A method of choice of the best alternative in the multiple solutions case in the Games Theory

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Abstract: In the game theory, there are a classic series of four criteria: Wald, Hurwicz, Savage, Laplace with a special importance in the choice of optimal decision in the situations of uncertainty. These criteria providing reasonable answers from different points of view. The question is to discern between different choices when the criteria do not provide a unique answer. In this paper, we present a possible way to reduce the final number of variants.

Keywords: games theory, Wald, Savage, Hurwicz, Laplace, decision

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1. Introduction

Let $a_1,...,a_n$ the alternatives and $b_1,...,b_m$ uncontrollable states. The payoffs for each pair (a_i,b_j) are c_{ij} and are find out in the following table:

	b_1	...	b_j	...	b_m
a_1	c_{11}	...	c_{1j}	...	c_{1m}
...	...	...	...	...	...
a_i	a_{i1}	...	a_{ij}	...	a_{im}
...	...	...	...	...	...
a_n	a_{n1}	...	a_{nj}	...	a_{nm}

In the process of decision making we have as well as principal methods the following:

- Wald's criterion (the maximin criterion)
- Laplace's criterion
- Hurwicz's optimist criterion
- Savage's regret criterion

The **Wald's criterion** suggest that for each alternative a_i the determination of the minimum of the quantities c_{ik}, $k=\overline{1,m}$ and after the selection of the greatest value.

For example, let the problem:

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	b_1	b_2	b_3	b_4	b_5	min
a_1	2	5	3	6	7	2
a_2	5	4	2	2	8	2
a_3	4	2	9	6	8	2
a_4	9	5	6	1	8	1

The maximum of the quantities from the last column is 2, therefore the best alternative is one of a_1, a_2 or a_3.

The **Laplace's criterion** suggest that for each alternative a_i the determination of the arithmetic mean of the quantities c_{ik}, $k=\overline{1,m}$ and after the selection of the greatest value.

The problem:

	b_1	b_2	b_3	b_4	b_5	min
a_1	2	5	3	6	7	4,6
a_2	5	4	2	2	8	4,2
a_3	4	2	9	6	8	5,8
a_4	9	5	6	1	8	5,8

give the best alternatives a_3 and a_4.

The **Hurwicz's criterion** consider a coefficient of optimism $\omega \in [0,1]$ and a mean value for each alternative between the maximum and minimum values with weight given by ω and $1-\omega$ respectively. Finally the best alternative comes from the greatest obtained value.

The problem, for $\omega=0,3$

	b_1	b_2	b_3	b_4	b_5	max	min	$0,3\cdot\max + 0,7\cdot\min$
a_1	5	9	1	2	1	9	1	3,4
a_2	8	6	7	0	6	8	0	2,4
a_3	1	4	7	2	10	10	1	3,7
a_4	3	1	1	10	4	10	1	3,7

give the best alternatives a_3 and a_4.

The **Savage's criterion** introduce the notion of regret like difference between how much can be win if we know apriori the appearance of one of the uncontrollable states and the real gain. Finally, the decision is taken after the minimax criterion, that is the minimum from the maximum of the values on lines.

Considering the problem:

	b_1	b_2	b_3	b_4	b_5

a_1	**10**	**2**	**2**	**3**	**7**
a_2	**6**	**5**	**9**	**2**	**6**
a_3	**1**	**1**	**7**	**6**	**10**
a_4	**7**	**1**	**0**	**9**	**5**

we have the regrets table:

	b_1	**b_2**	**b_3**	**b_4**	**b_5**	**max**
a_1	**0**	**3**	**7**	**6**	**3**	**7**
a_2	**4**	**0**	**0**	**7**	**4**	**7**
a_3	**9**	**4**	**2**	**3**	**0**	**9**
a_4	**3**	**4**	**9**	**0**	**5**	**9**

which give the best alternatives a_1 and a_2.

2. An alternative of the choice of the optimal

In what follows, in the case of multiple optimal choices, we will distinguish between them, considering the mean square deviation of the values corresponding at each alternative. The reason for such a choice is that a minimal mean square deviation signify that the distribution of the values around the mean value is close, therefore the alternative is less subject to changes in the uncontrollable states.

In the case of the Wald's criterion we have obtained in the first section that the best alternative is one of a_1, a_2 or a_3. But, for an alternative a_i we have $\sigma(a_i)=\sqrt{\frac{\sum_{j=1}^{m} c_{ij}^2}{m}-\left(\frac{\sum_{j=1}^{m} c_{ij}}{m}\right)^2}$. Therefore:

$$\sigma(a_1)=\sqrt{\frac{2^2+5^2+3^2+6^2+7^2}{5}-\left(\frac{2+5+3+6+7}{5}\right)^2}=1{,}855;$$

$$\sigma(a_2)=\sqrt{\frac{5^2+4^2+2^2+2^2+8^2}{5}-\left(\frac{5+4+2+2+8}{5}\right)^2}=2{,}227;$$

$$\sigma(a_3)=\sqrt{\frac{4^2+2^2+9^2+6^2+8^2}{5}-\left(\frac{4+2+9+6+8}{5}\right)^2}=2{,}561.$$

Because a_1 has the minimal mean square deviation it will be the optimal choice.

For the Laplace's criterion, we had the best alternatives a_3 and a_4. But:

$$\sigma(a_3)=\sqrt{\frac{4^2+2^2+9^2+6^2+8^2}{5}-\left(\frac{4+2+9+6+8}{5}\right)^2}=2{,}561;$$

$$\sigma(a_4)=\sqrt{\frac{9^2+5^2+6^2+1^2+8^2}{5}-\left(\frac{9+5+6+1+8}{5}\right)^2}=2{,}786.$$

Because a_3 has the minimal mean square deviation it will be the optimal choice.

For the Hurwicz's criterion, the best alternatives were a_3, a_4. Because:

$$\sigma(a_3)=\sqrt{\frac{1^2+4^2+7^2+2^2+10^2}{5}-\left(\frac{1+4+7+2+10}{5}\right)^2}=3{,}311;$$

$$\sigma(a_4)=\sqrt{\frac{3^2+1^2+1^2+10^2+4^2}{5}-\left(\frac{3+1+1+10+4}{5}\right)^2}=3{,}311.$$

In this case, when both mean square deviations are equal we will take the alternative with the greatest mean value, that is a_3.

For the Savage's criterion we have obtained that the best alternatives were a_1 and a_2. From:

$$\sigma(a_1)=\sqrt{\frac{10^2+2^2+2^2+3^2+7^2}{5}-\left(\frac{10+2+2+3+7}{5}\right)^2}=3{,}187;$$

$$\sigma(a_2)=\sqrt{\frac{6^2+5^2+9^2+2^2+6^2}{5}-\left(\frac{6+5+9+2+6}{5}\right)^2}=2{,}245.$$

we find that the best alternative is a_2.

3. Conclusion

The method presented above gives a refined choice of the best alternative in the case when the classical methods provide more than one answer.

4. References

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