Naira-Dollar Exchange Rate Volatility Modeling Using Quadratic Moving Average Conditional Heteroscedasticity (QMACH)

Odunayo Magret Olarewaju[1], Timilehin John Olasehinde[2]

Abstract: This study investigates possible alternative modeling of Naira-Dollar exchange rate volatility in Nigeria. This paper compares the performance of the new model specification (QMACH) with the ARCH-GARCH that are already in existence in volatility modeling literature**.** The paper makes use of the monthly data on Naira-Dollar exchange rates from 1991 to 2016 which was sourced from the Central Bank of Nigeria statistical bulletin. In order to realize the aim of this study, anewly proposed Quadratic Moving Average Conditional Heteroscedasticity (QMACH) model was employed to investigate the volatility of Naira-Dollar exchange rate. The ADF unit root test reveals that the Naira-Dollar exchange rate return isstationary and this permits the usage of Quadratic Moving Average Conditional Heteroscedasticity (QMACH) methodology. The empirical analysis indicates that Naira-Dollar exchange rate volatility indeed follows the QMACH movement just like it follows both ARCH and GARCH movement. In comparison with ARCH and GARCH modeling, QMACH outperforms both as shownthrough the loglikelihood statistics and the information criteria.

Keywords: Exchange rate; volatility; ARCH-GARCH; QMACH

JEL Classification: D51

1. Introduction

In economics and finance parlance, an exchange rate which is also known as Forex rate between two currencies is the rate at which one currency exchanges for another .It is regarded as the value of one country's currency in relation to another currency.Exchange rate can either be fixed or floating in nature.The apex bank of a country dictates the fixed exchange rate while the floating exchange rate is driven by the forces of demand and supply. Exchange rate can also be categorized as the spot rate which is the current rate or a forward rate which is the spot rate adjusted for interest rate differentials.

The exchange rate plays a momentous role in any type of economic system as it directly affects all the macroeconomic variables .The effect of exchange rate on home price index, merchandised profitability and investment decision cannot be overemphasized. Rodrik (2007)opines that poorly managed exchange rates can be disastrous for economic growth while sustaining a relativelystable exchange rate is important in boostingeconomic growth.Due to its impact on business and the economy, it cannot be argued against that prospective investors and dynamic businessmen or entrepreneurs would prefera stable exchange rate to a volatile exchange rate. A hysterical fluctuation of exchange rate, whichoften results in continuous depreciation of the domesticcurrency, is considered volatile in theexchange rate terminology. Volatility of exchange rateinduces uncertainty and risk in investment decisions with subverting impact on the macroeconomic performance (Mahmood & Ali, 2011).Mordi (2006) notes that private sector agents are markedly concerned about the exchange rate volatility because of its asymmetricaleffects on their investments which may be capital gains or losses. Also, the

[1] Durban University of Technology, Department of Management Accounting, South Africa, Corresponding author: odunayofayigbe@gmail.com/drmagolar@yahoo.com.
[2] Ekiti State University, Department of Economics, Nigeria, E-mail: timmexdareal@gmail.com.

impact of exchange rate volatility on export and import cannot be underestimated. Exchange rate appreciationincreasesimport and reduces export while exchange rate depreciation would aid export and discourage import which tends to cause the shift from foreign goods consumption to domestic goods consumption.In addition, exchange rate plays crucial roles in Nigeria monetary policy because of its vital impact on the economy and trade.

Based on the assertions above, it can be seen that the measurement and predictability of exchange rate volatility is a priority for both the public and the private sector agents.

The modeling of financial time series volatility like exchange rate volatility has taken different forms and specifications over the decades. The ARCH (Engle, 1982) model and impressive arrays of variance specifications belonging to the same class of model (e.g. GARCH, EGARCH) have been used consistently over the years. Unarguably, these had achieved very successful empirical evidences. Nevertheless, several empirical studies seemto show that the performance of ARCH and its variants are not always appropriate (Ventosa, 2002).In an attempt to bridge the gap in the specification of models and estimation of parameters in modeling the exchange rate volatility of the Nigerian currency, this paper, investigates the characteristics of exchange rate volatility in Nigeria and adopts a new specification; the Quadratic Moving Average Conditional Heteroscedasticity (QMACH) model developed by Ventosa (2002) and the performance of this model is compared with the ARCH (1), ARCH (2) and GARCH (1,1) performance through the likelihood maximization and information criteria. Their various graphs will be shown to see the closeness of their estimatesto the traditional conditional variance which is the square of the residual from the AR (1) log return equation of exchange rate of naira per dollar. This paper is divided into introduction, survey of literatures, data methodology, empirical findings, conclusion and references.

2. Survey of Empirical Literatures on Naira-Dollar Exchange Rate Volatility Modeling

Olowe (2009) investigates the Naira-Dollar exchange rate volatility using monthly data .He employsgeneralized autoregressive conditional heteroscedasticity (GARCH) modeling technique and five of its (GARCH) variants with the assumption of residuals normality. His empirical investigation reveals persistency of Naira-Dollar exchange rate volatility. The result of the study further shows non-feasibility of leverage effect in Naira-Dollar exchange ratevolatility. He concludes his study that the asymmetric model of TS-GARCH and APARCH are the best in modeling naira-dollar exchange rate volatility.

Oloba and Abogan (2013) investigate the volatility of Naira-Dollar exchange rate in Nigeria. They employmonthly data spanning 1986-2001and they adopt exponential generalized autoregressive conditional heteroscedasticity (EGARCH) modeling technique in their study.Theirfindings show the presence of volatility in Naira-Dollar exchange rate during the period of their study.

Ajao and Igbekoyi (2013) investigate the determinant of real exchange rate volatility in Nigeria. They employ ECM-GARCH modeling technique. The GARCH (1,1) model was used to filter volatility while the ECM is used to investigate the determinants of exchange rate volatility .Their study reveal that trade openness, government expenditures ,interest rate volatility and a period lag of exchange rate are the significant determinants of exchange rate volatility in Nigeria.

3. Methodology

i. ARCH-GARCH Model

One of the basic assumptions of classical regression model is the constancy of the error variance overtime. This phenomenon is termed homoscedasticity. The otherwise of this case is termed heteroscedasticity. Also, it is logically assumed that the issue of heteroscedasticity is associated with a definite or set of regressors. However, it is possible that the variance of the error term changes over time rather than systematic with one of the regressors. This phenomenon is term ARCH (Autoregressive Conditional Heteroscedasticity).

According to Wang (2008), a stochastic process is called ARCH if its time varying conditional variance is heteroscedasticity with autoregression. The general specification of ARCH is represented as:

$$y_t = \beta_0 + \beta X_t + \varepsilon_t \qquad (1a)$$
$$\varepsilon_t \sim N(0, \sigma_t^2) \qquad (2a)$$
$$\varepsilon_t = v_t \sigma_t^2 \qquad (3a)$$
$$v_t \sim N(0, 1) \qquad (4a)$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 \qquad (5a)$$

Equation **(1a)** is the conditional mean equation and ***X*** is the vector of regressors which may include the lag(s) of the regressors and dependent variable. Equation **(5a)** is the conditional variance equation which is an ARCH (q) process where**q**is the autoregressive order of the squared residual. The optimal selection of **q** may be based on inspection or through information criteria. However, the major shortcoming of the ARCH (**q**) process is the infinite nature of the autoregressive squared errors and this will consume much degree of freedom and make the result from the model to become shaky or unreliable. In order to circumvent this problem, (Bollerslev, 1990) develops GARCH model which is the parsimonious representation of ARCH(∞). According to Wang (2008), a stochastic process is called GARCH if its time varying conditional variance is heteroscedasticity with both autoregression and moving average. The general specification of GARCH is represented as:

$$y_t = \beta_0 + \beta X_t + \varepsilon_t \qquad (1b)$$
$$\varepsilon_t \sim N(0, \sigma_t^2) \qquad (2b)$$
$$\varepsilon_t = v_t \sigma_t^2 \qquad (3b)$$
$$v_t \sim N(0, 1) \qquad (4b)$$
$$\sigma_t^2 = \alpha_0 + \sum_{1}^{p} \alpha_p \varepsilon_{t-p}^2 + \sum_{1}^{q} \alpha_q \sigma_{t-q}^2 \qquad (5b)$$

Equation **(5b)** is the GARCH(**p, q**) representation where **p** is the lags lengthfor the volatility which is the squared error while **q** is the lags' length for the conditional variance of the error.One of the advantages of GARCH over ARCH is parsimonious, i.e. less lagsare required to capture the property of time-varying variance in GARCH. In empirical applications, a GARCH (**1, 1**) model is widely adopted.

For the GARCH (**p, q**) process to possess a finite variance, the following conditionmust be met:

$$\sum_{1}^{p}\alpha_p + \sum_{1}^{q}\beta_q < 1$$

In commonly used GARCH (**1, 1**) models, the condition is simply$\alpha_1 + \beta_1 < 1$. Many financial time series have persistent volatility, i.e. the sum of $\alpha_p$and $\beta_q$is close to being unity. A unity sum of $\alpha_p$and $\beta_q$leads to so-called IntegratedGARCH or IGARCH as the process is not covariance stationary. However, this does not pose as serious a problem as it appears (Wang, 2008). According to Nelson (1990); Bougerol and Picard (1992); and Lumsdaine (1991), even if a GARCH (IGARCH) model is not covariance stationary, it is strictly stationary orergodic, and the standard asymptotically based inference procedures are generally valid (Wang, 2008).

ii. Estimation of ARCH-GARCH

Let Ω be the information set available at the time t we can use conditional densities:

$$e_t|\boldsymbol{\Omega}\sim \boldsymbol{N}(\boldsymbol{0},\boldsymbol{h_t})$$

This property can be used to define the likelihood functions:

$$\boldsymbol{l} = \frac{\boldsymbol{1}}{\boldsymbol{T}}\sum_{\boldsymbol{t=1}}^{\boldsymbol{T}}\boldsymbol{l_t}$$

$$\boldsymbol{l_t} = -{}^{\boldsymbol{1}}\!/_{\boldsymbol{2}}\,\boldsymbol{log h_t} -{}^{\boldsymbol{1}}\!/_{\boldsymbol{2}}\,\boldsymbol{log}\frac{\boldsymbol{e_t^2}}{\boldsymbol{h_t}}$$

$$\boldsymbol{l_t} = -{}^{\boldsymbol{1}}\!/_{\boldsymbol{2}}\,(\boldsymbol{log h_t} + \boldsymbol{log}\frac{\boldsymbol{e_t^2}}{\boldsymbol{h_t}})$$

e is the residuals obtained from the OLS estimate of the conditional mean ,while $\boldsymbol{h}$ is the conditional variance of the residuals obtained from the OLS estimate of the conditional mean .The initial parameters used in the maximization processes are obtained from the OLS estimate as they are consistent in nature.

iii. QMACH Model

QMACH model was developed by Ventosa (2002) through the inspiration of Volterra expansion. QMACH was developed in the spirit of ARCH but different from it as QMACH follows nonlinear moving average specification. In QMACH estimation, it is not necessary to impose conditions on the parameters to ensure the existence of all moments unlike GARCH estimation. The general specification of QMACH(q) is presented thus;

$$\boldsymbol{y_t} = \boldsymbol{\beta_0} + \boldsymbol{\beta X_t} + \boldsymbol{\varepsilon_t} \qquad (\boldsymbol{1c})$$

$$\boldsymbol{\varepsilon_t}\sim \boldsymbol{N}(\boldsymbol{0},\boldsymbol{h_t}) \qquad (\boldsymbol{2c})$$

$$\boldsymbol{\varepsilon_t} = \boldsymbol{v_t}\sqrt{\boldsymbol{h_t}} \qquad (\boldsymbol{3c})$$

$$\boldsymbol{v_t}\sim \boldsymbol{N}(\boldsymbol{0},\boldsymbol{1}) \qquad (\boldsymbol{4c})$$

$$\boldsymbol{h_t} = (\boldsymbol{\delta_0} + \sum_{\boldsymbol{i=1}}^{\boldsymbol{q}}\boldsymbol{\delta_i v_{t-i}})^2 \qquad (\boldsymbol{5c})$$

The unconditional variance of equation **5c** is calculated as;

$$E(\varepsilon^2) = \sum_{i=0}^{q} \delta_i^2$$

Equation **(1c)**is the conditional mean equation and X is the vector of regressors which may include the lag(s) of the regressors and dependent variable. The last expression **(5c)** shows the variance equation in QMACH form. The square term on the expression **(5c)** justifies the tagged name quadratic.$v_t$is the standardized residual and has zero mean and unit variance. The simpler specifications of the QMACH(q) are the QMACH(1) and the QMACH(2).They are presented below as equation (6) and equation (7).

$$h_t = (\delta_0 + \delta_1 v_{t-1})^2 \qquad (6)$$

$$h_t = (\delta_0 + \delta_1 v_{t-1} + \delta_2 v_{t-2})^2 \qquad (7)$$

The unconditional variance ofQMACH(1) and QMACH(2) are $\delta_0^2 + \delta_1^2$and $\delta_0^2 + \delta_1^2 + \delta_2^2$. These estimates can easily be computed after the estimation of the maximum likelihood estimation.

iv. Estimation of QMACH

The maximum likelihood technique works parallel with the arch estimation. Let Ω be the information set available at the time t we can use conditional densities;

$$e_t|\Omega \sim N(0, h_t)$$

This property can be used to define the likelihood functions;

$$l = \frac{1}{T}\sum_{t=1}^{T} l_t$$

$$l_t = -{}^1\!/_2\, log h_t - {}^1\!/_2\, log \frac{e_t^2}{h_t}$$

$$l_t = -{}^1\!/_2\, log h_t - {}^1\!/_2\, log v_t^2$$

$$l_t = -{}^1\!/_2\, (log h_t + log v_t^2)$$

The maximum likelihood estimates are obtained by any of the expression. **L-BFGS-M (Limited Memory BFGS)** algorithm is used and the initial values for the procedure are obtained from the OLS estimate as they are consistent in nature. The robust standard errordue to Wooldridge and Bollerslev was reported for the final estimates. Unlike the ARCH-GARCH case, for QMACH, if all the parameters are of opposite sign, there is no problem since the volatility equation is squared and provides exactly the same result (Ventosa, 2002).

4. Empirical Result

i. Descriptive Analysis

Table i: Summary Statistics of EXR and log (EXR)

	EXR	**Log(EXR)**
Mean	128.1213	4.670793
Median	134.5650	4.902047
Maximum	462.0300	6.135630
Minimum	10.87000	2.386007
Std. Dev.	69.83884	0.694094
Coefficient of variation	0.545099	0.148593
Skewness	1.576411	-1.424290
Kurtosis	8.566792	5.227519
Jarque-Bera	532.0830	169.9913
Probability	0.000000	0.000000
Sum	39973.86	1457.287
Sum Sq. Dev.	1516891.	149.8293
Observations	312	312

Source: Authors' computation (2017)

Table i shows the descriptive statistics of EXR and log (EXR). It can be shown that the variables contained 312 observations. Also, log (EXR) is negatively skewed while EXR is positively skewed. Both EXR and log(EXR) are leptokurtic,that is, greater than three. This reveals one of the properties of financial time series data. The standard deviation statistics shows that there is lesser variation in log(EXR) than EXR. The coefficient of variation statistics is computed to show the unitless dispersion comparison of EXR and log (EXR) and it shows that there is lesser variation in log (EXR) than EXR as revealed by the standard deviation statistics. The probability of the Jarque-Bera shows that none of the variables are normally distributed. This is another property of financial time series data as they are bound to have fat tails.

ii. Unit Root Test

Before one pursues formal tests, it is always advisable to plot the time series under study as it may reveal the integrating nature of the series. These variables (EXR and log (EXR) are examined graphically below.

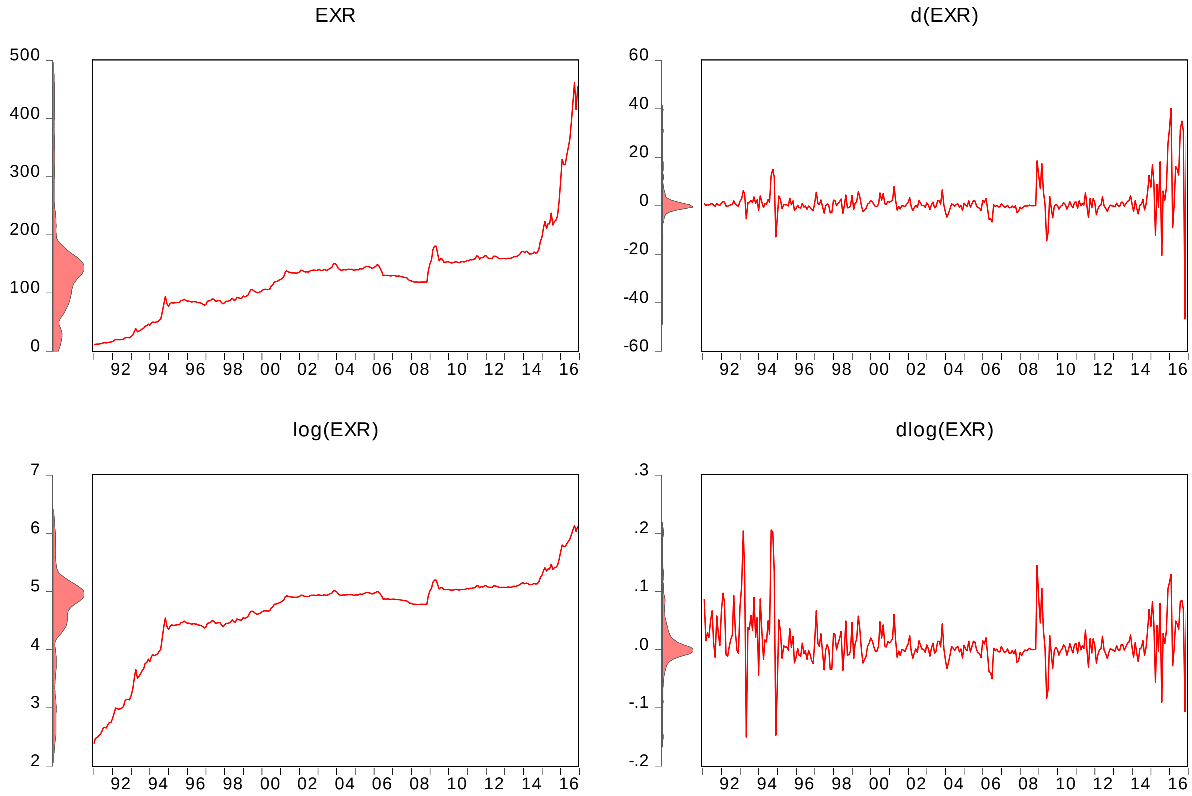

Figure i

Source: Authors' Computation (2017)

It can be shown from the table above that EXR and log(EXR) are both upward trended. There is no tendency for their mean reverting and variance constancy. (EXR) and dlog(EXR) look similar,however,d(EXR) shows diverging path towards the year 2014 which may affect the stationarity property. Log(EXR) shows similar hovering throughout the years. This suggeststhe mean reverting and variance constancy in exchange rate return. No statistical fact can be derived numerically from the graphical inspection of the variable in question. Based on this, ADF unit root test (Formal Test) is employed to investigate statistically the integration properties of EXR and its logarithm value.

Table ii. ADF Unit Root Test

	Level		First difference		
	ADF(c & t)	Prob	ADF(c)	Prob	Remark
EXR	0.4848	0.9993	-0.1751	0.9933	> I(1)
Log(EXR)	-3.0170	0.1292	-12.6690	0.0000***	I(1)

Source: Authors' computation (2017). Note: (**) (***) denotes significance at 10%, 5% and 1% respectively; c(Constant), t(Trend)*

The result of the ADF unit-root test is presented above. From the result, it can be shown that EXR is not stationary (as suggested by the graph) even after first difference and likely it possess a quadratic trend; I(2).Log(EXR) is I(1) at 5% and 10% level of significance. The stationarity nature of Log(EXR)has been suggested earlier by the graph above. The estimation of ARCH and GARCH required stationary data though not necessary for estimation QMACH(Ventosa, 2002) but it will be used for conventional sake.Inessence, exchange rate return (dlog(EXR)) will be used as the dependent variable as it meets the condition for estimation.

iii. Estimation of QMACH Model

Table iii. Dependent variable: dLog(EXR)

AR(1)-QMACH(1)				
Variable	Coefficient	S.E	t-stat	Prob.
Constant	0.0116	0.0046	2.522	0.0117**
dLog(EXR(-1))	0.4134	0.0926	4.462	0.0000***
Conditional Variance equation				
Constant	0.0528	0.0084	6.293	0.0000***
V(-1)	0.0182	0.0026	7.081	0.0000***
AR(1)-QMACH(2)				
Variable	Coefficient	S.E	t-stat	Prob.
Constant	0.0019	0.0016	1.211	0.2259
dLog(EXR(-1))	0.3178	0.0054	58.99	0.0000***
Conditional Variance equation				
Constant	0.0373	0.0044	8.998	0.0000***
V(-1)	0.0395	0.0044	8.998	0.0000***
V(-2)	0.1378	0.0140	9.825	0.0000***

Source: Author's computation (2017).

Note: (**) (***) denotes significance at 10%, 5% and 1% respectively*

Table iii depicts the estimation result of AR(1)-QMACH(1) and AR(1)-QMACH(2) model. It can be shown that the estimated coefficients of the AR(1)-QMACH(1) are all significant while only the constant coefficient in the AR(1)-QMACH(2) conditional mean equation is not significant. The significance of the coefficients of the conditional variance equation in the AR(1)-QMACH model shows the evidence that Naira-Dollar exchange rate volatility follows both QMACH(1) and QMACH(2) pattern. The unconditional variance for QMACH(1) is **0.003120** while for QMACH(2), it is **0.002949**.These revealagreement between the two models on the estimate of unconditional variance. 113

iv. Estimation of ARCH and GARCH Model

Table iv. Dependent variable: dLog(EXR)

AR(1)-ARCH(1)				
Variable	Coefficient	S.E	t-stat	Prob.
Constant	0.0034	0.0024	1.403	0.1605
dLog(EXR(-1))	-0.0327	0.1476	-0.2214	0.8248
Conditional Variance equation				
Constant	0.0005140	0.00016	3.087	0.0020***
RESID(-1)^2	0.9082	0.3843	2.364	0.0181**

AR(1)-ARCH(2)				
Variable	Coefficient	S.E	t-stat	Prob.
Constant	0.0026	0.0019	1.367	0.1716
dLog(EXR(-1))	0.1713	0.1960	0.8741	0.3821
Conditional Variance equation				
Constant	0.0003995	0.00021	1.927	0.0540*
RESID(-1)^2	0.4111	0.2174	1.891	0.0586*
RESID(-2)^2	0.565632	0.329468	1.717	0.0860*

AR(1)-GARCH(1,1)				
Variable	Coefficient	S.E	t-stat	Prob.
Constant	0.0030	0.0017	1.749	0.0803*
dLog(EXR(-1))	0.3238	0.0821	3.945	0.0000***
Conditional Variance equation				
Constant	8.54e-05	6.2e-05	1.367	0.1716
RESID(-1)^2	0.2416	0.1166	2.072	0.0382**
GARCH(-1)	0.7159	0.1361	5.259	0.0000***

Source: Author's computation (2017). Note: (**) (***) denotes significance at 10%, 5% and 1% respectively*

Table iv indicates the estimation result of AR(1)-ARCH(1),AR(1)-ARCH(2) and AR(1)-GARCH(1,1) model. It can be shown that only the estimated coefficients in the conditional variance of AR(1)-ARCH(1) are significant while none is significant in its conditional mean equation. Likewise, only the estimated coefficients in the conditional variance of AR(1)-ARCH(2) are significant while none is significant in its conditional mean equation. The case is different for the AR(1)-GARCH(1,1) model. In the AR(1)-GARCH(1,1) model, all the parameters estimated are significant both in the conditional mean and variance equation. This supports the theoretical and empirical evidence that GARCH model could model and predict volatility more accurately than the ARCH model. However ,the significance of the coefficients of the conditional variance equation in the AR(1)-ARCH-GARCH model shows the evidence that Naira-Dollar exchange rate volatility follows ARCH-GARCH pattern as well as that of AR(1)-QMACH model. The unconditional variance for ARCH(1) is **0.00560421**, for ARCH(2) it is **0.0171666** while for GARCH(1,1), it is **0.00201096**. There is little or no agreement between these models, however, in the estimates of their unconditional variance.

v. Model Performance Comparison

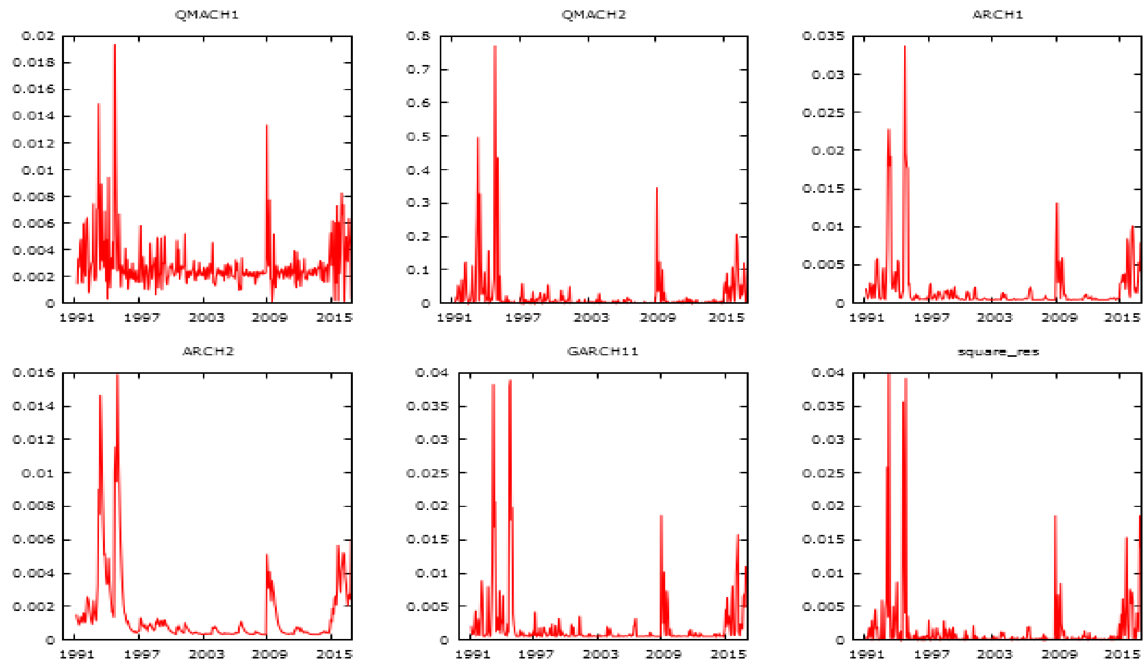

Figure 2.
Source: Author's computation (2017)

The graphs above show the squared residual from the AR(1) OLS estimate(this squared residual can be used to measure volatility), QMACH, ARCH and GARCH. The squared residual was used as the base comparison of volatility. It can be shown that QMACH(2) and GARCH(1,1) graphical estimates are very similar to the squared residual graph. This suggests likely behavioral performance of QMACH and GARCH modeling. In essence, GARCH(1,1) and QMACH(2) will forecast better than their lower respective counterparts. In order to investigate further in a formal way, there is need to cross check the log-likelihood statistics and the information criteria of the models considered so far.

Table v. Comparison of the Estimated Models

	QMACH(1)	QMACH(2)	ARCH(1)	ARCH(2)	GARCH(1,1)
logLikelihood	777.9687	755.0486	632.3488	646.2356	657.0807
Akaike info	−1547.937	-1500.097	-1254.698	-1280.471	-1302.161
Schwarz info	−1533.004	-1481.447	-1236.015	-1258.052	-1279.742
Hannan-Quinn info	−1541.967	-1492.640	-1247.229	-1271.509	-1293.199

Source: Author's computation (2017)

Tablevshows the performance statistics of the various conditional variance equations considered in this study.The likelihood statistics and the information criteria reveal thatARCH (2) outperforms ARCH(1)conditional variance model as expected. Likewise, the likelihood statistics and the information criteria reveal that GARCH(1,1) outperforms both ARCH(1) and ARCH(2) variance model. This result is not surprising as the GARCH model is expected to capture higher volatility effect than ARCH of any order. For the QMACH type conditional variance equation, QMACH(1) outperforms QMACH(2) as revealed by the likelihood statistics and the information criteria. However, it can be seen that the QMACH(1 and 2) conditional variance model outperforms both ARCH and the GARCH model. This finding coincides with the result of Ventosa (2002).

5. Conclusion

We have attempted to model Naira-Dollar exchange rate volatility using a newly proposed conditional variance specification, the QMACH model.It can be confirmed that QMACH specification fits as well as the ARCH and GARCH butQMACH has advantages by minimizing information losses and maximizing log likelihood than both the ARCH and GARCH and it does not necessarily require stationary data as that of ARCH and GARCH.It can thus be concluded that QMACH specification will have to compete with many variants belonging to ARCH class.

6. References

Ajao, M. G., & Igbekoyi, O. (2013). The determinants of real exchange rate volatility in Nigeria. *Academic Journal of Interdisciplinary Studies, 2*(1), 459-471.

Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model. *The review of economics and statistics*, 498-505.

Bougerol, P., & Picard, N. (1992). Stationarity of GARCH processes and of some nonnegative time series. *Journal of econometrics, 52*(1-2), 115-127.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica: Journal of the Econometric Society*, 987-1007.

Lumsdaine, R. L. (1991). Asymptotic properties of the maximum likelihood estimator in GARCH (1, 1) and IGARCH (1,l) models. *V Princeton University Department of Economics Manuscript*.

Mahmood, I., & Ali, S. (2011). Impact of exchange rate volatility on macroeconomic performance of Pakistan. *International Research Journal of Finance and Economics, 64*, 1450-2887.

Mordi, C. (2006). Challenges of exchange rate volatility in economic management in Nigeria. *The dynamics of exchange rate in Nigeria*, 17-25.

Nelson, D. B. (1990). Stationarity and persistence in the GARCH (l, l) model. *Econometric Theory, 6*(3), 318-334.

Oloba, O., & Abogan, O. (2013). Exchange Rate Volatility in Nigeria: Evidence from a Parametric Measure. *Australian Journal of Business and Management Research, 3*(5), 12.

Olowe, R. A. (2009). Modelling naira/dollar exchange rate volatility: application of GARCH and assymetric models. *International Review of Business Research Papers, 5*(3), 377-398.

Rodrik, D. (2007). The real exchange rate and economic growth: theory and evidence: Citeseer.

Ventosa, D. (2002). *A proposal for a new specification for a conditionally heteroskedastic variance model: the Quadratic Moving-Average Conditional Heteroskedasticity and an application to the D. Mark-US dollar Exchange Rate*. Retrieved from

Wang, P. (2008). *Financial econometrics*: Routledge.