

Long - Memory Persistence in African Stock Markets

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Abstract: Emerging stock markets are said to become efficient with time. This study seeks to investigate this assertion by analyzing long - memory persistence in 8 African stock markets covering the period from 28 August 2000 to 28 August 2015. The Hurst exponent is used as our efficiency measure which is evaluated by the Detrended Fluctuation Analysis (DFA). Our findings show strong evidence of long - memory persistence in the markets studied therefore violating the weak - form Efficient Market Hypothesis (EMH).

Keywords: Long – memory; Hurst exponent; DFA; Market efficiency

1. Introduction

Long - memory or long - range dependence or non - Gaussianity which is a stylized fact of financial data implies that observations which are apart in time are highly correlated. When there is persistence of long - memory on a stock market, it means a violation of the Efficient Market Hypothesis (EMH). The EMH states that information on a market is incorporated correctly and instantaneously in setting prices on that market so if long - memory persist, abnormal returns can be made by analyzing information on the market.

This paper investigates the degree of long - memory persistence over time with the use of the Hurst exponent which will be evaluated by the Detrended Fluctuation Analysis (DFA). When our efficiency measure, the Hurst exponent, shows persistence or anti - persistence, then the market is inefficient. On the other hand, when there is no persistence nor anti - persistence, then abnormal returns cannot be made by analyzing information on that market which goes to affirm the EMH. It is of belief that as stock markets develop, they become efficient with an increase in the number of research analysts and arbitrageurs alike hence anything that constitutes abnormal profit making should be identified and discarded immediately. The reason why arbitrageurs are seen on efficient markets is because Grossman and Stigitz (1980) argue that to bring about efficiency, there should be some level of inefficiency in order to make it incentive for arbitrageurs to find and trade the assets which are not correctly priced.

The aim of this paper is to investigate the degree of long-memory in African stock markets over time. African stock markets which are emerging have seen little attention when it comes to long-memory persistence, Barkoulas et al. (2000). Therefore, this work tends to build on the works on emerging markets in the literature by Tolvi (2003), Cajueiro and Tabak (2004a, 2004b), Jefferis and Smith (2005), McMillan and Thupayagale (2008), Jefferies and Thupayagale (2008), and Morris, Q et al (2009).

The rest of the paper will be sectioned as follows:

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Section two presents the literature review. Section three justifies the stock markets selected and the methods used in collecting data. The Detrended Fluctuation Analysis (DFA) is described in section four. Section five presents the research findings and discusses the findings with appropriate tables and graphs. Section six concludes the paper.

2. Literature Review

There has been considerable amount of research on long-memory persistence in financial time series. A time series that shows presence of long - memory implies it is serially dependent, a violation of the EMH in that the market is said not to correctly and instantaneously reflect all the available information on that market.

The literature makes mention of the works of Mandelbrot (1971) who argues that the presence of serial dependence in a financial time series shows that new market information is not incorporated in setting current prices. This assertion was affirmed by Mukherjee et al. (2011) who found that Indian stock returns were autocorrelated. He therefore concluded that markets response to new information is a gradual process and not instantaneous as the EMH suggests. Also, Los (2003) showed that the use (G) ARCH processes cannot correctly model long-term dependence. Barkoulas and Baum (1996) extended the argument by Los (2003). They stated that since techniques such as linear modelling, forecasting assume EMH-consistent Gaussian distributions, the presence of long - memory in financial time series would make conclusions based on these techniques invalid. The works of Costa and Vasconcelos (2003) found that the Brazilian stock market displayed a memory effect which lasted for up to six months. Barkoulas et al. (2000) and Panas (2001) both found significant and robust evidence of long memory processes in the Greek stock market. Rege and Martin (2007) discovered pronounced long memory effects in the Portuguese stock market. The work of Sadique and Silvapulle (2001) found evidence of long - memory persistence in emerging markets of Korea, Malaysia, Singapore and New Zealand but not in developed markets such as USA, UK and Japan. These findings suggest that emerging markets exhibit long – memory whiles developed markets do not. The reasons behind this assertion were given much emphasis in the paper by Chan and Hameed (2006) who found out that long - memory persists in emerging markets because flow of information of a firm to potential investors is poorer than the flow of information in developed markets. Also, Tolvi (2003) observed that serial dependence in smaller markets make them less efficient than developed markets, a conclusion which was affirmed by Kim and Shamsuddin (2008).

But Cajueiro and Tabak (2004b) makes a conclusion that emerging markets move towards efficiency with time. This is one of the aims of this work, to investigate if African stock markets are becoming efficient with time. Cajueiro and Tabak (2004b) give reasons such as increase in foreign capital inflow and increase in trading volumes as the drivers of emerging markets towards efficiency.

The Hurst exponent developed by Hurst (1951) to compute the optimum size of a dam on River Nile will be used to measure the degree of long - memory in the stock markets. The exponent will be evaluated by the use of a technique for detecting long range autocorrelations in time series, the Detrended Fluctuation Analysis (DFA) developed by Peng et al. (1994).

3. Data

The stock indices selected for the countries were based on the availability of data for the 15 year period chosen. Data was obtained for each country from DataStream denominated in their respective local currency units between 28/8/2000 to 28/8/2015. Each of the eight countries; Botswana, Egypt, Kenya, Mauritius, Morocco, Nigeria, South Africa, and Tunisia had a stock index representing it.

Table 1. Displays the countries and its corresponding stock index.

Country	Representative Index
Botswana	S&P Botswana BMI
Egypt	EGX 30
Kenya	NSE 20
Mauritius	SE Semdex
Morocco	Morocco All Share (MASI)
Nigeria	Nigeria All Share
South Africa	FTSE JSE All Share
Tunisia	Tunindex

Source: Authors own construct

4. Methodology – Detrended Fluctuation Analysis

Long – memory is defined based on frequency or time domains. It is commonly defined with respect to its autocorrelation function (ACF). Ding et al (1993) describes a series to have long – memory if its ACF decline slowly and an infinite spectrum at zero frequency.

Given a time series X_t , the frequency domain definition of long – memory is when a spectral density $f(\lambda)$ of low frequencies obeys

$$f(\lambda) \sim C_f |\lambda|^{-2d} \text{ as } \lambda \rightarrow 0 \tag{1}$$

Where λ is the frequency, C is a strictly positive constant and d is the degree of integration.

In the domain, long – memory is said to be present in a series X_t if there exist a real number H and finite constant C such that the ACF, $\rho(k)$ has the following rate of decay:

$$\rho(k) \approx Ck^{2H-2} \text{ as } k \rightarrow \infty \tag{2}$$

Where $H \in (0,1)$, is called the Hurst exponent, which measures the degree of long – memory of the time series.

This paper seek to measure the degree of long – memory in the time domain. Therefore, the Hurst exponent will be evaluated by the Detrended fluctuation Analysis (DFA).

Detrended Fluctuation Analysis

The Detrended fluctuation analysis (DFA) is an important tool for detecting long –range autocorrelations in time series with non – stationarities which affects experimental data.

We follow Peng et. al (1994) who proposed the DFA.

Suppose $X(t)$ an integrated financial time series of logarithmic returns, i.e.

$$X(t) = \ln(P_t) - \ln(P_{t-1}) \tag{3}$$

With $t = 1, \dots, N$. in this method, the integrated time series is divided into blocks of the same length n .

The ordinary least squares method is used to estimate the trend in each block. In each block, the ordinary least square line is expressed as $X_n(t)$. The trend of the series is removed by subtracting $X_n(t)$ from the integrated series $X(t)$ in each block.

This procedure is applied to each block and the fluctuated magnitude is defined as

$$\sigma_{DFA} = \sqrt{\frac{1}{N} \sum_{t=1}^N (X(t) - X_n(t))^2} \tag{4}$$

This step is repeated for every step n and to estimate Hurst exponent, the following scaling relationship is defined

$$\sigma_{DFA} \propto n^H \tag{5}$$

Taking logs of both sides of equation (5), we get

$$\log(\sigma_{DFA}) \propto H \log(n) \tag{6}$$

This linear relationship between σ_{DFA} and n on a log – log plot support the presence of a power law (fractal) scaling which indicate there is self – similarity in the series. This means the fluctuation over small time scale are related to fluctuations over larger time scales. The following are to be noted about H :

If $H = 0.5$, it means that all autocorrelations tend rapidly to zero and the time series is a random walk. The market is thus weak - form efficient.

If $H > 0.5$, the memory effect is stronger which means persistence. Thus, an increase (decrease) of asset price is likely to follow another increase (decrease).

If $H < 0.5$, it suggests anti – persistence or mean reversion in the time series.

5. Empirical Results

In Table 2, the summary statistics of the return series for the markets show the series are non-Gaussian. The kurtosis coefficients are large and mostly skewed negatively. The Jarque – Bera test statistic null hypothesis is rejected at the 1% level of significance. The Augmented Dickey Fuller test shows that the return series are stationary.

Table 2. Summary Statistics for returns

Market	Number of observations	Mean	Std Deviation	Skewness	Kurtosis	Jarque-Bera	ADF
Botswana	3915	0.0006	0.0078	5.8178	124.9661	2572200	-14.823
Egypt	3915	0.0006	0.0169	-0.4309	10.1907	17084	-14.491
Kenya	3915	0.0002	0.0093	0.3047	31.6902	164070	-13.932
Mauritius	3915	0.0004	0.0067	0.2802	22.7257	84396	-13.543
Morocco	3915	0.0003	0.0077	-0.4954	7.1333	7712	-14.117
Nigeria	3915	0.0003	0.0133	-0.6872	368.4107	22164000	-14.862
South Africa	3915	0.0005	0.0121	-0.1164	3.6536	2190.4	-15.743
Tunisia	3915	0.0003	0.0053	-0.4453	11.2584	20833	-14.500

In Table 3, the Hurst exponent for each of the years is computed. It was observed that there were years the Hurst exponent, H was less than 0.5 for some markets. This means the memory effect during that year was less and possibly moving towards efficiency. However, a close look at the DFA plots of log (RMSE) versus log (scale) in figures 1, 2, 3 and 4 of the return series for the 15 year period showed a different feature. The DFA plots indicate the degree of long - memory persistence by estimating the Hurst exponent. The Hurst exponents for the markets show persistence of long - memory in the return series because all have H greater than 0.5.

Table 3. Hurst exponent for returns

Year	Botswana	Egypt	Kenya	Mauritius	Morocco	Nigeria	South Africa	Tunisia
2000	0.4829	0.4655	0.5969	0.5604	0.5712	0.6150	0.4623	0.4117
2001	0.6704	0.6072	0.5525	0.6916	0.5318	0.4972	0.5785	0.4754
2002	0.5145	0.5278	0.6944	0.6153	0.6257	0.5808	0.4473	0.6726
2003	0.6128	0.6347	0.5305	0.7955	0.6395	0.4203	0.4885	0.5879
2004	0.5980	0.6521	0.7889	0.7569	0.6156	0.4173	0.6455	0.7385
2005	0.5756	0.5498	0.6049	0.5666	0.5595	0.6753	0.4793	0.6483
2006	0.5551	0.4929	0.5714	0.6390	0.6551	0.6494	0.5342	0.7036
2007	0.6007	0.5887	0.4632	0.6283	0.4282	0.6159	0.4662	0.5907
2008	0.8045	0.7127	0.6228	0.6512	0.4905	0.6949	0.4717	0.6044
2009	0.5171	0.4395	0.6631	0.5784	0.5324	0.5287	0.4139	0.7519
2010	0.6283	0.5735	0.5683	0.6235	0.4549	0.5037	0.5298	0.4925
2011	0.6017	0.5437	0.5883	0.5252	0.5609	0.5315	0.3857	0.5728
2012	0.7107	0.4417	0.5492	0.6727	0.5618	0.6803	0.5091	0.5222
2013	0.6504	0.6663	0.5128	0.6644	0.4692	0.6398	0.4858	0.5753
2014	0.7952	0.4375	0.5673	0.6032	0.4916	0.5406	0.4254	0.6527
2015	0.6435	0.4476	0.5763	0.6233	0.4917	0.5507	0.4752	0.6321
All years	0.6532	0.6007	0.6290	0.6667	0.5879	0.6081	0.5279	0.6450

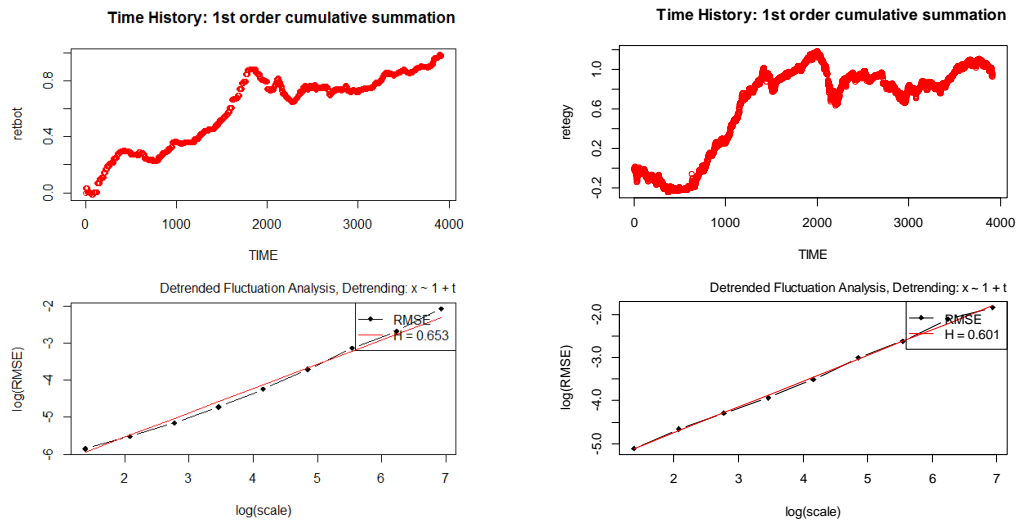


Figure 1. Extended Data Analysis Plots for Botswana and Egypt

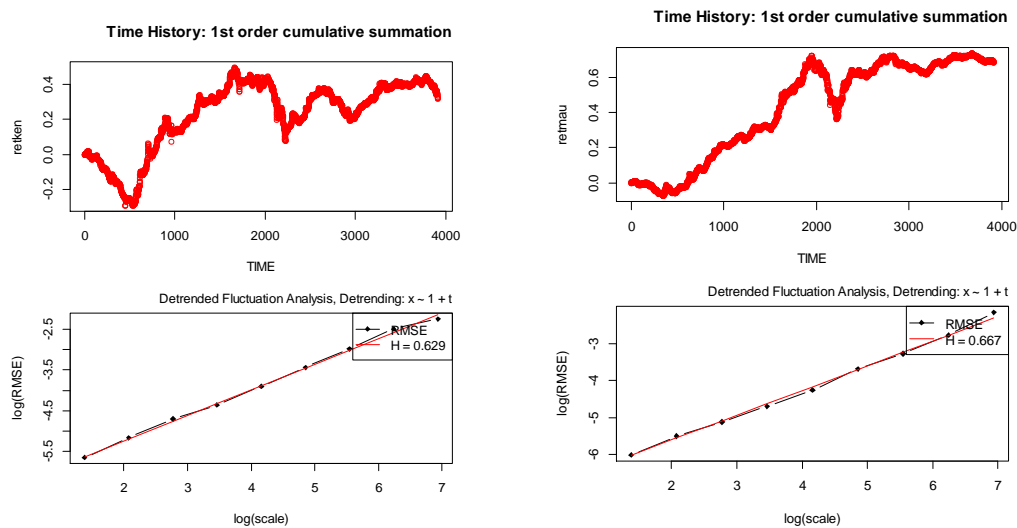


Figure 2. Extended Data Analysis Plots for Kenya and Mauritius

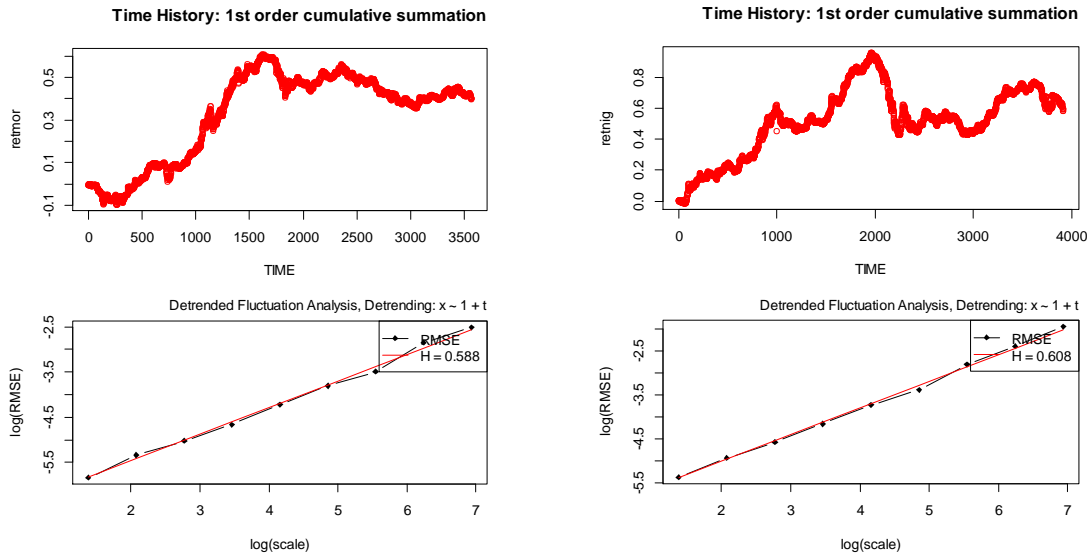


Figure 3. Extended Data Analysis Plots Morocco and Nigeria

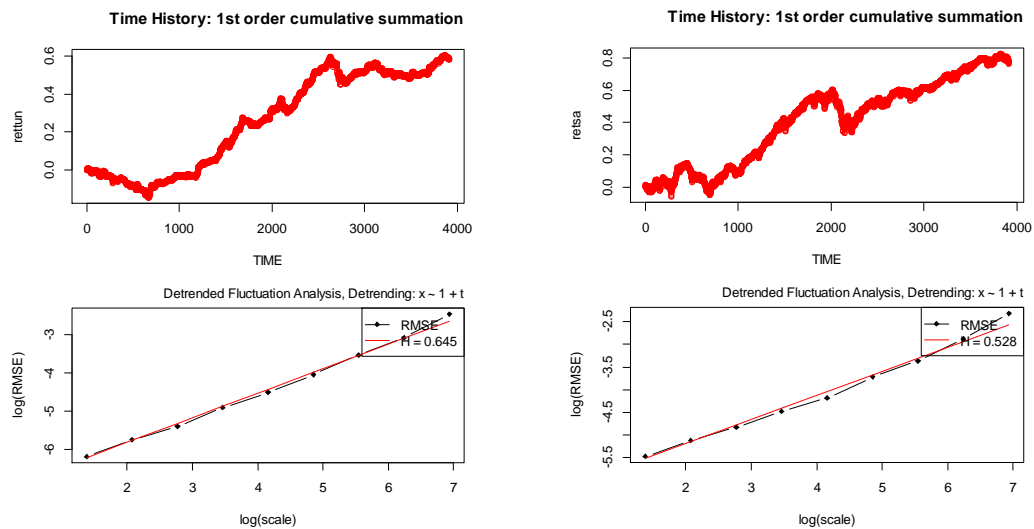


Figure 4. Extended Data Analysis Plots for South Africa and Tunisia

The plot of the return series in figures 1, 2, 3 and 4 show an upward trend which is in contrast to Cajueiro and Tabak (2004b) who found a downward slope as evidence of emerging markets becoming efficient with time. This means the markets are not becoming efficient with time.

In ranking the 8 markets studied using the all year Hurst exponents in Table 3, it was observed that the market with less Hurst exponent and hence less memory effect was South Africa followed by Morocco, Egypt, Nigeria, Kenya, Tunisia, Botswana and finally Mauritius.

6. Conclusions

In this paper, we have investigated the degree of long - memory persistence in return series over time in 8 African stock markets. We used the Hurst exponent as our measure of long -memory persistence which was evaluated by the Detrended Fluctuation Analysis (DFA).

Our work builds on the works on emerging markets in the literature by Tolvi (2003), Cajueiro and Tabak (2004a, 2004b), Jefferis and Smith (2005), McMillan and Thupayagale (2008), Smith (2008), Jefferies and Thupayagale (2008), and Morris, Q. et al (2009).

It was observed that there is strong evidence of long - memory persistence in the markets studied which is contrary to the works of Cajueiro and Tabak (2004b), Jefferies and Smith (2005) but agrees with the work of Smith (2008) who concluded that none of the 11 African markets studied was weak - form efficient.

These findings give strong evidence against the weak - form Efficient Market Hypothesis (EMH) because long - memory persistence in these markets means existence of arbitrage opportunities.

7. References

- Barkoulas, J. & Baum, C. (1996). Long-term dependence in stock returns, *Econ. Lett.* 53 (3):253-259.
- Barkoulas, J.; Baum, C. & Travlos, N. (2000). Long memory in the Greek stock market. *Appl. Financ. Econ.* 10 (2): 177-184.
- Cajueiro, D. & Tabak, B. (2004a). Ranking efficiency for emerging markets, *Chaos, Solitons Fractals* 22: 349-352.
- Cajueiro, D. & Tabak, B. (2004b). The Hurst exponent over time: testing the assertion that emerging markets are becoming more efficient, *Phys. A Stat. Theor. Phys.* 336 (3):521-537.
- Chan, K. & Hameed, A. (2006). Stock price synchronicity and analyst coverage in emerging markets. *J. Financ. Econ.* 80:115-147.
- Costa, R. & Vasconcelos, G. (2003). Long-range correlations and nonstationarity in the Brazilian stock market. *Phys. A Stat. Mech. Appl.* 329:231-248.
- Ding, Z.; Granger, C. & Engle, R. (1993). A long memory property of stock returns and a new model. *Journal of Empirical Finance*, 1:83-106.
- Fama, E. (1970). Efficient capital markets: a review of theory and empirical work. *J. Financ.* 25 (2):383-417.
- Grossman, S. & Stiglitz, J. (1980). On the impossibility of informationally efficient markets. *Am. Econ. Rev.* 70:393-408.
- Hurst, H. (1951). Long term storage capacity of reservoirs. *Trans. Am. Soc. Civil Eng.* 116: 770-808.
- Jefferis, K. & Smith, G. (2005). Capitalisation and weak-form efficiency in the JSE Securities Exchange. *South African Journal of Economics*, 72(4):684-707.
- Jefferis, K. & Thupayagale, P. (2008). Long memory in Southern African stock markets. *South African Journal of Economics*, 76(3):384-398.
- Kim, J. & Shamsuddin, A. (2008). Are Asian stock markets efficient? Evidence from new multiple variance ratio tests. *J. Empir. Finance* 15 (3):518-532.
- Los, C.A. (2003). *Financial Market Risk: Measurement and Analysis*. Routledge.
- Mandelbrot, B.B. (1971). When can price be arbitrated efficiently? A limit to the validity of the random walk and martingale models. *Rev. Econ. Stat.* 53:225-236.
- McMillan, D.G & Thupayagale, P. (2008). Efficiency of the South African equity market. *Applied Financial Economics Letters*, 4:327-330.



Morris, Q.; Vuuren, V. & Styger, P. (2009). Further Evidence of long memory in the South African Stock market. *South African Journal of Economics*, 77 (1):81 – 101.

Mukherjee, I.; Sen, C., & Sarkar, A. (2011). Long memory in stock returns: insights from the Indian market. *Int. J. Appl. Econ. Finance* 5:62-74.

Panas, E. (2001). Long memory and chaotic models of prices on the London Metal Exchange. *Resources Policy* 27:235-246.

Peng et al (1994). Mosaic organization of DNA nucleotides. *Phys. Rev.*49, pp. 1685-1689.

Siddique, S. & Silvapulle, P. (2001). Long-term memory in stock market returns: international evidence. *Int. J. Finance Econ.* 6:59-67.

Tolvi, J. (2003). Long memory and outliers in stock market returns. *Appl. Financ. Econ.* 13:495 – 502.