# The Proportionality Hypothesis in Capital Theory: an Assessment of the Literature

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***Abstract***

*It is found that the hypothesis of a constant replacement investment capital stock ratio has several fundamental shortcomings. It conflicts with most of the available theoretical and empirical evidence. It is alien to researchers in other fields of economics and related areas; and, perhaps most importantly, it has restrained progress in economic theory and econometric applications based on more realistic conceptualizations of the time structure of capital. On these grounds it is concluded that its abandonment is long overdue.*

***Keywords:*** *Capital longevity, replacement, depreciation, maintenance, utilization, obsolescence.*



## 1. Introduction

Once durable goods are put in place, in the overwhelming majority of cases their earning capability starts to decline. This happens for many reasons. One is the intensity with which they are used, because frequently it is responsible for their wear and tear or physical deterioration. Another is that all durable goods are designed and built for normal usage under certain conditions of maintenance; so if owners cut corners with regard to manufactures recommendations for proper maintenance, the quantity and at times the quality of their services decline. Lastly a third reason is that with the passage of time older durables become economically inferior because there appear newer ones that are able to produce the same amount of services with less resources, since they embody the most recent advances in science and technology. However, in as much as the owners of durables have significant control over these and other influences, to transform the non-stationary replacement problem that Preinreich (1940) had posed, Terborgh (1949) introduced initially two simplifications. These were that the operating costs of durables in place increase and that the operating costs of newer vintages decline at constant rates per unit of time. As a result, he did managed to derive the optimal useful life of durables in the steady state of a perpetual stream of reinvestments, but at the cost of quashing the effects of utilization, maintenance and technological obsolescence on the processes of replacement and scrapping. The field remained in the above state until Smith (1960) revisited it in a truly magnificent contribution to the theory of capital using enterprise. As Terborgh did over a decade earlier, he continued to approximate the operating cost and salvage value functions involved in the perpetual replacement problem with linear forms. But his modeling of the process by which market and engineering factors combine to reduce the efficiency of capital services was ingenious. In particular, he hypothesized that these factors work through two main channels. The one of them is the useful live and other is the multitude of non-age related forces that are responsible for the normal wear and tear of durable goods. Thus, to capture their impact on capital services, he postulated that:[1]

$$C=(m+bT)x+\left(\hat{\delta}+\alpha T+q/T+rq\right)K, \qquad (1)$$

where the various symbols have the following meanings: $C=$ total current cost; $x=$ variable input like the amount of energy consumed; $K=$ stock of durable goods; $T=$ useful life of the stock of durable goods; $m=$ unit cost of variable input; $q=$ purchase cost of the stock of durable goods; $b=$ age related rate of deterioration in the usage of the variable input; $a=$ age related rate of deterioration in the services from the incumbent durables due to embodied technological change in newer vintages; $r=$ a constant rate of interest, and $\hat{\delta}$ = a constant non-age related proportional rate of deterioration in capital services. Now from (1) it is clear that the so-called proportionality hypothesis was adopted for the first time by Smith (1960, p.166) and it was motivated by his concern to account for the impact on capital services of the numerous non-age related factors. By contrast to the above, in a very influential paper that appeared three years later, Jorgenson (1963) stipulated in different but equivalent terms that:

$$C=m\,x+\left(\hat{\delta}+rq\right)K=mx+q(r+\delta)K. \qquad (2)$$

Clearly this conceptualization constituted a major break from all past endeavors in this area, which centered primarily on the role of useful life in the services of the stock of durables. Therefore, the justifications that warranted this far-reaching departure from the received theory were of particular importance. In this regard, here is how Jorgenson (1963) supported his assertion that the rate of deterioration of capital services is a constant proportion $\delta$ of the stock of durables involved:

> "… The justification for this assumption is that the appropriate model for replacement is not the distribution of replacements of a single investment over time but rather the infinite stream of replacements generated by a single investment; in the language of probability theory, replacement is a recurrent event. It is a fundamental result of renewal theory that replacements for such an infinite stream approach a constant proportion of capital stock for (almost) any distribution of replacements for a single investment and for any initial age distribution of capital stock. This is true for both constant and growing capital stocks…" (p.251).

Thus, in view of its apparent grounding in renewal theory and the determination with which Jorgenson (1965) returned with further details to demonstrate its validity, the proportionality hypothesis gained quickly respectability and eventually came to dominate theoretical and applied economics. However, in the meantime, there accumulated voluminous evidence, which raised serious doubts about its underpinnings. Some of this evidence emanated from theoretical considerations, Some other derived from empirical studies of replacement investment and the price-age profiles and scrappage of various types of durables; and still some other sprung from the nature of theories and practices in several

[1] Note that the symbols used below may not correspond to those employed in the original sources.

neighboring scientific fields. Yet this evidence has not been assessed so far in any systematic way and as a result the objections to the proportionality hypothesis have not received a fair hearing. For this reason the goal here is to conduct a meticulous review of the relevant literature, so as to gauge whether this hypothesis is valid or not. At the end of this endeavor it is concluded first, that the evidence is overwhelmingly against the proportionality hypothesis and, second, that because of its adverse implications for economic theory and policy its abandonment is long overdue. The paper is organized as follows. Section 2 reviews the theoretical literature. It does so by focusing on the evidence from the theories of replacement, economic growth and business cycles, industrial organization, and other scientific fields like operations research, operations management, capital budgeting, and accounting. Section 3 reviews three groups of empirical literature. These include studies of replacement investment, economic depreciation, and scrappage. Section 4 provides an overall assessment of the available evidence together with certain methodological remarks, and, lastly, Section 5 closes with a synopsis of the main findings and conclusions.

## 2. The theoretical evidence

This may be classified into direct and indirect. The direct evidence comes from research in the areas of replacement investment and macroeconomics, including economic growth and business cycles, whereas the indirect evidence stems from research in industrial organization, capital budgeting and other neighboring scientific fields. This section highlights the standing of the proportionality hypothesis in the theoretical literature.

### 2.1 Indications from the theory of replacement investment

As it was pointed out above, even though Smith (1960) introduced the proportionality hypothesis for purpose of analytical convenience, Jorgenson (1963, 1965) was the first to elevate it to a proposition of general validity by asserting that it constituted a fundamental result of renewal theory. With the exception of certain isolated attempts, for several years this claim went unchallenged. In particular, while the empirical evidence that was reported did cast doubts about its validity, a poof that refuted it by recourse to theory was missing. This situation lasted until Feldstein and Rothchild (1974) in a widely cited paper argued that:

> "…Except for numerical accidents of no economic interest, … a constant replacement ratio will emerge only if either: (i) each piece of equipment is subject to output decay at the same constant exponential rate or (ii) the entire capital stock, and therefore both net and gross investment, grow at a constant exponential rate" (p. 397).

Based on the theorems from which both arguments derived, the balance of professional opinion was expected to tip on the side of the conclusion that the proportionality hypothesis lacked theoretical foundations. But in the same year Jorgenson (1974) provided a step-by-step counterproof by showing that, if the coefficients of output efficiency from the one vintage of an investment to the next follow the geometric distribution, their series converge to a constant. So the answer to the question as to who had won the debate boiled down to whether there existed convincing reasons to expect that the coefficients in question would follow the geometric distribution or not. Crucial in this regard is the role of utilization, maintenance and embodied technological change. For if the owners of an investment vary deliberately the respective policies in response to changing market and technological conditions, most likely the coefficients of output efficiency from the one vintage to the next will vary and thus they may not follow the geometric distribution. Therefore, the issue can be resolved by looking into the relevance of these processes for replacement investment. To this end it suffices to mention that by the mid 1970s, i.e. when the debate broke open, there existed already a large volume of theoretical and empirical literature establishing that utilization, maintenance and repair costs, and obsolescence determine significantly the deterioration of capital services, and hence replacement. Just to cite a few examples, Smith (1957) had ascertained this linkage in the case of trucks; Thompson (1968) and Kamien and Schwartz (1971) had highlighted the relationship of maintenance to the sale date of a machine under conditions of stochastic failure and deterioration, respectively; Bitros (1976a, 1976b) had found that the decisions of maintenance and repair expenditures, utilization, and retirements from and gross additions to the capital stock are interrelated in a statistically significant way; Taubman and Wilkinson (1970) had shown how utilization affects gross investment via replacement, whereas the exhaustive survey by Winston (1974) regarding capital utilization and idleness left no margin of uncertainty that output efficiency varies with utilization, and Malcomson (1975) had demonstrated how serious was the omission of obsolescence from the analysis of replacement investment. In short, all indications at the time were that the distribution of output coefficients from one vintage to the next would follow the geometric distribution only by numerical accident. Consistent with this view were also the findings by most other replacement investment theorists. For a few examples, consider first the results obtained by Nickell (1975). In the concluding remarks to the section where he investigated the implications of a constant scrapping age to the ratio of replacement investment to capital stock he writes:

> "…It is perhaps worth mentioning that the above analysis indicates that the conditions under which the replacement/capital ratio is constant are very restrictive. It therefore seems very unlikely that it would be constant in reality" (p. 63).

Next, take the widely acclaimed study by Rust (1987), which focused on the relationship of maintenance of bus-engines to the timing of their replacement. By setting up a stochastic dynamic programming model of bus-engine replacement and testing it with monthly data from 104 buses over a 10 year period, he found that mileage and maintenance and repair expenditures explained most of the variance in the decisions of bus-engine replacement. Finally, it is worth noting that in Bitros and Flytzanis (2005, 2007) and Bitros, Hritorenko and Yatsenko (2007) we have traced the influences that reinvestment opportunities exercise on the decisions to replace or scrap under active utilization and maintenance policies, as well as embodied technological change. In conclusion, to derive the proportionality hypothesis from renewal theory it would require adopting several heroic assumptions. That is it would be necessary to postulate that the coefficients of output efficiency from one vintage of an investment to the next are invariant with respect to utilization, maintenance, technological change, and the reinvestment opportunities that market conditions afford to enterprises. But doing so would amount to imposing on the theory of capital policies overly unrealistic restrictions that preempt significantly its explanatory power.



### 2.2 Views from macroeconomics, including the theories of economic growth and business cycles.

A look at today's advanced textbooks in the captioned fields of economics would create the impression that a consistent economy-wide capital stock does exist and that its replacement takes place at a constant proportional rate. But from Bitros (2008) it turns out that a) the proportionality hypothesis is not indispensable in the construction of such an aggregate, and b) in recent years more and more economic theorists in the above areas prefer to adopt models based on non-exponential representations of the decay of durables goods. Hence, the significance of the following key literature favoring the decoupling of the theory of capital from the proportionality hypothesis can hardly be overstressed. In two significant papers Miller (1990) and Barnhart and Miller (1990) assessed the proportionality hypothesis from a theoretical and empirical point of view and made a strong case for its abandonment. Two researchers who did so in the area of macroeconomics in the early 1990s were

Cooper and Haltiwanger (1993). By focusing on the way in which the state of the economy influences the decision on machine replacement at the firm level, they were able to show that replacement occurs near the end of downturns and just prior to upturns, i.e. at the time when the resource cost of replacement is lowest and the benefits highest. But this evidence was not an isolated incident. For in the following years there appeared a barrage of contributions in which the decay of durables was an economic, not an engineering process. To appreciate the shift that took place, consider a representative of such research efforts. Cooley, Greenwood and Yorukoglu (1997) analyzed the decision to replace old technologies with new ones in a vintage model of growth and found that the transitional dynamics differed markedly from the standard neoclassical growth model. Cooper, Haltiwanger and Power (1999) investigated the implications for aggregate investment of the apparent lumps and bumps of investment at the plant level and found that "… the behavior of aggregate investment can be highly dependent on the cross-sectional distribution of the age of capital stock" (p.921). Licardo and Puch (2000), Boucekkine and Tamarit (2003) and Chatterjee (2005) highlighted the implications for economic growth and business cycles when deterioration is introduced as a function of utilization and/or maintenance; and last, but not list, Gylfanson and Zoega (2002) placed the emphasis of their analysis on the effects of obsolescence and durability. In short, from these studies, as well as the voluminous literature to which they refer, it follows that the outlook in these areas of research has turned against the proportionality hypothesis.

### 2.3 Evidence from the theory of industrial organization

While the debate about the proportionality hypothesis raged among capital and investment theorists, another group of researchers working independently investigated the factors that determine the durability of durable goods. To be sure this literature was not concerned with the question when is it optimal to discard or replace a durable. But since a more durable good would last longer than an identical good of lesser durability, the two goods could not be expected to deteriorate at the same exponential rate, as the proportionality hypothesis would predict. The reason behind that if, ceteris paribus, the two durables deteriorated at the same exponential rate, the demand for the more durable and presumably more costly good would case and only the less durable would be offered. Hence, this literature had a crucial implication for the issue under consideration. This is that the amount of durability built into producer's durables is not a technological datum but an attribute determined by market forces. More specifically, in the 1960s the model presented by Kleiman and Ophir (1966) established that under perfect competition a rise in the interest rate reduces durability, increases the number of units produced by the manufacturing firms, but may either increase or decrease the total number of units produced by the industry. Then in the 1970s Swan (1972, 1977) Coase (1972), Barro (1972), Schmalensee (1974), Parks (1974, 1979) and others, investigated the relationship of market structure to durability and maintenance. From their studies emerged several results. One was that, irrespective of whether a monopoly produces durables of higher or lower durability than the firms under perfect competition, the structure of the market does influence the amount of durability produced. Another was that the nature of ownership affects durability. In particular, if manufactures chose to sell rather than rent their durables, the durability they build into them would be different. Last, but not least, was the result that the ability to change the useful lives of durables through maintenance influences the choice of durability at the time of their production. Therefore, this literature left no doubt about the endogenous nature of durability and hence the rate of deterioration of producers durables. Moreover, other research efforts parallel to the above reinforced this view even further. Bulow (1986), Rust (1986) and Waldman (1993) added significantly to the results that had been achieved earlier by Swan (1972) in the front of planned obsolescence. Contrary to the views held by neoclassical theorists, who insist on the like-for-like perpetual inventory replacement of capital goods, Mann (1992) showed that when used durables are relatively good substitutes for new ones, a durable goods monopolist is better of repair market structure on the choice of durability; and a sizable group of researchers focused on the intricacies introduced in the analysis of durability by normal hazard and adverse selection in the selling or renting of durables by their manufacturers.

### 2.4 The proportionality hypothesis in other fields

Theorists in other fields neighboring to economics have ignored the proportionality hypothesis altogether. Because to those who know the importance attached by firms to maintenance and repair, shift work and various other operating and capital policies, its implication that economic agents do not control the output deterioration of their durables is conceptually misplaced and observationally counterfactual. To ascertain it, consider the following indicative summaries.

***Operations research***

In the period during which Jorgenson (1963, 1974) launched and defended the proportionality hypothesis, in the literature of operations research there appeared several surveys. Dean (1962) reviewed the literature on replacement theory and noted that the subject involved two problems, i.e. a) determining the optimum point in time or cumulative usage to replace, and b) choosing the best available equipment to be purchased for replacement. Moreover, he classified the various studies into three categories, i.e. a) the replacement of durables that deteriorate, b) the replacement of durables that fail, and c) the mathematics of failure. Three years later McCall (1965) reviewed the literature on the maintenance of stochastically failing equipment and shortly thereafter Jorgenson et al. (1967) summarized the literature on optimal replacement policy for both deteriorating and failing equipment. Lastly, shortly thereafter, Pierskalla and Voelker (1967) reviewed the literature regarding maintenance models of deteriorating systems. Having gone through all these surveys rather carefully, I failed to find even a single reference to the proportionality hypothesis. Hence, given that a) all known replacement models at the time aimed at the determination of the optimal useful life of equipment, and b) the primary focus was on the reliability, maintainability, and reparability of equipment and systems therefore, I surmised that the hypothesis under consideration is alien to operations research analysts. To make sure that this is the case, consider the process of replacement as perceived by experts in engineering economics. Helpful in this regard is the analytical framework put forth by Fraser and Posey (1989). From this it follows that the replacement decision in all its phases is dominated by economic, not engineering considerations. In particular, the emphasis is placed on the process of comparing the present values from alternative replacement investments. But as the studies by Wang (2002), Hartman (2004), and Dobbs (2004) confirm, in this process operations research theorists stress the importance of maintenance and repair, the intensity of utilization, and various channels of uncertainty. Furthermore, in the context of this literature, two significant results are worth noting. The first of them is due to Howe and McCabe (1983), who demonstrated that the optimal useful life of a durable depends on the available reinvestment opportunities. If the re-investment opportunities are expected to continue ad infinitum, the infinite-cycle replacement model that leads to a constant economic life over all investment-cycles may be a good approximation.[2] On the contrary, if the reinvestment opportunities are not expected to repeat or if due to bounded rationality or other reasons enterprises behave as if reinvestment opportunities will not repeat, the abandonment or scrapping model would be appropriate. In this event, as shown in Bitros (2007), the optimal useful life would be higher than that computed from the replacement model, and hence the straight-line replacement investment opportunities. In Bitros and Flytzanis (2005) we found that the number of reinvestment

[2] As a matter of fact, if the horizon of re-investment opportunities is allowed to go infinity, one can show that the useful lives in the series of investment cycles converge to a constant. This proves the optimality of the equal life policy.

cycles and the useful lives of durables in each one of them depend on the shape of the revenue and salvage value functions and the market and engineering parameters that enter through them. Thus we established that, irrespective of whether one adopts a replacement or scrapping model, it is forces over which economic agents have significant control that determine the optimal useful lives of durables.

***Operations management***

According to the theory of reliability and maintenance, as expounded, say in Jardine and Tsang (2006), firms take replacement decisions on the basis of purely economic calculations. Critical in these calculations are the comparisons of the cost of maintenance as a percentage of the estimated replacement value. The latter may be calculated more or less accurately by revising past information in the light of current quotations from the market. But the answer to the question what is optimal for a firm in a particular industry to spend for maintenance depends on the objectives that it seeks to accomplish and the resource constraints that it faces. For if, for example the firm is willing to accept at various rates of utilization more down time for its productive facilities relative to its competitors, the firm may be able to economize on the resources that it devotes to maintenance.

***Capital budgeting***

According to Fabozzi (1978), among other tasks, capital budgeting deals with the identification, analysis and selection of investments in the following four areas: 1) the replacement of productive facilities and equipment, either because they have been worn-out or because they have become technologically obsolete; 2) the addition of facilities and equipment for the purpose of expanding the productive capacity in the existing product lines; 3) the acquisition of the necessary facilities and equipment in order to launch new products and services, and 4) the upgrading and modernization of existing facilities and equipment. While pursuing these objectives a central issue that arises invariably concerns the useful lives of durables and if one following the proportionality hypothesis recommended the adoption of infinite useful lives, that one would be considered either ignorant or suspect. For this reason, capital budgeting experts adopt finite useful lives by striving to balance between those based on past experiences and those that can be computed with the help of appropriate models developed by theoretically minded researchers in economics and operations research.

***Accounting***

Depreciation is the amount of value that producer durables lose from the one period to the next due to usage, technological obsolescence and other reasons. From the accounting point view there are two approaches for reckoning depreciation, i.e. those of book depreciation and tax depreciation. Book depreciation is employed by firms to prepare financial statements to their stockholders and other related people, whereas tax depreciation is used for the purpose of calculating taxes. So, if we ignore tax depreciation, which must conform to the useful lives of durables that are mandated by the tax authorities, the question is how should firms calculate book depreciation. The answer given by financial theorists is that firms should approximate as best as possible the effects of utilization, maintenance, technological obsolesce and other relevant factors to the useful lives of their durables. In practice firms do so by employing one of the following methods: straight line (SL), declining balance (DB), double-declining balance (DDB), sum-of-year's digits (SOYD), and units-of-production (UOP). Occasionally they combine the DDB and the SL methods to match the book value of durables with their salvage value at the end of their depreciable life. However, irrespective of the particular method that they adopt, accounting correctly for book depreciation entails an effort to compute the changes in the useful lives of durables that are brought about by changes in the aforementioned determinants. This implies in turn that accounting for book depreciation is driven by economic forces.

### 3. The proportionality hypothesis in the empirical literature

Hulten and Wykoff (1996) reviewed the criticisms of the proportionality hypothesis by economic replacement theorists and suggested that their arguments may be wrong because of a fallacy of composition. That is, they may be right with regard to the deterioration of individual producer durables, but when aggregating short-lived and long-lived assets and performing econometric estimations, the deterioration looks nearly exponential. This argument takes for granted that the available empirical evidence from such aggregate tests contradicts the objections of economic replacement theorists. But is it true? According to the presentation below the answer is negative.

#### 3.1 Evidence from replacement investment

Feldstein and Foot (1971) were among the first to raise doubts the validity of the proportionality hypothesis from an empirical point of view. Using annual data over the 1948-1968 period from the McGraw-Hill *Survey of Business Plans for New Plants and Equipment*, in conjunction with the U.S. Department of Commerce series of planned gross investment, they found that the aggregate replacement/capital stock ratio varied significantly under the influence of conventional economic forces. But two years later Jorgenson (1974) evaluated the consistency with which they had constructed the critical variables in their model and concluded that:

> "…Feldstein and Foot have not successfully avoided the necessity for direct observation of both replacement investment and capital stock in studying the validity of the geometric approximation to the replacement distribution…"

As it would be expected, this verdict undermined seriously the credibility of Feldstein and Foot results. However, the criticism did not apply to Eisner (1972), where the variables of both replacement investment and capital stock had been constructed consistently using data at the firm level from the same survey. Hence Eisner's finding that expenditure planned for replacement and modernization: a) were not a constant proportion of capital, and b) related to changes in past and expected sales, previous depreciation charges and profits, corroborated strongly those obtained by Feldstein and Foot. Moreover, since they were already on record and Jorgenson (1974) did not make even a passing reference to them, at least Eisner's results provided solid empirical evidence against the proportionality hypothesis.



#### 3.2 Evidence from economic depreciation

In a very influential paper Hulten and Wykoff (1981) investigated the evolution of price-age profiles for vintages from four classes of commercial and industrial structures using actual transaction prices. In doing so they regressed first the prices of the four structures on their ages and time by applying the Box-Box power transformation. They found that, with the possible exception of factories, the depreciation rates varied significantly, particularly in the early years. Then they regressed the logarithm of the fitted prices from the above regression on the ages and time and found an average depreciation rate, which they called Best Geometric Approximation (BGA). Finally, by drawing on these results they concluded that:

> "…a constant rate of depreciation can serve as a reasonable statistical approximation to the underlying Box-Cox rates even though the latter are not geometric." (p.387).

However, according to Miller (1990) and Barnhart and Miller (1990), if the BGA were used to calculate perpetual inventory measures of capital stocks, the errors between the BGA rate and the Box-Cox rates would prohibit the estimation of such standard parameters as elasticities of substitution, rates of productivity growth and biases in technical change; The reason being that conventional statistical methods estimate these parameters from covariances of asset prices with other variables. Hence, the conclusion that Hulten and Wykoff derive from their results is unjustified. The same criticism applies also to the results obtain by Hulten, Robertson and Wykoff (1989) as well as to the state-of-the-art papers by Prucha and Nadiri (1996) and Nadiri and Prucha (1996). To this effect, consider for example the latter one. Starting with some initial values of the stocks for physical and R&D capital in the U.S. Total Manufacturing sector, what these authors do is that they estimate simultaneously input demand functions for labor and materials in which the corresponding rates of depreciation and capital stocks are estimated so as to be consistent with the series of gross investment. Hence the estimated depreciation rates are averages that differ from those that apply in each year of the sample. As a result the variances of the depreciation rates will be correlated with the variances of the other variables in the estimated input demand functions, thus making the estimation of the desired parameters impossible. Moreover, the evidence from the above studies is highly precarious and uncertain for yet another reason. To highlight it, consider the results obtained by Nelson and Caputo (1997). These authors adapted the model presented by Parks (1979) in the light of the flexible functional form introduced by Hulten and Wykoff (1981) to explain the depreciation rates of two types of aircraft over four five year periods from 1971 to 1991. From the meticulous tests they run they found that, even though the rates of depreciation implied by the Box-Cox rates were not geometric, they could be approximated reasonably well by a constant rate of depreciation. This finding confirmed the results of Hulten and Wykoff (1981). But at the same time it turned out that maintenance expenditures related negatively to depreciation rates. This is turn implied that the price-age profiles shifted upwards (downwards) as maintenance expenditures increased (declined). Therefore, since the depreciation rate shifted every time maintenance changed, the depreciation rate could not be possibly geometric. That is why the authors concluded that depreciation rates respond systematically to key economic variables.



### 3.3 Evidence from scrappage

Under the proportionality hypothesis producer durables are predicted to remain in production ad infinitum without scrappage ever taking place. This prediction is patently counterfactual and the hypothesis survives only because those who support it argue that it holds as an approximation. But if as predicted by the models, say, of Taubman and Wilkinson (1970) and Schmalensee (1974), the deterioration of producer durables varies with the intensity of utilization, maintenance and others economic variables, then not only will we observe scrappage but also their owners may have significant discretion in determining their deterioration. This prospect, in conjunction with the realization that scrappage constitutes a sizeable component of replacement investment, led empirically minded researchers to inquire whether the scrappage rate related to such variables in a statistically significant way; For if it did, in all probability the same would be true with respect to the rate of replacement. To this effect Walker (1968) investigated the determinants of auto scrappage. He found that the deviations from an age determined trend in the auto scrappage rate were very well explained by the rate of turnover in automobile ownership and the level of used car prices relative to costs of representative car repair prices. However, because of various data limitations this evidence was viewed as tentative. For thid reason in Bitros and Kelejian (1974) we run a test based on high quality data from the electricity generating capacity in the United States. The results enabled us to conclude that:

> "…, a component of the replacement ratio, namely, the scrappage ratio, is significantly related to such economic variables as gross investment, maintenance expenditures, and the interest rate. Therefore the replacement rate should be also related to these variables" (p. 277).

Since then the literature on scrappage has been enriched significantly. For example, Cowing and Smith (1977) refined further on our data from the electric utilities in the United States and with their estimates re-iterated the above conclusion. Parks (1977) revisited the determinants of scrapping rates for postwar vintages of automobiles and found that the probability of a car to be scrapped related significantly to such variables as its age, relative repair costs, and various characteristics of durability. Lioukas (1982) extended further the results on scrappage by electric utilities in the United Kingdom by including in the estimations retirement backlogs. Cockburn and Frank (1992) investigated the retirement of oil tankers and established that it is driven by markets conditions. Last but not least Goolsbee (1998) researched the retirement of airplanes from the fleet of Boeing 707s and found that fuel costs, the business cycle, the cost of capital and firm financial performance are very important factors for capital retirement decisions. In short, I am not aware of a single study that has looked into the determinants of scrappage that has not shown that this major component of replacement investment is not related systematically to key economic variables.

**4. Overall assessment and implications**

With the exception of Jorgenson (1963, 1965, 1966, 1974), the view of replacement theorists is that the conditions for a constant replacement/capital stock ratio are highly restrictive and unlikely to hold in reality. In the area of economic growth and business cycles the hypothesis is being abandoned in favour of an economic theory of replacement. All models in industrial organization show that how sturdy producer durables are built is decided at the time of their production on the basis of economic criteria and eventually such deliberate economic processes as the intensity of utilization and maintenance determine their useful lives; and last, but not least, the implication that firms cannot affect the manner in which their durables decay is completely alien to the modes of thinking in neighboring fields like operations research, operations management, capital budgeting and accounting. Consequently, the proportionality hypothesis lacks theoretical foundations. Now, if one subscribed to the position of ultra empiricists, according to whom scientific propositions do not need to derive from some system of axioms, this finding would not be damaging. After all what would matter in this event would be whether the proposition was grounded firmly in empirical evidence. But as it was found above the empirical evidence is also overwhelmingly against the proportionality hypothesis. In particular, the replacement investment/capital stock ratio varies over the business cycle under the influence of key economic variables. Contrary to the claims based solely on the research work by Hulten and Wykoff and their associates, the age-price profiles of durables do not support the view that depreciation rates are geometric; and, lastly, the scrappage rate, which is main component of the replacement rate, is determined to a significant extent by market forces. In conclusion, the hypothesis in question should be abandoned. One reason for doing so springs from the finding that it is conflict with the available theoretical and empirical evidence. But this is hardly the only one. Another reason emanates from the need to preserve the uniformity of science. For it is a profound contradiction for replacement theorists in economics to insist on a hypothesis that is patently unacceptable to economists and research scientists in other neighboring fields. Still a third reason is because of the advances that can be achieved in economic theory and econometric applications by returning to a theory of replacement investment centered in the determinants of the useful lives of durables. Indicative of how powerful the theoretical advances might turn out to be is the progress that has been achieved already in the fronts of economic growth and business cycles, where the adoption of the proportionality hypothesis has retreated. However, the returns in terms of precision and robustness can be expected to be even higher in growth accounting, productivity studies, and various other econometric applications, where presently researchers employ estimates of capital stocks based on the perpetual inventory method.[3]

**5. Summary of findings and conclusions**

The proportionality hypothesis has two great advantages: a) it facilitates the construction of models that result in simple and elegant solutions, and b) it permits aggregation and measurement of capital stock series through the perpetual inventory method.[4] However, it has several fundamental shortcomings. That is, it conflicts with most of the available theoretical and empirical evidence. It is alien to researchers in other fields of economics and related areas; and, perhaps most importantly, it has restrained the development of theories and econometric applications based on more realistic conceptualizations of the time structure of capital. For these reasons its abandonment is long overdue.

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[3] To appreciate the differences that they may result in these areas from the new age of capital centered research approach, compare the study by Jorgenson (1966) with those, for example, of McHugh and Lane (1987) and more recently Whelan (2005).
[4] Here is how Hulten et al. (1989) expressed the usefulness of the proportionality hypothesis: "We have found that a major event like energy crises, …did not in fact result in a systematic change in age-price profiles. This lends confidence to procedures that assume stationarity in order to achieve a major degree of simplification (and because of nonstationarity is so difficult to deal with empirically). Or, put simply, the use of a single number to characterize the process of depreciation (of a given type of capital asset) seems justified in light of the results…" (p. 255).

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