poverty in these regions. The information conducted by the Tunisian Statistical Institute in 2010 prove

that regional disparities are exacerbated by the concentration of public services, investment and economic activities in the coastal region, therefore the interior regions are less well served in terms of public services. It is the same with regard to the number of hospital beds, and the illiteracy rate is 30percent in rural areas against 15percent in urban areas and 23percent national. Excessive concentration of Directors attended the neglect of public services, including basic services in areas of the interior. Similarly, the concentration of economic activity in the regions of North East and Central East has led to regional disparities, the coastal region with 75percent of non-farm employment. The governorate of Tunis has absorbed 23percent of all job vacancies in 2010 against only 4percent in the South east, although this region accounts for 9percent of the population. Without a doubt, Provision and equal access to high-quality public services and strengthening social protection programmes will help provide equal opportunities for all citizens. For many developing countries and emerging economies, reducing inequalities will require tackling informality, expanding the social security system, and reducing underemployment.

Development of rural and interior regions infrastructure is a key of rural and economic life. But nowadays, Tunisian's government gave less priority to rural and interiors regions infrastructure in promoting agriculture production rural Non-agricultural employment and improved living standards for the rural population. To develop the interior regions, we must introduce electricity, it can profoundly affect the interior rural regions. It may expend the production hours in the day. The television and radios extend accessible free entertainment and education indeed government must invest too in telecommunication. In fact, government spending on roads electrification, education and other public investment in rural areas contributed to rapid growth in agricultural production. Consequently, policies to promote agriculture production and to promote growth led to increase inequality in the North West regions.

Many strategies should rest on most important pillar which is the reforming tax and benefit policies can also progress the distribution of income. We must go slowly for fear of practicing a tax optimization policy so if we tax the rich a lot it encourages them to go abroad or refrain from investing. Therefore, we can reduce inequality through the creation of employment, encouraging investors to invest and to ameliorate employment in Tunisia and following an appropriate fiscal policy. Certainty, taxes coming from rich are not always efficient and adequate but they are necessary from the political and social point of view. Consequently, the main solution is the use of a redistributive policy to promote the redistribution of wealth in favor of the poorest, to curb inequality of opportunity, to a high quality education for the most disadvantaged and to facilitate access to employment. We can also reduce inequality by giving more chance to jobless to create their own business through tax havens and encouraging them with micro credit with low interest rates .the government is obliged to use these main solutions to reduce unemployment therefore reducing inequality.

5 Conclusion

The analysis of inequality is frequently seen as a unidimensional form. In fact, it is interest to consider the level of inequality in a multidimensional form they should in principle account different aspects of economic and social life. Nowadays, the choice of the welfare indicator depends on the standard of living and other factors, such as health, education level and transport. In this paper, a multidimensional form of inequality is considered. We have presented a systematic treatment of the conceptual frame work of the measurement of inequality augmented by the fuzzy approach. Fuzzy logic is a type of multivalued logic consequential from fuzzy set theory to deal with probable reasoning rather than the

precise one. Our main anxiety in this search is the urgent need to reduce the levels of inequalities and to develop the quality of services. Development policies may help our country to develop the middle class. Proposed solutions range from the practical but weak, like raising the minimum wage, to the fanciful, like global wealth tax. Our country need to design whole-of-government strategies to reduce inequalities. Any of these strategies should rest on three most important pillars. The first challenge is to create new productive and rewarding jobs. Second, we have to investment in education and skills development to promote employment and employability. In fact, if we invest in education, we realize the wage gain over the long term and my reduce poverty too. While the national perspective, this investment increase the economic growth rate and reduce inequality. Finally, reforming tax and benefit policies can also progress the distribution of income.

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