Proposal of a Semi Fuzzy Poverty Index

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Abstract. Certainly poverty is a phenomenon easy to understand and to describe, but difficult to measure and to determine. In fact, to measure poverty, we are first in front of the problem of choice of the threshold which depends itself on the choice of adapted approach, then in the choice of the indicator which must be faithful, and reflects clearly the real state of the population in study, in aim to optimize the planning of socio-economic policies authorizing the reduction of the poverty intensity.

This paper aims to avoid part of these weaknesses and difficulties. We present a new reading of the FGT (Foster, Greer and Thorbecke) index with a human dimension instead of the monetary, then we propose a combination between the fuzzy approach, and a classic measure of poverty, by defining a semi-fuzzy indicator which we generalize at the end of this paper.

Keywords: Fuzzy logic, poverty line, measure of poverty, index FGT, confidence interval.

1 Introduction. Different Approaches to Poverty and Threshold Problem

Poverty is a notion socially precised, if it relates to the non-satisfaction of basic needs, but economically fuzzy (Betti and Verma, 2004), (Betti and al, 2004), (Hajek, 2001), (Kaufmann & Gupta, 1991), (Klasen, 2000), (Makdissi & Wodon, 2004, (Zadeh, 1995). Several approaches have been developed in order to characterize and measure it by integrating various dimensions differing from one country to another and between regions within the same country. We can distinguish three main approaches to poverty (Marniesse, 1999): The monetary poverty, poverty of conditions of existence, subjective poverty.

The first one considers poor people whose resources are below a certain level says poverty threshold. This approach is not devoided of interest in companies, where most goods and services are traded as commodities. The greater availability of data on income distribution also explains, in part, that this approach is the most common. These monetary thresholds may be absolute thresholds or thresholds related.

The second approach identifies a number of difficulties, lack or deprivation in different areas of living conditions of households. These areas may refer to a poverty of an existential nature (food, shelter ...) and social (relationships, employment, social security,...).

The subjective approach doesn't consist to refer to a minimum threshold of resources conventionally defined, or to objectives conditions of existence, but to interrogate directly the households about their perception of these realities, from questions about their income.

Different methods are then used on the basis of these responses, to establish a subjective poverty line, households with incomes below this threshold will then be considered poor (insecurity objective existence). Another indicator (insecurity of subjective existence) is to count households that report having financial difficulties to buckle their budget. (Cahiers Français, n° 286)

Several authors such as Townsend and recently Cerioli and Zani developed multi-dimensional approaches in the study of poverty ((Cerioli & Zani, 1990) (Townsend, 1979)). These approaches treat poverty as a multiple deprivation. They state that it is important to take into account monetary and non-monetary dimensions in the analysis and measurement of poverty, which is seen as a multiple deprivation that can not be reduced only to lack of resources (Touhami & Ejjanaoui, 2009), in other words, an individual is poor if deprived of a set of basic goods and basic services deemed necessary to achieve a certain quality of life (Asselin, 2002),

We note that the concept of different classical approaches declare that an individual is poor compared to an attribute where the implementation of this attribute is below a fixed threshold. The binary coding can therefore determine by an area if the individual is poor or not. Thus, if we define a function of deprivation φ(xij ;zj), we have (Delhausse, 2002) (Alexandre, 2007)

$$\varphi(x_{ij}, z_j) = \begin{cases} 1 \text{ if } x_{ij} \geq z_j \rightarrow \text{non-deprivation} \\ 0 \text{ if } x_{ij} < z_j \rightarrow \text{deprivation} \end{cases} \qquad (1)$$

Where x_{ij} the level of functioning made by individual i for the attribute j, and z_j the deprivation threshold for the attribute j.

The study of poverty with the mentioned approaches, or others similar, requires to make choices of a poverty line that separates population into two sub-categories: poor and the non-poor people, and often requires the researchers to postulate this threshold. Besides that, there is no consensus on the establishment of the threshold (Belhadj, 2005) it may be doubtful even to establish it exactly, and therefore establish a clear demarcation between the poor and the non-poor. Thus can be difficult to argue that two people whose assessment of poverty indicators, according to the adopted approach, are nearly equal, could be below and above the threshold, so that one is considered poor and other as non-poor. Therefore some authors (Chiappero, 2000) (Lelli, 2001) noted that the passage of the state of deprivation and the non-deprivation really is not so sudden, but gradually. For this, the use of fuzzy set theory, allows to account for this deficiency, and avoid the gap between the two states considered.

2 Poverty Analysis Using FGT Indices

Once poverty line is determined according to a conventional approaches, several indices have been developed as a basis for measuring and comparing poverty between countries and regions. To do this, it is not confined to a single index, but we calculate several, the simplest and most commonly used is the Headcount ratio (Notes techniques,V1, 2002):

$$H = q/n \qquad (2)$$

Where q is the number of poor people, and n the size of the study population.

H is the number of people in poverty, it is the share of population living in poverty whichever consumption or income noted Y is below poverty line denoted z.

Second index often used is the ratio of the income gap I that can be seen as the average distance separating the poor from poverty line. It is defined by:

$$I = \frac{z - \overline{Y}_q}{z} \text{ where } \overline{Y}_q = \frac{1}{q}\sum_{i=1}^{q} Y_i \qquad (3)$$

where $\overline{Y}_q$ is the average income of the poor, which is the arithmetic average income of the poor.

Although these two indices suffer from several weaknesses, they are always calculated as they allow a first overview of the situation in the country easy to interpret. The other indexes are then studied to understand better the differences within the poor population.

The FGT class developed by Foster, Greer and Thorbecke (Foste and al, 1984) is very important in the poverty study. Indeed, they have proposed an index that satisfies the axiom of decomposition, the three axioms proposed by Sen, the one proposed by Kakwani (Kakwani, 1980) (Sen, 1976), and also satisfies other axioms proposed by some other researchers, where it is considered an index of the most advanced and most used in many empirical work.

This index was defined as the squared poverty gap that is a weighted sum of the income shortfalls of the poor, where each deficit is weighted by the value of itself deficit. Its original formula was:

$$P = \frac{1}{nz^2} \sum_{i=1}^{q} g_i^2 \qquad (4)$$

With $g_i = z - y_i$

Then, to satisfy the axiom of sensitivity to transfers (Kakwani, 1980), Foster, Greer and Thorbecke suggest a generalization of the above index which is the definitive version of this index:

$$FGT_\alpha = \frac{1}{n} \sum_{i=1}^{q} \left(\frac{g_i}{z} \right)^{\alpha} \qquad (5)$$

The FGT_α index satisfies the transfer axiom (Kakwani, 1980) for values of α above 1, and the axiom of sensitivity to transfers for values of α above 2. Besides, it is easy to see that the index H becomes a special case of the FGT_α index for α= 0, and the index I is obtained with α □= 1.

Even more important is that this index is decomposable for any value of α parameter that reflects the importance given to the poorest people. Indeed, assuming that the population is divided into several subgroups, and noting n_k the number of people belonging to the k-th group, and m_k the number of poor in the same group, then the level of poverty of this group will be:

$$FGT_\alpha^k = \frac{1}{n_k} \sum_{i=1}^{m_k} \left(\frac{g_i}{z} \right)^{\alpha} \qquad (6)$$

3 A need to a Fuzzy Approach of Poverty

3.1 Degree of Membership

Fuzzy logic was born from the realization that most of the phenomena can not be represented using Boolean variables (Gilles, 2002), which can take only two values 0 or 1 (poor, non-poor). In fact, some phenomena can assume a continuous change from one state to its complement, hence the existence of a subset of intersection between two complementary states, which is impossible in the classical reasoning, this feature has been a strong point for interpreting poverty in a fuzzy way.

The introduction of this theory in the poverty study, allows to distinguish individuals in to poor and non poor in a stepwise manner, we will not speak of poverty and non poverty, but poverty or non poverty levels, (degree membership of a household or individual to all poor).

The introduction of this theory in the poverty study, allows to distinguish individuals poor and non poor in a stepwise manner, we will not speak of poverty and non poverty, but poverty levels or non poverty, (degree of membership of a household or individual to the set of poor population). Thus we can see a gradual transition between extreme poverty (misery) and the limited resources (Hajek, 2001) (Makdissi & Wodon, 2004) (Zadeh, 1995). A household may be considered poor but its degree of belonging to the subset of poor is less than 1 , which represents the core of all the poor, and it is the same for a household in the subset of non-poor

Since all the classical approaches do not estimate poverty line with certainty, some authors like Basma Belhadj (Belhadj, 2005) have reconstructed bounds within which guaranteed the existence of this threshold (Alcaraz and al, 2001) (Alcaraz & Gonzalers, 2002).

3.2- Confidence Interval in the Fuzzy Approach of Poverty

The construction of the confidence interval, containing the poverty line is a very delicate task (Belhadj & Matoussi, 2007) because it is always dependent on the socioeconomic context in which it is located, and should take into account the specific characteristics of the chosen indicator of deprivation

In our work we start from the assumption that we know the interval within which lies the poverty line Z. In fact, considering the minimum value Z_{min} that we want to raise poverty line, and its maximum value Z_{max} (Ravallion, 1994) (Ravallion, 2003), then we have:

$$Z \in [Z_{\min}, Z_{\max}]$$

Basma Belhadj (Belhadj, 2005) has proposed a determination of terminals Z_{min} and Z_{max} decomposing them, using the LES demand system, in some food and other non-food part.

The upper poverty line Z_{max} is the level of per capita total expenditure required to enable households to achieve, without sacrificing their basic food needs. This poverty line, which can be obtained by iteration, to estimate the maximum non-food expenditures that match the food poverty line (Belhadj, 2005).

The determination of these two terminals to define a membership function defining the different fuzzy sets spanning the population studied (example: strong Deprivation, average deprivation and Low deprivation). In this framework, several membership functions were proposed by the decomposition described and the dimensions databases integrated for the measure of poverty in this population. Besmah and al. (Belhadj & Matoussi, 2007) proposed the following membership function:

$$\mu_Q(i) = \begin{cases} 1 & \text{if } 0 \le x_i < z_{\min} \\ \dfrac{-2}{z_{i\max} - z_{i\min}} x_i + \dfrac{2 z_{i\max}}{z_{i\max} - z_{i\min}} & \text{if } z_{i\min} \le x_i < z_{i\max} \\ 0 & \text{if } x_i > z_{i\max} \end{cases} \quad (7)$$

Where x_i is the income or expenditure of the i^{th} household, Alexandre Bertin and al (Alexandre & Leyle, 2007) have used a membership function, based on the work of Chiappero-MARTINETTI (Chiappero, 2000)(Chiappero, 2007), through which they have proposed a multidimensional measurement of poverty:

$$f_i(\varphi_j) = \begin{cases} 0 & \text{if } \varphi \le \varphi_{\min} \\ \dfrac{\varphi_j - \varphi_{\min}}{\varphi_{\max} - \varphi_{\min}} & \text{if } \varphi_{\min} \le \varphi < \varphi_{\max} \\ 1 & \text{if } \varphi_{\max} \le \varphi \end{cases} \qquad (8)$$

With φ is the value of the score (Included between 0 and 1) for each selected attribute (the dimensions of well-being), φ_{min} and φ_{max} and extreme values of the score on the general population, and many more others who have been proposed to define fuzzy indices for measuring poverty.

After reading and analysis of some traditional approaches of poverty, the fuzzy approach and its benefits, also FGT indices and their peculiarities in poverty measure, we base the fundamental idea of our paper on the combination of the fuzzy approach, given the deficiencies of the classical ones, and a transcription of FGT indices, thus we construct an index that we call semi-fuzzy index of poverty.

4 Construction of a New Semi-fuzzy Poverty Index

4.1 A New Reading of Poverty Gap Index: From Monetary Dimension to Human Dimension

The FGT index for α = 1, noted I, is defined in particular as poverty gap, it is like to mention before hand, the average distance between the population of poverty line when a distance zero is attributed to non-poor. We can still analyze the index simplifying its expression as follows:

$$I = \frac{1}{n}\sum_{i=1}^{q}\frac{z-y_i}{z} = \frac{1}{n}\sum_{i=1}^{q}(1-\frac{y_i}{z}) = \frac{1}{n}(q-\sum_{i=1}^{q}\frac{y_i}{z})$$

Therefore

$$I = \frac{1}{n}(q-\sum_{i=1}^{q}\frac{y_i}{z}) \qquad (9)$$

n is the size of the population under study, y_i is the income of household i and q the number of poor, so we can say that :

$$Y_q = \sum_{i=1}^{q} y_i$$

represents the wealth or total income of all poor, therefore the ration A defined by:

$$\frac{Y_q}{z} = \frac{\sum_{i=1}^{q} y_i}{z} = A \qquad (10)$$

reflects the number of people can live in dignity with an equal income to the threshold z from a wealth $Y_q = \sum_{i=1}^{q} y_i$. It follows from this simplification that:

- The value of A satisfies

$0 \leq A < q$

if not, there are some households classified as poor when their incomes exceed the threshold z.

- The difference $q - A = (q - \frac{Y_q}{z})$ is the number of people estimated to live with nearly zero income.

Hence the new writing of I via A and q:

I = (q - A) / n (11)

It is the weight of people supposed living with an income near zero compared to the population study. Thus the index I is read in human dimension instead of those monetary.

4.2 Construction from Four Classes

To construct the index we have chosen to start with the simple case of generating four classes that every interval, like the confidence interval of poverty line, is characterized by its boundaries and its center.

In the formula (11) A represents a number of people, but it is not necessarily an integer, therefore, to keep its meaning, we replace it by its integer part [A], which checks in (10) Euclidean division of Y_q by z such that,

We have

$$A=Y_q/z,$$

this involves:

$$Y_q=A.\ z$$

but

$$A= [A] + r' \quad \text{with} \quad 0\leq r' < 1$$

therefore

$$Y_q= [A].z + r'.z \quad \text{with} \quad 0\leq r' < 1$$

hence: by taking r=r'Z, we find :

$$Y_q = [A].z + r \quad \text{with} \quad 0 \leq r < z \qquad (12)$$

Which can be obtained by the Euclidean division of Y_q by z.

Now lets project this writing in the context of fuzzy approach. For this, we consider a confidence interval of poverty line fuzzy [Z_{min}, Z_{max}], where Z_{min} is the minimum value that we want to raise the threshold of poverty, and its maximum value is Z_{max}. Then by using a membership function (see paragraph 3), so we can determine the number of poor q_f.

Let μ_p be the membership function chosen[1], and B the subset defined by:

$$B = \{ i \in \Omega : \mu_p(i) > 0 \}$$

B is also said the support of the membership function μ_p.

Then q_f is given by:

$$q_f = \text{Cardinal}(B)$$

Let $Y_{qf} = \sum_{i=1}^{qf} y_i$ and $Z_B = (Z_{max} + Z_{min})/2$

Note [x] the integer part of x.

Let a, b and c be integers, and r 1, r2 and r3 in IR^+ such that:

$$\begin{cases} Y_{qf} = a.\ Z_{max} + r1 & \text{with} \quad 0 \le r1 < Z_{max} \quad (13) \\ r1 = b.\ Z_B + r2 & \text{with} \quad ; 0 \le r2 < Z_B \quad (14) \\ r2 = c.\ Z_{min} + r3 & \text{with} \quad ; 0 \le r3 < Z_{min} \quad (15) \end{cases}$$

If equality (13) is natural, equality (14) does not lose meaning if $r1 < Z_B$ since it suffices to take b = 0 and r1 = r2. In this case the equality (15) remains possible even if r2 <Zmin, because in this case c = 0 and r3 = r2 valid writing. Other situations are algebraic consequences of the Euclidean division.

Given $Y_{qf} = \sum_{i=1}^{qf} y_i$ total income of poor people, equality (13) implies that the value "a" represents the number of people estimated to live on an income Z_{max}. Equation (14) can be deduced that the value "b" is the number of people estimated to live with an income that is $(Z_{max} + Z_{min}) / 2$. The last equality of the system implies that the number of estimated people to live with an income Z_{min} is equal to "c". Considering these three classes of the poor population, the remains is:

((q – (a+b+c)) people supposed to live with an income close to zero[2].

Thus the poor population is represented by four sets:

$$B = B_a \cup B_b \cup B_c \cup B_0$$

Where

B_a: is the set of poor people supposed to live with an income equal to Z_{max}.

B_b: : is the set of poor people supposed to live with an income equal to Z_B

B_c: : is the set of poor people supposed to live with an income equal to Z_{min}

The subsets B_a, B_b, B_c and B_0 are subsets of B.

We propose a new semi-fuzzy PG_f index defined by:

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[1] For selecting membership function, experts can make their choices depending on the dimensions they want to integrate (income, illiteracy, welfare ...).

[2] Income close to zero: It's in the sense that income is worth less than r3/(q-(a+b+c)).

$$PG_f = \frac{q-(a+b+c)}{n} \qquad (16)$$

This index reflects the weight of people supposed unable to meet even the basic minimum needs of life. So it reflects the degree of misery in population Ω, it is an effective tools to measure frailty and hardship of population life. Indeed, the analysis of poverty with the FGT index gives an idea about how much money needed for that missing people so they can get out of poverty, but the differences in a currency value, and its daily change, gives relativity in understanding and reading this "money". The PG_f index is unchanged to change currencies, and its reading does not influence each other by the differences in living standards in the country, because it reflects a number of people, not money, thus we shifted the analysis of poverty based on monetary data remains a highly variable, can be misleading because of purely financial factors (inflation, exchange rate, ...) to a human dimension which is more reliable.

To simplify the PG_f index advantage, let us considering this example. Suppose that the statistics of 1999 in a region R adapting a poverty line Z, and according to FGT index, it needs 2Millions$ to exceed the poverty line. Seen the changes, socioeconomic development and living standards in the region R on 2008, statistics adapts an other poverty line Z'(≠Z), So results according the same index confirms that this region needs 3Millions$ to get out poverty. From this data, we can not study poverty variation between 1999 and 2008, it may be unfaithful to reality because:

- Poverty lines adapted are not same,
- The continuous variation of exchange rates.
- The standard of living changing over time.

But according to our approach, using semi-fuzzy PG_f index, the comparison is simple and measuring the impact of strategies and programs is easy, because as a result you can have 7690 people can not satisfy any of their basic food needs in 1999 against 4000 in 2008, then it is clear that the development program followed in this region had a good impact.

An analysis of this index allows constructing a vector index MI_f (I_1, I_2, I_3) based on boundaries Z_{max}, Z_{min} and Z_B selected. Each component of this vector represents the weight of a specific subclass of the poor. Specifically:

$$I_1 = \frac{(q-a)}{n} = \frac{1}{n}\left(q - \left[\sum_{i=1}^{q_f} \frac{y_i}{Z_{\max}} \right] \right) \qquad (17)$$

It reflects the weight of poor people supposedly unable to live with an income Z_{max} compared to the population Ω under study. In other words, this is the amount of money that must be injected into the subpopulation poor for all its people manage to live on an income Z_{max}.

The index I_2 defined by:

$$I_2 = \frac{(q-a-b)}{n} = \frac{1}{n}\left(q - \left[\sum_{i=1}^{q_f} \frac{y_i}{Z_{\max}} \right] - \left[\frac{r1}{Z_B} \right] \right) \qquad (18)$$

Where $r1 = Y_{qf} - a.\ Z_{max}$

I_2 reflects the weight of poor people supposedly unable to live with an income Z_B over the entire study population Ω.

While the index where I_3:

$$I_3 = \frac{(q - a - b - c)}{n}$$

i.e

$$I_3 = \frac{1}{n}\left(q - \left[\sum_{i=1}^{q_f} \frac{y_i}{Z_{max}} \right] - \left[\frac{r_1}{Z_B} \right] - \left[\frac{r_2}{Z_{min}} \right] \right) \qquad (19)$$

With $r2= r1 - b. Z_{min}$

I_3 reflects the weight of poor people supposedly unable to live even with Z_{min} income over the entire study population Ω. In other words it is the weight of people thought to live with an income near zero, or crushed by poverty.

4.3 Generalizing the Construction of the Semi fuzzy Vector Index of Poverty

Recall that Y_{qf} is the total income of all the poor sub-population:

$$Y_{qf} = \sum_{i=1}^{qf} y_i$$

with

q_f = Cardinal(B) where $B = \{ i \in \Omega : \mu_p(i) > 0 \}$

Consider $n \in IN^*$ the discretization order, and $(h_1, h_2, h_3 \ldots h_n) \in IR^{n^*}+$ stepsize of the confidence interval [Zmin, Zmax], those steps h_i represent the differences that an expert

considers reasonable, to measure the degradation of income, as it is known for the evaluation and the devaluation of wages.

The first stage of process construction consists on Euclidean division of Y_{qf} by Zmax, which gives us:

$Y_{qf}= a_0. Z_{max} + r_0$ where $0 \le r_0 < Z_{max}$

If $Z_{max} – h_0 < r0$, we make again the following division::

$r_0= a_1.(Z_{max} – h_0) + r_1$ where $0 \le r_1 < Z_{max} – h_0$

Even if $Z_{max} – h_1 < r_1$, we can write

$r_1= a_2 (Z_{max} – h_1) + r_2$ where $0 \le r_2 < Z_{max} – h_1$

………

……..

If $Z_{max} – h_m < rn$, we can write:

$r_{m-2} = a_{m-1} (Z_{max} – h_{m-2}) + r_{m-1}$ where $0 \le r_{m-1} < Z_{max} – h_{m-2}$

until the last division we can do if $Z_{min} < r_{m-1}$

$r_{m-1} = a_m . Z_{min} + r_m$ where $0 \leq r_m < Z_{min}$

So similarly to the four classes we have a_0 people supposed to live with an income Z_{max}.

The second Euclidean division there are a_1 people thought to have an income (Z_{max}-h_0), and so on untill the last equality implies the existence of a_m people thought to live with an income Z_{min}. The rest of the population of qf poor people is (q_f-(a_0+a_1+….+a_m)) people supposed to live with an income near zero , we note it the set B*.

The choice of steps and the order of the discretization depends on the extended interval [Z_{min}, Z_{max}] initially selected, as also depends on the description and meaning associated with each terminal Z_i , with:

$Z_i = Z_{max} - h_i$

If we choose the fixed stepsize

$h_i = i.h$; $i \in \{1,2,\ldots n\}$

Classes will be equidistant, but cardinals a_i are different according to data from the population study. Therefore we get a vector MI defined by :

$$MI = \begin{pmatrix} I_1 \\ I_2 \\ \cdots \\ \mathrm{Im} \end{pmatrix}$$

Such as each component I_j (j=1,2…m) is determined by

$$I_j = \frac{q_f - \sum_{k=1}^{j} a_k}{n} \qquad (20)$$

Where a_k (k=,…j) values obtained above.

We note $P_{Gf} = I_m$

By construction, the indices I_j (j=1,2…m) are decreasing in the sense that the passage of the calculation from I_j to I_{j+1} is given by:

$$I_j - I_{j+1} = \frac{a_{j+1}}{n} \geq 0$$

what is the weight of the (j+1)th set (B_{j+1}) relative to the entire population, so we have built a system of weights giving the depth of each subset of the poor.

The last class B* is a special class because it represents the misery in the society studied, it's

characterized by $$PG_f = \frac{q_f - \sum_{i=1}^{m} a_i}{n} \qquad (21)$$

5 Analysis and Extreme Cases

5.1 An Analysis on the Basis of the Main Axioms of Poverty Indices

By analyzing the indexes semi fuzzy Ij (j=1 ,2…m}, especially the PGf index, we found that they satisfy all the three main axioms:

1- Focus Axiom: The measurement of poverty remains unchanged if the income of a person who is above the poverty line increases

2- Monotony Axiom: All other things being equal, a reduction of income of a person who is below the poverty line must increase the poverty measure.

3- Transfer Axiom: All things being equal, a transfer of income between a person who is below the poverty line and a person who is richer must increase the poverty measure.

Indeed, for the first axiom, the indices Ij (j)1 ,2…m} and PGf are not influenced by the increase of income of non-poor people, their expressions do not involve any data related to income of people ranked above the poverty line.

For the monotony axiom, suppose that the income yi of a person i who is below the poverty line decreases,

Then the quantity $Y_{qf} = \sum_{i=1}^{q_f} y_i$ also decreases,

So $a_0 = \left[\frac{Y_{qf}}{Z_{\max}} \right]$ becomes lower,

There fore $\sum_{i=1}^{j} a_i$ decreased turn,

Result the quantity $q - \sum_{i=1}^{j} a_i$ augment, and then for any index Ij $j \in \{1,2,...m\}$,

$I_j = \frac{(q - \sum_{i=1}^{j} a_i)}{n}$ increases.

In particular when j = m, the semi fuzzy index PG_f increases if the income of an individual classified poor decreases.

To verify compliance with the axiom of transfer, let i be an individual who is below the poverty line, and k a richer person.

Suppose there was a transfer of income between the person I and the person k. Then there will be a reduction of income yi for individual i, and an increase of income for the person k..

Since the expression indices Ij, components of the vector index semi fuzzy MIf, and in particular the PG_f index, are independent of income of individuals above the poverty line, then these indexes are not affected by the increased income of the person k . While the decrease of income for the poor person i causes, as seen by the front, that the indices Ij increases and consequently the measurement of poverty increases.

5.2 Analyse of Extreme Cases

Generally we have

$$0 \leq q_f \leq n$$

And a_1 , $a_{2,\ldots.}$ a_m are positive integers whose sum is less than q_f by construction, which implies that:

$$0 \leq q_f - (a_1 + a_2 + \ldots a_m) \leq n$$

Therefore

$$0 \leq P_{Gf} \leq 1$$

By analyzing the PG_f index, we distinguish the three extreme cases:

1- $PG_f = 0 \quad \Leftrightarrow \quad q_f = \sum_{i=0}^{m} a_i$

This reflects the case where the set B_0 of people thought to live with an income close to zero is an empty set, therefore we can say that it is a poor population but misery is absent among its members.

2- $PG_f = q/n$: $\Leftrightarrow$ $a_1 = a_2 = \ldots = a_m = 0$

In this case the set B_0, $B_1 \ldots$ and B_m which cardinals respectively a_0, $a_{1..}$ and a_m are all empty, while the set B^* is dominant:

$$B^* = B$$

$$Card(B^*) = q_f - \sum_{i=0}^{m} a_i = q_f = Card(B)$$

This is a serious case where all the poor of the study population are living in a misery.

3- $PG_f = 1$: $\Leftrightarrow$ $a_0 = a_1 = \ldots = a_m = 0$ and $q = n$

This is the case where the whole population identified as poor are all living misery. This case is mathematically possible but not easy to find in reality.

6 Utility & advantages of the vector index MI: Case of four classes

According to the class that represents those different indices, components of the vectors MI_f, they allow a specific analysis of each one of the four classes, thus the accumulation of measures of these indices over time in a region or country to measure the progress and slippage of poor subpopulations, and also the translations or the crash of each compared to others, namely for example:

- The poor subpopulation denoted B_a, people supposed unable to meet their food and non food basic needs without sacrifice compared to the entire study population Ω.
- The subpopulation of poor people, noted B_b, supposed unable to meet the majority of their food and non food basic needs without sacrifice compared to the entire study populationΩ.
- The poor subpopulation denoted B_c, people supposed unable to meet most of their food and non food basic needs over the entire study populationΩ.
- The poor subpopulation, denoted B_0, people thought unable to meet any food and non food basic needs regarding the entire population under study. In other words, they are people supposed to live with an income close to zero and crushed by poverty.

On strategies and programs these indices remain a very remarkable interest. Indeed, knowledge of the numbers of people from each of the classes we have mentioned enables leaders to build an idea of the type of socio-economic development projects suitable for mounting in each study area, and hence measure over time the impact of its program in this area. For example, if the majority of inhabitants of a region is in the second subpopulation, then we must develop projects that must satisfy the minimum income equal to Z_B, even as people in this class are assumed unable to meet majority of their food and non food basic needs without sacrifice over the entire study population, it gives an overall idea about the level of health, access to education, … in this population.

5 Conclusion

To maximize the economic and social development programs, and to have more impact on the poor, we proposed a new method of analysis of poverty through our new index semi-fuzzy To construct this index we combined between:

- The fuzzy approach of poverty, since it allows on the one hand, with its confidence interval to avoid the problems had to choose the poverty line, on the other hand, it allows with its membership function to integrate different dimensions: monetary, non-monetary, qualitative and quantitative.
- The qualities and properties of the FGT classical index, namely the respect of the axioms of Sen, Kakwani and others.

This new index will replace the analysis with a monetary dimension, by an analysis with human dimension remains invariant to the currency change, variation modes and living standards between countries or within the same country over time, this which gives an intrinsic character.

Another strong point of this semi fuzzy index is that it gives a certain permeability reading of the inner poverty of the poor class, since it allows to write the class of poor as a union of subsets forming a constituent system. The interest of this writing is the analysis of slip between the poor subpopulations, which reflect the evolution or degradation of the socio-economic status of poor.

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