A method of choice of the importance coefficients in the Electre method

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Abstract. The Electre method aims as objective the choosing of the best variant in the conditions of existence of some decision criteria. A subjective factor in the method consists in choosing the coefficients of importance. This paper proposes an objective method of determining the coefficients of importance using the standard deviation of the utilities for each criterion.

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1 Introduction

The Electre method which was discovered in the years 1965-1966 by a team of European researchers attached to the Consultancy Company SEMA (Benayoun, 1966) is mainly aimed at determining the best choice of action in the condition of existence of some decision criteria.

Many problems arising in this method but also extensions of this have generated a series of subsequent development (Mousseau, 2001), (Buchanan, 2007), (Almeida, 2008).

The method consists of a number of action' s variants V_1, V_2, ..., V_n whose choice is faced with a decision maker. Let also, be a number of m criteria C_1, C_2,..., C_m which have each an importance coefficient (*usually determined subjectively*) k_1, k_2,..., k_m. To each pair (V_i,C_j) we assign a numerical value v_{ij} (*if it is a qualitative appreciation we will convert it in a number of hierarchy*). To determine the optimal action, the coefficients of importance will be normalize by the relationship: $\nu_j=\frac{k_j}{\sum_{p=1}^{m} k_p}$, $j=\overline{1,m}$ getting: $\sum_{j=1}^{m}\nu_j=1$.

2 The Electre Method

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After these considerations, we will determine the nature of the method (maximizing or minimizing) harmonizing the data in that, in the case of a criterion of contrary nature to the problem, the corresponding range change its sign.

Also, at least two corresponding values of a criterion must be different, otherwise the selection criterion becoming insignificant.

We, then, determine the utilities U_{ij} that correspond to the pairs (V_i, C_j) as follows: for the problem of maximizing: $U_{ij}=\dfrac{v_{ij}-\min\limits_{k=1,\ldots,n} v_{kj}}{\max\limits_{k=1,\ldots,n} v_{kj}-\min\limits_{k=1,\ldots,n} v_{kj}}$ and for the minimization: $U_{ij}=\dfrac{\max\limits_{k=1,\ldots,n} v_{kj}-v_{ij}}{\max\limits_{k=1,\ldots,n} v_{kj}-\min\limits_{k=1,\ldots,n} v_{kj}}$ and after we will construct their table.

The importance of considering the utilities is special in that, on the one hand, they are dimensionless (*being obtained as ratios between the sizes of the same nature*) which allows comparison of sizes of different nature, and on the other hand, it gives an overview on the quantities of each criterion meaning how they are closer to the requirement of the problem (*maximization or minimization*) the utility being closer to 1 and otherwise, to 0.

Also, it should be noted that utilities are quantities located always between the interval: [0,1].

From the definition of the utilities, we note that multiplication of the values matrix by an arbitrary strictly positive factor and also an arbitrary additive amount will conserve the utilities.

We will compute after the concordance indicators according to the following:

$$c(V_i,V_j)=\sum_{\substack{p=1,\ldots,m\\ U_{ip}\geq U_{jp}}} \nu_p$$

and the discordance indicators:

$$d(V_i,V_j)=\max_{p=1,\ldots,m}(U_{jp}-U_{ip},0)$$

We will establish two values p and q such that $p,q\in(0,1)$ and $p+q=1$ to measure limits of concordance and discordance. We will say that a variant V_i is preferred to a variant V_j if:

$$\begin{cases} c(V_i,V_j)\geq p \\ d(V_i,V_j)\leq q \end{cases}$$

Taking into account that q=1-p we have therefore:

$$\begin{cases} c(V_i, V_j) \geq p \\ 1 - d(V_i, V_j) \geq p \end{cases}$$

So we get:

$$p \leq \min\left(c(V_i, V_j), 1 - d(V_i, V_j)\right)$$

A variant V_i will satisfy the optimal condition if for a given p:

$$p \leq \min\left(c(V_i, V_j), 1 - d(V_i, V_j)\right) \forall j = \overline{1,n}$$

We thus determine p by condition:

$$p = \min_{i=1,n}\left(\min_{j=1,n} c(V_i, V_j), 1 - \max_{j=1,n} d(V_i, V_j)\right)$$

the optimal variant (variants) corresponding to this value.

The problem that will be discussed below, is how we will select the importance coefficients. It is noted that their role is definitive in determining the optimal variant, having an influence on concordance indicators.

If the nature of the discordance indicators is objective one, being independent of the decision maker, the role of the indicators of concordance may be essential in selecting a variant or another.

As principle, it might try to determine the optimal variant according to arbitrary values of the coefficients of importance. The problem which appears, is that of an existence of impossibility of an analytical approach.

We can overcome this situation, in order to establish a rule of allocation for the importance coefficients to remove the note of subjectivity.

We will propose therefore the proportionality of the coefficients of importance with the standard deviation of utilities for each criterion. The logic of this choice is that square deviation closed to 0 means a reduced variability of data which leads to a weak dependence of the optimal variant from the data.

A large square deviation means a wide margin of variation of data in the criterion (through the coefficient of importance) leading to a greater instability of the optimal choice.

We will choose:

$$k_j = \sqrt{\sum_{i=1}^{n}\left(U_{ij} - \frac{\sum_{p=1}^{n} U_{pj}}{n}\right)^2} = \frac{\sqrt{n\sum_{i=1}^{n} U_{ij}^2 - \left(\sum_{i=1}^{n} U_{ij}\right)^2}}{n}, \; j = \overline{1,m}$$

and after the normalization:

$$\nu_j = \frac{\sqrt{n\sum_{i=1}^{n} U_{ij}^2 - \left(\sum_{i=1}^{n} U_{ij}\right)^2}}{\sum_{p=1}^{m}\sqrt{n\sum_{i=1}^{n} U_{ip}^2 - \left(\sum_{i=1}^{n} U_{ip}\right)^2}},\ j=\overline{1,m}$$

We note from the first expression of k_j, the fact that from the definition of the utilities, we have (for a maximization problem, but similar it happens for a minimization):

$$k_j = \sqrt{\sum_{i=1}^{n}\left(U_{ij} - \frac{\sum_{p=1}^{n} U_{pj}}{n}\right)^2} = \sqrt{\sum_{i=1}^{n}\left(\frac{v_{ij} - \min\limits_{k=1,\dots,n} v_{kj}}{\max\limits_{k=1,\dots,n} v_{kj} - \min\limits_{k=1,\dots,n} v_{kj}} - \frac{\sum_{p=1}^{n}\frac{v_{pj} - \min\limits_{k=1,\dots,n} v_{kj}}{\max\limits_{k=1,\dots,n} v_{kj} - \min\limits_{k=1,\dots,n} v_{kj}}}{n}\right)^2} =$$

$$\frac{1}{\max\limits_{k=1,\dots,n} v_{kj} - \min\limits_{k=1,\dots,n} v_{kj}}\sqrt{\sum_{i=1}^{n}\left(v_{ij} - \min\limits_{k=1,\dots,n} v_{kj} - \frac{\sum_{p=1}^{n} v_{pj} - n\min\limits_{k=1,\dots,n} v_{kj}}{n}\right)^2} =$$

$$\frac{1}{\max\limits_{k=1,\dots,n} v_{kj} - \min\limits_{k=1,\dots,n} v_{kj}}\sqrt{\sum_{i=1}^{n}\left(v_{ij} - \frac{\sum_{p=1}^{n} v_{pj}}{n}\right)^2}.$$

Following these calculations, we see that if the determining of the coeficients of importance would be considered the absolute values, but not the utilities, then the answer would have been dependent to the difference: $\max\limits_{k=1,\dots,n} v_{kj} - \min\limits_{k=1,\dots,n} v_{kj}$ as with the normalization it would introduce large distortions between criteria.

3 Example

A company want to manufacture a product. For this, there are more variants of technological process V_1, V_2, V_3, V_4 and V_5, and as criteria are considered: the profit (C_1), the quality (C_2) and the manufacturing time (C_3). We will appreciate with numerically the qualities, as: low quality – 0, medium quality – 1, good – 2 and very good – 3.

The obtained table is:

Criterion Variant	C_1	C_2	C_3
V_1	1000	0	1
V_2	800	1	4
V_3	600	3	5
V_4	500	3	7

The nature of the problem is obviously of maximizing.

After transformation of C_3 column, we have:

Criterion Variant	C_1	C_2	C_3
V_1	1000	0	-1
V_2	800	1	-4
V_3	600	3	-5
V_4	500	3	-7
min	500	0	-7
M-m	500	3	6

The table of utilities is thus:

Criterion Variant	C_1	C_2	C_3
V_1	1	0	1
V_2	0,6	0,33	0,5
V_3	0,2	1	0,33
V_4	0	1	0

The coeficients of importance are:

$$k_1=\frac{\sqrt{4(1+0{,}36+0{,}04+0)-(1+0{,}6+0{,}2+0)^2}}{4}=0{,}384$$

$$k_2=\frac{\sqrt{4(0+0{,}1089+1+1)-(0+0{,}33+1+1)^2}}{4}=0{,}433$$

$$k_3=\frac{\sqrt{4(1+0{,}25+0{,}1089+0)-(1+0{,}5+0{,}33+0)^2}}{4}=0{,}361$$

Normalizing the coefficients of importance:

$$\nu_1=\frac{k_1}{k_1+k_2+k_3}=\frac{0{,}384}{1{,}178}=0{,}326$$

$$\nu_2=\frac{k_2}{k_1+k_2+k_3}=\frac{0{,}433}{1{,}178}=0{,}368$$

$$\nu_3=\frac{k_3}{k_1+k_2+k_3}=\frac{0{,}361}{1{,}178}=0{,}306$$

The table of concordance and discordance indicators becomes:

	V_1		V_2		V_3		V_4	
V_1	1	0	0,632	0,33	0,632	1	0,632	1
V_2	0,368	0,5	1	0	0,632	0,67	0,632	0,67
V_3	0,368	0,8	0,368	0,4	1	0	1	0
V_4	0,368	1	0,368	0,6	0,368	0,33	1	0

For the determination of p we have:

	$\min\limits_{j=1,n} c(V_i,V_j)$	$1-\max\limits_{j=1,n} d(V_i,V_j)$	min
V_1	0,632	0	0
V_2	0,368	0,33	**0,33**
V_3	0,368	0,2	0,2
V_4	0,368	0	0

The chosen option is that for which is obtained the maximum from the last column, meaning V_2.

4 Conclusion

The method of choice for the coeficients of importance, described above, has the advantage of eliminating the subjective factor leading to a final decision more objective. On the other hand, when there are only two variants, as in each criterion exists two distinct values, the two utilities will have the values 0 and 1. Therefore, all coefficients of importance will be equal, which will remove the optimal decision of a correlation with the reality.

We therefore recommend this method when there are at least three types of variants of action.

5 References

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