The evolution of GDP in USA using cyclic regression analysis

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Abstract: Based on the four major types of economic cycles (Kondratieff, Juglar, Kitchin, Kuznet), the paper aims to determine their actual length (for the U.S. economy) using cyclic regressions based on Fourier analysis.

Keywords: Kondratieff, Juglar, Kitchin, Kuznet, cycle.

1 Introduction

The development and growth analysis takes into account different states where the economy goes, states which are called phases of the economic cycle. In an economic cycle, its phases are knitting and relations between them are causal, resulting in qualitative and quantitative changes in the economic life of society.

The description of processes taking place in the economic activity involves understanding the causes and how economic variables included in wave motion changes its own dynamics thus making the transition from one phase to another economic cycle.

Even though economists say, belonging to different current is unanimous in defining the economic cycle, the diversity of opinion occurs about the causes of the economic cycle.

In the second half of the nineteenth century were raised short-term crises, decades, being explain by either exogenous or endogenous factors, then the researches pointed on long-term secular.

Schumpeter believes that the business cycle can have two phases (expansion and depression) and four phases (prosperity, recession, depression and revival). Schumpeter argues that evolutionary process dividing into two or four phases is not just purely descriptive. Each phase of the economic cycle is a distinct phenomenon not through economical characteristics but also through the set of forces dominating and generating these characteristics.

In the early twentieth century, Schumpeter and other economist, on the previous researches and conclusions, try a first typology of the economic cycle. Thus, depending on the duration and generating factor, Schumpeter presents four types of economic cycles that coexist in the economy and the existence of a cycle, not excluding the other.

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Name	Period (years)	Generator factor
KITCHIN	3-5	Variations in stocks
JUGLAR	7-11	Variations in the fixed investment Credit expansion
KUZNETS	15-25	Variations in infrastructure
KONDRATIEFF	50-60	The discovery and implementation of new technologies

2 Review of Literature

Kitchin cycle

In the early twentieth century (1923), Joseph Kitchin, an English statistician, based on interest rates and other variables analysis (the analysis being performed on the economies of the United States and England) discovers a short business cycle of about 40 months. Joseph Kitchin cycle described by the laws neoclassical equilibrium is considered to be based on fluctuations in the stock business. During economic progress, enterprises react by increasing production, inventory cycle, increasing labor employment. After a certain period of time (3-5 years) commodity market is oversaturated and the gap between supply and demand expectations lead to un-timing of the expectations of companies with market realities.

The Kitchen cycle has two phases: expansion and economic downturn, the transition the from the expansion phase to the slowing expansion being done without causing any major crisis.

Juglar cycle

A first analysis of the economic cycle, in terms of recurrence phenomenon was due to the French economist and statistician Clement Juglar, who studied fluctuations in interest rates and the price on which he discovered in 1860, an economic cycle (also called decennial cycle) with alternating periods of prosperity and decline over a period of 8-11 years.

Juglar cycles are endogenous cycles (a result of forces within the economic system), being generated by the investment process variations and long-term bank policies.

Following the analysis undertaken by Juglar (the economies of France, England and the United States) he captures a common factor which makes these countries being susceptible to an economic crisis, namely the uncontrolled expansion of credit. Development credit expansion generates economic activity but also makes the economy concerned to become increasingly unstable. Symptoms that generates economic crisis are even signs of a period of prosperity.

Kuznets cycle

Another relevant business cycle is those of Kuznets, associated with fluctuations in the rate of investment in construction and lasts between 15 and 25 years. The first tests of Kuznets on "secondary secular movements" occur in 1927 and 1930, the conclusion drawn is that fluctuations in production are similar to those of prices. Kuznets, with the later surveys (1966,1971) demonstrated a direct link between the business cycle, economic growth and economic development, the complexity of the cycle phenomenon being associated with a diversified economic system.

Kondratieff cycle

The idea of a long cycle in capitalism was not in the economic research center than at the beginning of the twentieth century, when the pace of economic development was abruptly followed by long periods of stagnation and contraction.

The first economist who brought to the attention of the international scientific community a long cycle, was the Russian economist Nikolai Kondratieff. In the economic literature, the Kondratieff cycle, named also super-cycle describes the oscillations of economic activity accompanying the capitalist system with an average duration of 50 years, ranging between 40 and 60 years.

Based on statistical research on long-term fluctuations in production and prices (analysis was performed on the economy of the United States, France and England) Kondratieff observed periods of accelerated growth of industries in the economy, alternated with slower growth. The test results covering the period 1770-1920 were published in "The Major Economic Cycles" published in Moscow in 1925.

Kondratieff long cycle theory, although not unanimously accepted, represents a solid foundation of future research on fluctuations in economic activity. Among the economists who accepted the existence of Kondratieff cycle is not always a consensus on the start and the end of the super-cycle.

3 Mathematical preliminaries

Let consider a function $f:\mathbf{R}\to\mathbf{R}$, with f and f' continue. Suppose also that f is periodic of period T, i.e. $f(x+T)=f(x)\ \forall x\in\mathbf{R}$. The periodicity means that the function values are repeated after a delay equal to T.

Consider also the associated Fourier series of f:

$$F(x)=\frac{a_0}{2}+\sum_{k=1}^{\infty}\left(a_k\cos\frac{2k\pi x}{T}+b_k\sin\frac{2k\pi x}{T}\right)$$

where: $a_k=\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}f(x)\cos\frac{2k\pi x}{T}dx$, $k\geq 0$, $b_k=\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}f(x)\sin\frac{2k\pi x}{T}dx$, $k\geq 1$.

It is noted that:

$$F(x+T)=\frac{a_0}{2}+\sum_{k=1}^{\infty}\left(a_k\cos\frac{2k\pi(x+T)}{T}+b_k\sin\frac{2k\pi(x+T)}{T}\right)=$$

$$\frac{a_0}{2}+\sum_{k=1}^{\infty}\left(a_k\cos\left(\frac{2k\pi x}{T}+2k\pi\right)+b_k\sin\left(\frac{2k\pi x}{T}+2k\pi\right)\right)=$$

$$\frac{a_0}{2}+\sum_{k=1}^{\infty}\left(a_k\cos\frac{2k\pi x}{T}+b_k\sin\frac{2k\pi x}{T}\right)=F(x)\ \forall x\in\mathbf{R}$$

so F is also periodic of period T.

Dirichlet's theorem (Spiegel, 1974) states that, under the above conditions, series converges to f at every point of continuity. This does not mean anything other than that sum of Fourier series is equal for every $x\in\mathbf{R}$ with the value of the function f at x.

Taking the partial sums of Fourier series of order n (obtained by taking into account only the first n terms), we obtain Fourier polynomials of order n:

$$F_n(x)=\frac{a_0}{2}+\sum_{k=1}^{n}\left(a_k\cos\frac{2k\pi x}{T}+b_k\sin\frac{2k\pi x}{T}\right)$$

It is obvious that for these polynomials: $F_n(x)=F_n(x+T)\ \forall x\in\mathbf{R}$.

Due to convergence, Fourier polynomials are able to approximate the function f by one periodic, the absolute error (difference between the exact value and those approximate) tending to zero with increasing n.

Because many economic phenomena are cyclical (in this case GDP) we want to determine the best approximation for a given number of terms of the development corresponding to a data set.

Let therefore, the dataset: (x_i,y_i), $i=\overline{1,m}$ and the Fourier polynomial: $F_n(x)=\frac{a_0}{2}+\sum_{k=1}^{n}\left(a_k\cos\frac{2k\pi x}{T}+b_k\sin\frac{2k\pi x}{T}\right)$. We shall determine the coefficients a_k, $k=\overline{0,n}$ and b_k, $k=\overline{1,n}$ using the method of least squares.

Let therefore:

$$\varepsilon(a_0,a_k,b_k)=\sum_{i=1}^{m}\left(\frac{a_0}{2}+\sum_{k=1}^{n}\left(a_k\cos\frac{2k\pi x_i}{T}+b_k\sin\frac{2k\pi x_i}{T}\right)-y_i\right)^2$$

the sum of squared differences between approximate values and the exact amount.

In order that $\varepsilon(a_0,a_k,b_k)$ have a minimum point, must that the partial derivatives with respect to a_k, $k=\overline{0,n}$ and b_k, $k=\overline{1,n}$ vanish. Therefore:

$$\begin{cases} \dfrac{\partial \varepsilon}{\partial a_0} = \sum\limits_{i=1}^{m}\left(\dfrac{a_0}{2} + \sum\limits_{k=1}^{n}\left(a_k \cos\dfrac{2k\pi x_i}{T} + b_k \sin\dfrac{2k\pi x_i}{T}\right) - y_i\right) = 0 \\ \dfrac{\partial \varepsilon}{\partial a_j} = \sum\limits_{i=1}^{m}\left(\dfrac{a_0}{2} + \sum\limits_{k=1}^{n}\left(a_k \cos\dfrac{2k\pi x_i}{T} + b_k \sin\dfrac{2k\pi x_i}{T}\right) - y_i\right)\cos\dfrac{2j\pi x_i}{T} = 0,\ j = \overline{1,n} \\ \dfrac{\partial \varepsilon}{\partial b_j} = \sum\limits_{i=1}^{m}\left(\dfrac{a_0}{2} + \sum\limits_{k=1}^{n}\left(a_k \cos\dfrac{2k\pi x_i}{T} + b_k \sin\dfrac{2k\pi x_i}{T}\right) - y_i\right)\sin\dfrac{2j\pi x_i}{T} = 0,\ j = \overline{1,n} \end{cases}$$

Noting: $A_{ik}=\cos\dfrac{2k\pi x_i}{T}$, $B_{ik}=\sin\dfrac{2k\pi x_i}{T}$, $i=\overline{1,m}$, $k=\overline{1,n}$, the system is written as:

$$\begin{cases} \dfrac{n}{2}a_0 + \sum\limits_{k=1}^{n}\left(a_k \sum\limits_{i=1}^{m} A_{ik} + b_k \sum\limits_{i=1}^{m} B_{ik}\right) = \sum\limits_{i=1}^{m} y_i \\ \dfrac{\sum\limits_{i=1}^{m} A_{ij}}{2} a_0 + \sum\limits_{k=1}^{n}\left(a_k \sum\limits_{i=1}^{m} A_{ik}A_{ij} + b_k \sum\limits_{i=1}^{m} B_{ik}A_{ij}\right) = \sum\limits_{i=1}^{m} y_i A_{ij},\ j = \overline{1,n} \\ \dfrac{\sum\limits_{i=1}^{m} B_{ij}}{2} a_0 + \sum\limits_{k=1}^{n}\left(a_k \sum\limits_{i=1}^{m} A_{ik}B_{ij} + b_k \sum\limits_{i=1}^{m} B_{ik}B_{ij}\right) = \sum\limits_{i=1}^{m} y_i B_{ij},\ j = \overline{1,n} \end{cases}$$

Noting now, for simplicity:

$$\alpha_k=\sum_{i=1}^{m} A_{ik},\ \beta_k=\sum_{i=1}^{m} B_{ik},\ \gamma_{kj}=\sum_{i=1}^{m} A_{ik}A_{ij},\ \delta_{kj}=\sum_{i=1}^{m} B_{ik}B_{ij},\ \varepsilon_{kj}=\sum_{i=1}^{m} A_{ik}B_{ij},$$

$$\mu=\sum_{i=1}^{m} y_i,\ \nu_j=\sum_{i=1}^{m} y_i A_{ij},\ \lambda_j=\sum_{i=1}^{m} y_i B_{ij},\ k,j=\overline{1,n}.$$

the system becomes:

$$\begin{cases} \dfrac{n}{2}a_0 + \sum\limits_{k=1}^{n}\left(\alpha_k a_k + \beta_k b_k\right) = \mu \\ \dfrac{\alpha_j}{2}a_0 + \sum\limits_{k=1}^{n}\left(\gamma_{kj} a_k + \varepsilon_{jk} b_k\right) = \nu_j,\ j = \overline{1,n} \\ \dfrac{\beta_j}{2}a_0 + \sum\limits_{k=1}^{n}\left(\varepsilon_{kj} a_k + \delta_{kj} b_k\right) = \lambda_j,\ j = \overline{1,n} \end{cases}$$

Solving the system of 2n+1 equations with 2n+1 unknowns: ak, k=$\overline{0,n}$ and bk, k=$\overline{1,n}$ it follows that for a given period T>0 and n≥1, the Fourier polynomial: Fn(x)= $\dfrac{a_0}{2} + \sum\limits_{k=1}^{n}\left(a_k \cos\dfrac{2k\pi x}{T} + b_k \sin\dfrac{2k\pi x}{T}\right)$ is the best cyclical approximation in the terms of the method of least squares. We call Fn thus determined, the cyclic regression of order n and period T corresponding to the dataset**.**

4. The analysis

Let consider now, the values GDP_k, k=$\overline{1,m}$ corresponding to a period of m consecutive years and the growth rate: $r_k=\frac{GDP_k - GDP_{k-1}}{GDP_{k-1}}$, k=$\overline{2,m}$.

The cyclical analysis (through the theory above) of U.S. GDP for the period 1792-2012 taking into account the growth rate does not provide acceptable results, especially in terms of the current period. For this reason, we shall consider the absolute variation of it: $v_k=\Delta r_k=r_k-r_{k-1}$, k=$\overline{3,m}$. This indicator provides more direct information on the phenomenon of crisis, meaning that $v_k<0$ indicates a decrease in the growth rate, while $v_k>0$ is equivalent to exit the crisis phenomenon.

For our analysis, we consider therefore value pairs: (k,vk), $\overline{3,m}$ for the period 1792-2012.

Table 1

The evolution of GDP, growth rate and absolute change in growth rate for U.S. economy during 1792-2012

Year	GDP_k	r_k	$v_k=\Delta r_k$	Year	GDP_k	r_k	$v_k=\Delta r_k$
1792	4.58	0	0	1903	481.8	0.029	-0.0224
1793	4.95	0.0808	0.0808	1904	464.8	-0.0353	-0.0643
1794	5.6	0.1313	0.0505	1905	517.2	0.1127	0.148
1795	5.96	0.0643	-0.067	1906	538.4	0.041	-0.0717
1796	6.15	0.0319	-0.0324	1907	552.2	0.0256	-0.0154
1797	6.27	0.0195	-0.0124	1908	492.5	-0.1081	-0.1337
1798	6.54	0.0431	0.0236	1909	528.1	0.0723	0.1804
1799	7	0.0703	0.0272	1910	533.8	0.0108	-0.0615
1800	7.4	0.0571	-0.0132	1911	551.1	0.0324	0.0216
1801	7.76	0.0486	-0.0085	1912	576.9	0.0468	0.0144
1802	8	0.0309	-0.0177	1913	599.7	0.0395	-0.0073
1803	8.14	0.0175	-0.0134	1914	553.7	-0.0767	-0.1162
1804	8.45	0.0381	0.0206	1915	568.8	0.0273	0.104
1805	8.9	0.0533	0.0152	1916	647.7	0.1387	0.1114
1806	9.32	0.0472	-0.0061	1917	631.7	-0.0247	-0.1634
1807	9.33	0.0011	-0.0461	1918	688.7	0.0902	0.1149
1808	9.35	0.0021	0.001	1919	694.2	0.008	-0.0822
1809	10.07	0.077	0.0749	1920	687.7	-0.0094	-0.0174
1810	10.63	0.0556	-0.0214	1921	671.9	-0.023	-0.0136
1811	11.11	0.0452	-0.0104	1922	709.3	0.0557	0.0787
1812	11.55	0.0396	-0.0056	1923	802.6	0.1315	0.0758
1813	12.21	0.0571	0.0175	1924	827.4	0.0309	-0.1006

1814	12.72	0.0418	-0.0153	1925	846.8	0.0234	-0.0075
1815	12.82	0.0079	-0.0339	1926	902.1	0.0653	0.0419
1816	12.82	0	-0.0079	1927	910.8	0.0096	-0.0557
1817	13.12	0.0234	0.0234	1928	921.3	0.0115	0.0019
1818	13.6	0.0366	0.0132	1929	977	0.0605	0.049
1819	13.86	0.0191	-0.0175	1930	892.8	-0.0862	-0.1467
1820	14.41	0.0397	0.0206	1931	834.9	-0.0649	0.0213
1821	15.18	0.0534	0.0137	1932	725.8	-0.1307	-0.0658
1822	15.76	0.0382	-0.0152	1933	716.4	-0.013	0.1177
1823	16.33	0.0362	-0.002	1934	794.4	0.1089	0.1219
1824	17.3	0.0594	0.0232	1935	865	0.0889	-0.02
1825	18.07	0.0445	-0.0149	1936	977.9	0.1305	0.0416
1826	18.71	0.0354	-0.0091	1937	1028	0.0512	-0.0793
1827	19.29	0.031	-0.0044	1938	992.6	-0.0344	-0.0856
1828	19.55	0.0135	-0.0175	1939	1072.8	0.0808	0.1152
1829	20.3	0.0384	0.0249	1940	1166.9	0.0877	0.0069
1830	22.16	0.0916	0.0532	1941	1366.1	0.1707	0.083
1831	23.99	0.0826	-0.009	1942	1618.2	0.1845	0.0138
1832	25.61	0.0675	-0.0151	1943	1883.1	0.1637	-0.0208
1833	26.4	0.0308	-0.0367	1944	2035.2	0.0808	-0.0829
1834	26.85	0.017	-0.0138	1945	2012.4	-0.0112	-0.092
1835	28.27	0.0529	0.0359	1946	1792.2	-0.1094	-0.0982
1836	29.11	0.0297	-0.0232	1947	1776.1	-0.009	0.1004
1837	29.37	0.0089	-0.0208	1948	1854.2	0.044	0.053
1838	30.59	0.0415	0.0326	1949	1844.7	-0.0051	-0.0491
1839	31.37	0.0255	-0.016	1950	2006	0.0874	0.0925
1840	31.46	0.0029	-0.0226	1951	2161.1	0.0773	-0.0101
1841	32.17	0.0226	0.0197	1952	2243.9	0.0383	-0.039
1842	33.19	0.0317	0.0091	1953	2347.2	0.046	0.0077
1843	34.84	0.0497	0.018	1954	2332.4	-0.0063	-0.0523
1844	36.82	0.0568	0.0071	1955	2500.3	0.072	0.0783
1845	39.15	0.0633	0.0065	1956	2549.7	0.0198	-0.0522
1846	42.33	0.0812	0.0179	1957	2601.1	0.0202	0.0004
1847	45.21	0.068	-0.0132	1958	2577.6	-0.009	-0.0292
1848	46.73	0.0336	-0.0344	1959	2762.5	0.0717	0.0807
1849	47.38	0.0139	-0.0197	1960	2830.9	0.0248	-0.0469
1850	49.59	0.0466	0.0327	1961	2896.9	0.0233	-0.0015
1851	53.58	0.0805	0.0339	1962	3072.4	0.0606	0.0373
1852	59.76	0.1153	0.0348	1963	3206.7	0.0437	-0.0169

1853	64.65	0.0818	-0.0335	1964	3392.3	0.0579	0.0142
1854	66.88	0.0345	-0.0473	1965	3610.1	0.0642	0.0063
1855	69.67	0.0417	0.0072	1966	3845.3	0.0652	0.001
1856	72.47	0.0402	-0.0015	1967	3942.5	0.0253	-0.0399
1857	72.84	0.0051	-0.0351	1968	4133.4	0.0484	0.0231
1858	75.79	0.0405	0.0354	1969	4261.8	0.0311	-0.0173
1859	81.28	0.0724	0.0319	1970	4269.9	0.0019	-0.0292
1860	82.11	0.0102	-0.0622	1971	4413.3	0.0336	0.0317
1861	83.57	0.0178	0.0076	1972	4647.7	0.0531	0.0195
1862	93.95	0.1242	0.1064	1973	4917	0.0579	0.0048
1863	101.18	0.077	-0.0472	1974	4889.9	-0.0055	-0.0634
1864	102.33	0.0114	-0.0656	1975	4879.5	-0.0021	0.0034
1865	105.26	0.0286	0.0172	1976	5141.3	0.0537	0.0558
1866	100.43	-0.0459	-0.0745	1977	5377.7	0.046	-0.0077
1867	102.15	0.0171	0.063	1978	5677.6	0.0558	0.0098
1868	106.13	0.039	0.0219	1979	5855	0.0312	-0.0246
1869	109.02	0.0272	-0.0118	1980	5839	-0.0027	-0.0339
1870	112.3	0.0301	0.0029	1981	5987.2	0.0254	0.0281
1871	117.6	0.0472	0.0171	1982	5870.9	-0.0194	-0.0448
1872	127.5	0.0842	0.037	1983	6136.2	0.0452	0.0646
1873	138.3	0.0847	0.0005	1984	6577.1	0.0719	0.0267
1874	140.8	0.0181	-0.0666	1985	6849.3	0.0414	-0.0305
1875	140.6	-0.0014	-0.0195	1986	7086.5	0.0346	-0.0068
1876	146.4	0.0413	0.0427	1987	7313.3	0.032	-0.0026
1877	153.7	0.0499	0.0086	1988	7613.9	0.0411	0.0091
1878	158.6	0.0319	-0.018	1989	7885.9	0.0357	-0.0054
1879	177.1	0.1166	0.0847	1990	8033.9	0.0188	-0.0169
1880	191.8	0.083	-0.0336	1991	8015.1	-0.0023	-0.0211
1881	215.8	0.1251	0.0421	1992	8287.1	0.0339	0.0362
1882	227.3	0.0533	-0.0718	1993	8523.4	0.0285	-0.0054
1883	233.5	0.0273	-0.026	1994	8870.7	0.0407	0.0122
1884	229.7	-0.0163	-0.0436	1995	9093.7	0.0251	-0.0156
1885	230.5	0.0035	0.0198	1996	9433.9	0.0374	0.0123
1886	249.2	0.0811	0.0776	1997	9854.3	0.0446	0.0072
1887	267.3	0.0726	-0.0085	1998	10283.5	0.0436	-0.001
1888	282.7	0.0576	-0.015	1999	10779.8	0.0483	0.0047
1889	290.8	0.0287	-0.0289	2000	11226	0.0414	-0.0069
1890	319.1	0.0973	0.0686	2001	11347.2	0.0108	-0.0306
1891	322.8	0.0116	-0.0857	2002	11553	0.0181	0.0073

1892	339.3	0.0511	0.0395	2003	11840.7	0.0249	0.0068
1893	319.6	-0.0581	-0.1092	2004	12263.8	0.0357	0.0108
1894	304.5	-0.0472	0.0109	2005	12638.4	0.0305	-0.0052
1895	339.2	0.114	0.1612	2006	12976.2	0.0267	-0.0038
1896	333.6	-0.0165	-0.1305	2007	13228.9	0.0195	-0.0072
1897	348	0.0432	0.0597	2008	13161.9	-0.0051	-0.0246
1898	386.1	0.1095	0.0663	2009	12703.1	-0.0349	-0.0298
1899	412.5	0.0684	-0.0411	2010	12615	-0.0069	0.028
1900	422.8	0.025	-0.0434	2011	12982	0.0291	0.036
1901	445.3	0.0532	0.0282	2012	13351	0.0284	-0.0007
1902	468.2	0.0514	-0.0018				

* GDP-billion $-2005 US

Source: http://www.usgovernmentrevenue.com

First, we determine the Fourier coefficients corresponding to the Kondratieff cycle. Thus, we obtain successive developments with respect to all periods integers in the range [40,75] (corresponding to the classical theory) for a variable number of terms of the development ($n=\overline{1,20}$). Finally, we shall take that development which corresponds to the smallest mean square deviation: $\sigma = \sqrt{\frac{\sum_{k=1}^{m}(z_k - v_k)^2}{m}}$ where z_k are the values obtained in the simulation, and m is the number of data considered.

Computer analysis revealed minimal error σ=0.04572 for a period T=44 and a total of 20 terms of development.

The Fourier coefficients have the values:

Table 2

a_0	$8.002175\cdot10^{-04}$						
a_1	$1.422694\cdot10^{-03}$	a_{11}	$1.390276\cdot10^{-03}$	b_1	$1.631444\cdot10^{-04}$	b_{11}	$-4.722482\cdot10^{-03}$
a_2	$-1.783924\cdot10^{-03}$	a_{12}	$3.664564\cdot10^{-03}$	b_2	$-2.307899\cdot10^{-03}$	b_{12}	$-5.943708\cdot10^{-04}$
a_3	$2.610124\cdot10^{-03}$	a_{13}	$2.178575\cdot10^{-03}$	b_3	$-1.059022\cdot10^{-03}$	b_{13}	$-1.458957\cdot10^{-02}$
a_4	$-1.708445\cdot10^{-03}$	a_{14}	$-7.375367\cdot10^{-03}$	b_4	$1.79701\cdot10^{-05}$	b_{14}	$-8.483229\cdot10^{-04}$
a_5	$1.197551\cdot10^{-02}$	a_{15}	$1.178741\cdot10^{-03}$	b_5	$-5.418106\cdot10^{-04}$	b_{15}	$8.983581\cdot10^{-03}$
a_6	$-3.957016\cdot10^{-03}$	a_{16}	$1.195034\cdot10^{-02}$	b_6	$6.491387\cdot10^{-03}$	b_{16}	$-1.004112\cdot10^{-02}$
a_7	$5.04061\cdot10^{-03}$	a_{17}	$9.020037\cdot10^{-03}$	b_7	$-3.938911\cdot10^{-03}$	b_{17}	$8.178469\cdot10^{-03}$
a_8	$4.233568\cdot10^{-03}$	a_{18}	$1.913765\cdot10^{-03}$	b_8	$-6.424004\cdot10^{-03}$	b_{18}	$3.785044\cdot10^{-03}$
a_9	$-5.520029\cdot10^{-03}$	a_{19}	$5.627982\cdot10^{-03}$	b_9	$4.881333\cdot10^{-03}$	b_{19}	$-1.538891\cdot10^{-02}$
a_{10}	$-3.21995\cdot10^{-04}$	a_{20}	$-1.848497\cdot10^{-03}$	b_{10}	$2.23776\cdot10^{-03}$	b_{20}	$-7.354639\cdot10^{-03}$

Substituting in the expression of F_{20}, the values $k=\overline{1,221}$ (for the period 1792-2012) we obtain the new values (periodic, of period 44) of v_k.

Table 3

The absolute variation in the growth rate for the U.S. economy during 1792-2012 obtained from Fourier analysis (corresponding to Kondratieff cycles)

Year	$v_k=\Delta r_k$ real	$v_k=\Delta r_k$ Fourier	Year	$v_k=\Delta r_k$ real	$v_k=\Delta r_k$ Fourier	Year	$v_k=\Delta r_k$ real	$v_k=\Delta r_k$ Fourier
1792	0	-0.0279	1866	-0.0745	-0.0366	1940	0.0069	-0.0036
1793	0.0808	0.017	1867	0.063	0.0272	1941	0.083	0.0213
1794	0.0505	0.0033	1868	0.0219	0.014	1942	0.0138	0.0114
1795	-0.067	-0.0249	1869	-0.0118	-0.0191	1943	-0.0208	-0.0247
1796	-0.0324	-0.0168	1870	0.0029	-0.0282	1944	-0.0829	-0.0128
1797	-0.0124	0.0175	1871	0.0171	0.0413	1945	-0.092	-0.0298
1798	0.0236	-0.0206	1872	0.037	0.028	1946	-0.0982	-0.0063
1799	0.0272	0.0126	1873	0.0005	-0.0403	1947	0.1004	-0.0022
1800	-0.0132	-0.0056	1874	-0.0666	0.0391	1948	0.053	-0.0034
1801	-0.0085	0.0146	1875	-0.0195	-0.0348	1949	-0.0491	0.0157
1802	-0.0177	0.0424	1876	0.0427	0.0081	1950	0.0925	0.0519
1803	-0.0134	-0.0345	1877	0.0086	-0.0095	1951	-0.0101	-0.0387
1804	0.0206	0.011	1878	-0.018	0.012	1952	-0.039	-0.0385
1805	0.0152	-0.0382	1879	0.0847	0.0401	1953	0.0077	0.0398
1806	-0.0061	-0.0121	1880	-0.0336	-0.0279	1954	-0.0523	-0.0366
1807	-0.0461	0.0584	1881	0.0421	0.017	1955	0.0783	0.0272
1808	0.001	-0.0036	1882	-0.0718	0.0033	1956	-0.0522	0.014
1809	0.0749	0.0213	1883	-0.026	-0.0249	1957	0.0004	-0.0191
1810	-0.0214	0.0114	1884	-0.0436	-0.0168	1958	-0.0292	-0.0282
1811	-0.0104	-0.0247	1885	0.0198	0.0175	1959	0.0807	0.0413
1812	-0.0056	-0.0128	1886	0.0776	-0.0206	1960	-0.0469	0.028
1813	0.0175	-0.0298	1887	-0.0085	0.0126	1961	-0.0015	-0.0403
1814	-0.0153	-0.0063	1888	-0.015	-0.0056	1962	0.0373	0.0391
1815	-0.0339	-0.0022	1889	-0.0289	0.0146	1963	-0.0169	-0.0348
1816	-0.0079	-0.0034	1890	0.0686	0.0424	1964	0.0142	0.0081
1817	0.0234	0.0157	1891	-0.0857	-0.0345	1965	0.0063	-0.0095
1818	0.0132	0.0519	1892	0.0395	0.011	1966	0.001	0.012
1819	-0.0175	-0.0387	1893	-0.1092	-0.0382	1967	-0.0399	0.0401
1820	0.0206	-0.0385	1894	0.0109	-0.0121	1968	0.0231	-0.0279
1821	0.0137	0.0398	1895	0.1612	0.0584	1969	-0.0173	0.017
1822	-0.0152	-0.0366	1896	-0.1305	-0.0036	1970	-0.0292	0.0033
1823	-0.002	0.0272	1897	0.0597	0.0213	1971	0.0317	-0.0249

1824	0.0232	0.014	1898	0.0663	0.0114	1972	0.0195	-0.0168
1825	-0.0149	-0.0191	1899	-0.0411	-0.0247	1973	0.0048	0.0175
1826	-0.0091	-0.0282	1900	-0.0434	-0.0128	1974	-0.0634	-0.0206
1827	-0.0044	0.0413	1901	0.0282	-0.0298	1975	0.0034	0.0126
1828	-0.0175	0.028	1902	-0.0018	-0.0063	1976	0.0558	-0.0056
1829	0.0249	-0.0403	1903	-0.0224	-0.0022	1977	-0.0077	0.0146
1830	0.0532	0.0391	1904	-0.0643	-0.0034	1978	0.0098	0.0424
1831	-0.009	-0.0348	1905	0.148	0.0157	1979	-0.0246	-0.0345
1832	-0.0151	0.0081	1906	-0.0717	0.0519	1980	-0.0339	0.011
1833	-0.0367	-0.0095	1907	-0.0154	-0.0387	1981	0.0281	-0.0382
1834	-0.0138	0.012	1908	-0.1337	-0.0385	1982	-0.0448	-0.0121
1835	0.0359	0.0401	1909	0.1804	0.0398	1983	0.0646	0.0584
1836	-0.0232	-0.0279	1910	-0.0615	-0.0366	1984	0.0267	-0.0036
1837	-0.0208	0.017	1911	0.0216	0.0272	1985	-0.0305	0.0213
1838	0.0326	0.0033	1912	0.0144	0.014	1986	-0.0068	0.0114
1839	-0.016	-0.0249	1913	-0.0073	-0.0191	1987	-0.0026	-0.0247
1840	-0.0226	-0.0168	1914	-0.1162	-0.0282	1988	0.0091	-0.0128
1841	0.0197	0.0175	1915	0.104	0.0413	1989	-0.0054	-0.0298
1842	0.0091	-0.0206	1916	0.1114	0.028	1990	-0.0169	-0.0063
1843	0.018	0.0126	1917	-0.1634	-0.0403	1991	-0.0211	-0.0022
1844	0.0071	-0.0056	1918	0.1149	0.0391	1992	0.0362	-0.0034
1845	0.0065	0.0146	1919	-0.0822	-0.0348	1993	-0.0054	0.0157
1846	0.0179	0.0424	1920	-0.0174	0.0081	1994	0.0122	0.0519
1847	-0.0132	-0.0345	1921	-0.0136	-0.0095	1995	-0.0156	-0.0387
1848	-0.0344	0.011	1922	0.0787	0.012	1996	0.0123	-0.0385
1849	-0.0197	-0.0382	1923	0.0758	0.0401	1997	0.0072	0.0398
1850	0.0327	-0.0121	1924	-0.1006	-0.0279	1998	-0.001	-0.0366
1851	0.0339	0.0584	1925	-0.0075	0.017	1999	0.0047	0.0272
1852	0.0348	-0.0036	1926	0.0419	0.0033	2000	-0.0069	0.014
1853	-0.0335	0.0213	1927	-0.0557	-0.0249	2001	-0.0306	-0.0191
1854	-0.0473	0.0114	1928	0.0019	-0.0168	2002	0.0073	-0.0282
1855	0.0072	-0.0247	1929	0.049	0.0175	2003	0.0068	0.0413
1856	-0.0015	-0.0128	1930	-0.1467	-0.0206	2004	0.0108	0.028
1857	-0.0351	-0.0298	1931	0.0213	0.0126	2005	-0.0052	-0.0403
1858	0.0354	-0.0063	1932	-0.0658	-0.0056	2006	-0.0038	0.0391
1859	0.0319	-0.0022	1933	0.1177	0.0146	2007	-0.0072	-0.0348
1860	-0.0622	-0.0034	1934	0.1219	0.0424	2008	-0.0246	0.0081
1861	0.0076	0.0157	1935	-0.02	-0.0345	2009	-0.0298	-0.0095
1862	0.1064	0.0519	1936	0.0416	0.011	2010	0.028	0.012

1863	-0.0472	-0.0387	1937	-0.0793	-0.0382	2011	0.036	0.0401
1864	-0.0656	-0.0385	1938	-0.0856	-0.0121	2012	-0.0007	-0.0279
1865	0.0172	0.0398	1939	0.1152	0.0584			

For the Juglar cycles, we shall proceed similarly, determining the Fourier coefficients corresponding to a period of 11 years (the classical theory stating the duration between 7 and 15 years) for a variable number of terms of the development ($n=\overline{1,20}$). The Choose of this period was made simply to fit an integer number of Juglar cycles in one of type Kondratieff, noting that between different periods (in the range 7-15 years) the mean square deviation being not significantly different. Thus, the computer analysis revealed the error σ=0.05034 for the period T=11 and a number of 11 terms of development.

Finally, we obtain the new values (periodic, of period 11) of v_k.

Table 4

The absolute variation in the growth rate for the U.S. economy during 1792-2012 obtained from Fourier analysis (corresponding to Juglar cycles)

Year	$v_k=\Delta r_k$ real	$v_k=\Delta r_k$ Fourier	Year	$v_k=\Delta r_k$ real	$v_k=\Delta r_k$ Fourier	Year	$v_k=\Delta r_k$ real	$v_k=\Delta r_k$ Fourier
1792	0	-0.0232	1866	-0.0745	-0.0186	1940	0.0069	0.0044
1793	0.0808	-0.0003	1867	0.063	0.0088	1941	0.083	-0.0154
1794	0.0505	-0.0021	1868	0.0219	0.0152	1942	0.0138	0.018
1795	-0.067	-0.0016	1869	-0.0118	-0.0223	1943	-0.0208	-0.0185
1796	-0.0324	0.0131	1870	0.0029	0.0002	1944	-0.0829	0.0095
1797	-0.0124	0.0031	1871	0.0171	-0.0014	1945	-0.092	0.0159
1798	0.0236	-0.017	1872	0.037	-0.0006	1946	-0.0982	-0.0217
1799	0.0272	0.0163	1873	0.0005	0.0137	1947	0.1004	0.0014
1800	-0.0132	-0.0194	1874	-0.0666	0.0039	1948	0.053	-0.0013
1801	-0.0085	0.0084	1875	-0.0195	-0.016	1949	-0.0491	-0.0005
1802	-0.0177	0.0146	1876	0.0427	0.0175	1950	0.0925	0.0144
1803	-0.0134	-0.0231	1877	0.0086	-0.0191	1951	-0.0101	0.0045
1804	0.0206	-0.0003	1878	-0.018	0.0089	1952	-0.039	-0.0153
1805	0.0152	-0.002	1879	0.0847	0.0153	1953	0.0077	0.0181
1806	-0.0061	-0.0015	1880	-0.0336	-0.0222	1954	-0.0523	-0.0184
1807	-0.0461	0.0132	1881	0.0421	0.0008	1955	0.0783	0.0096
1808	0.001	0.0031	1882	-0.0718	-0.0019	1956	-0.0522	0.016
1809	0.0749	-0.0169	1883	-0.026	-0.0011	1957	0.0004	-0.0216
1810	-0.0214	0.0163	1884	-0.0436	0.0138	1958	-0.0292	0.0015
1811	-0.0104	-0.0193	1885	0.0198	0.004	1959	0.0807	-0.0001

1812	-0.0056	0.0086	1886	0.0776	-0.0159	1960	-0.0469	0.0007
1813	0.0175	0.0147	1887	-0.0085	0.0176	1961	-0.0015	0.0156
1814	-0.0153	-0.0231	1888	-0.015	-0.019	1962	0.0373	0.0035
1815	-0.0339	0	1889	-0.0289	0.009	1963	-0.0169	-0.0164
1816	-0.0079	-0.0019	1890	0.0686	0.0154	1964	0.0142	0.0171
1817	0.0234	-0.0014	1891	-0.0857	-0.0221	1965	0.0063	-0.0183
1818	0.0132	0.0132	1892	0.0395	0.0009	1966	0.001	0.0097
1819	-0.0175	0.0034	1893	-0.1092	-0.0018	1967	-0.0399	0.0161
1820	0.0206	-0.0168	1894	0.0109	-0.001	1968	0.0231	-0.0215
1821	0.0137	0.0164	1895	0.1612	0.0139	1969	-0.0173	0.0016
1822	-0.0152	-0.0193	1896	-0.1305	0.0041	1970	-0.0292	0
1823	-0.002	0.0087	1897	0.0597	-0.0158	1971	0.0317	0.0008
1824	0.0232	0.0148	1898	0.0663	0.0177	1972	0.0195	0.0157
1825	-0.0149	-0.023	1899	-0.0411	-0.0189	1973	0.0048	0.0036
1826	-0.0091	0.0001	1900	-0.0434	0.0091	1974	-0.0634	-0.0163
1827	-0.0044	-0.0018	1901	0.0282	0.0155	1975	0.0034	0.0172
1828	-0.0175	-0.0013	1902	-0.0018	-0.022	1976	0.0558	-0.0182
1829	0.0249	0.0133	1903	-0.0224	0.001	1977	-0.0077	0.0098
1830	0.0532	0.0035	1904	-0.0643	-0.0017	1978	0.0098	0.0162
1831	-0.009	-0.0167	1905	0.148	-0.0009	1979	-0.0246	-0.0214
1832	-0.0151	0.0165	1906	-0.0717	0.014	1980	-0.0339	0.0017
1833	-0.0367	-0.0189	1907	-0.0154	0.0042	1981	0.0281	0.0001
1834	-0.0138	0.0085	1908	-0.1337	-0.0157	1982	-0.0448	0.0009
1835	0.0359	0.0149	1909	0.1804	0.0178	1983	0.0646	0.0158
1836	-0.0232	-0.0226	1910	-0.0615	-0.0188	1984	0.0267	0.0037
1837	-0.0208	-0.0001	1911	0.0216	0.0092	1985	-0.0305	-0.0162
1838	0.0326	-0.0017	1912	0.0144	0.0156	1986	-0.0068	0.0173
1839	-0.016	-0.0009	1913	-0.0073	-0.0219	1987	-0.0026	-0.0181
1840	-0.0226	0.0134	1914	-0.1162	0.0011	1988	0.0091	0.0099
1841	0.0197	0.0036	1915	0.104	-0.0016	1989	-0.0054	0.0163
1842	0.0091	-0.0169	1916	0.1114	-0.0008	1990	-0.0169	-0.0213
1843	0.018	0.0166	1917	-0.1634	0.0141	1991	-0.0211	0.0018
1844	0.0071	-0.0188	1918	0.1149	0.0042	1992	0.0362	0.0002
1845	0.0065	0.0086	1919	-0.0822	-0.0156	1993	-0.0054	0.001
1846	0.0179	0.015	1920	-0.0174	0.0179	1994	0.0122	0.0159
1847	-0.0132	-0.0225	1921	-0.0136	-0.0187	1995	-0.0156	0.0038
1848	-0.0344	0	1922	0.0787	0.0093	1996	0.0123	-0.0161
1849	-0.0197	-0.0016	1923	0.0758	0.0157	1997	0.0072	0.0174
1850	0.0327	-0.0008	1924	-0.1006	-0.0219	1998	-0.001	-0.018

1851	0.0339	0.0135	1925	-0.0075	0.0012	1999	0.0047	0.01
1852	0.0348	0.0037	1926	0.0419	-0.0015	2000	-0.0069	0.0163
1853	-0.0335	-0.0168	1927	-0.0557	-0.0007	2001	-0.0306	-0.0212
1854	-0.0473	0.0167	1928	0.0019	0.0142	2002	0.0073	0.0019
1855	0.0072	-0.0187	1929	0.049	0.0043	2003	0.0068	0.0003
1856	-0.0015	0.0087	1930	-0.1467	-0.0155	2004	0.0108	0.0011
1857	-0.0351	0.0151	1931	0.0213	0.0179	2005	-0.0052	0.016
1858	0.0354	-0.0224	1932	-0.0658	-0.0186	2006	-0.0038	0.0039
1859	0.0319	0.0001	1933	0.1177	0.0094	2007	-0.0072	-0.016
1860	-0.0622	-0.0015	1934	0.1219	0.0158	2008	-0.0246	0.0175
1861	0.0076	-0.0007	1935	-0.02	-0.0218	2009	-0.0298	-0.0179
1862	0.1064	0.0136	1936	0.0416	0.0013	2010	0.028	0.0101
1863	-0.0472	0.0038	1937	-0.0793	-0.0014	2011	0.036	0.0164
1864	-0.0656	-0.0167	1938	-0.0856	-0.0006	2012	-0.0007	-0.0211
1865	0.0172	0.0168	1939	0.1152	0.0143			

The evolution's graphs of the pair Kondratieff-Juglar cycles corresponding to the 5 so determined are:

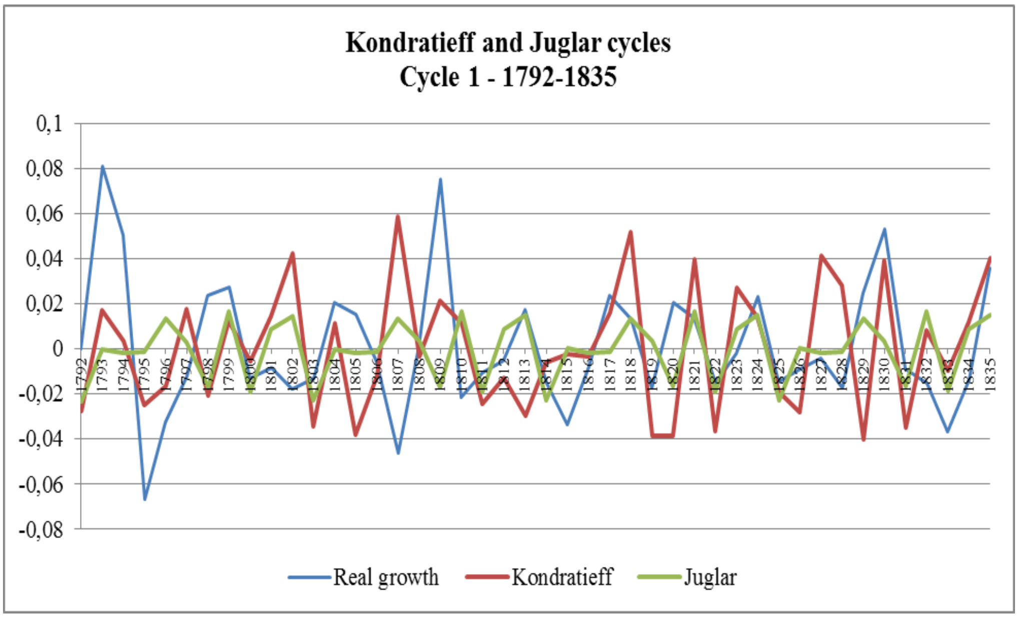

Figure 1

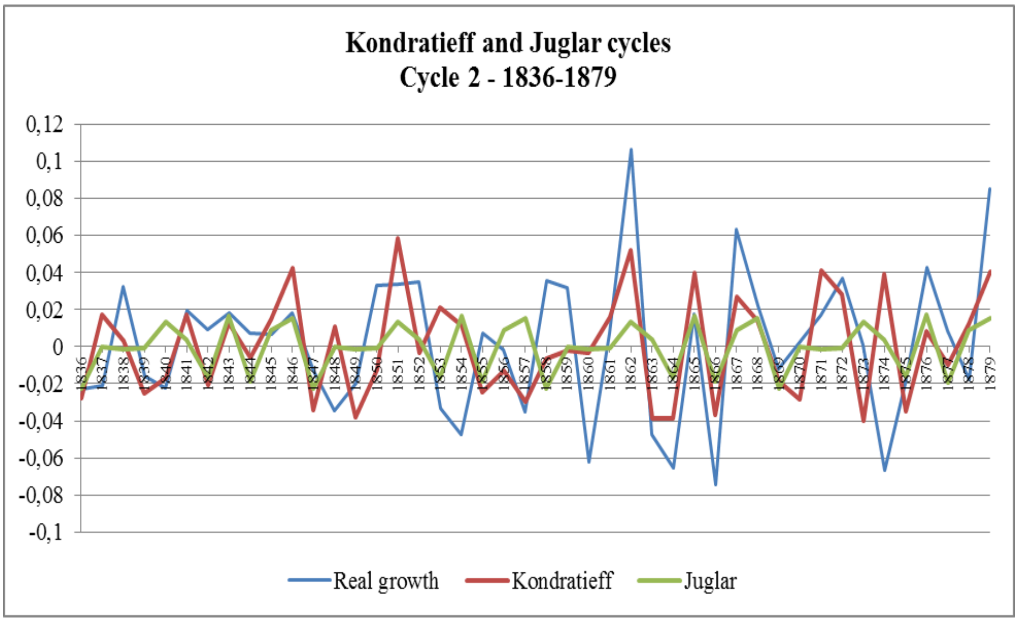

Figure 2

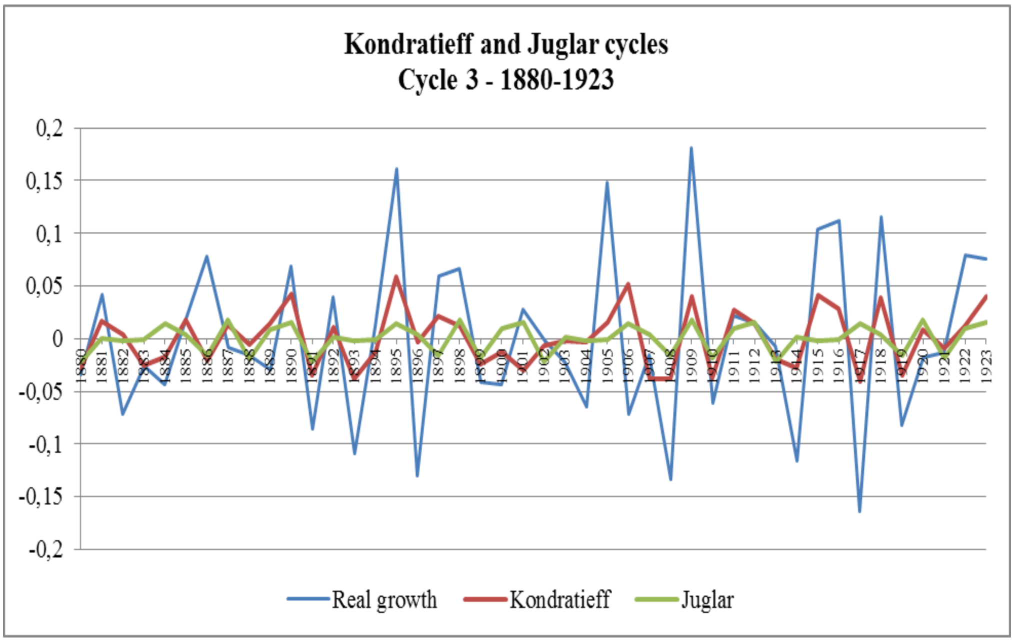

Figure 3

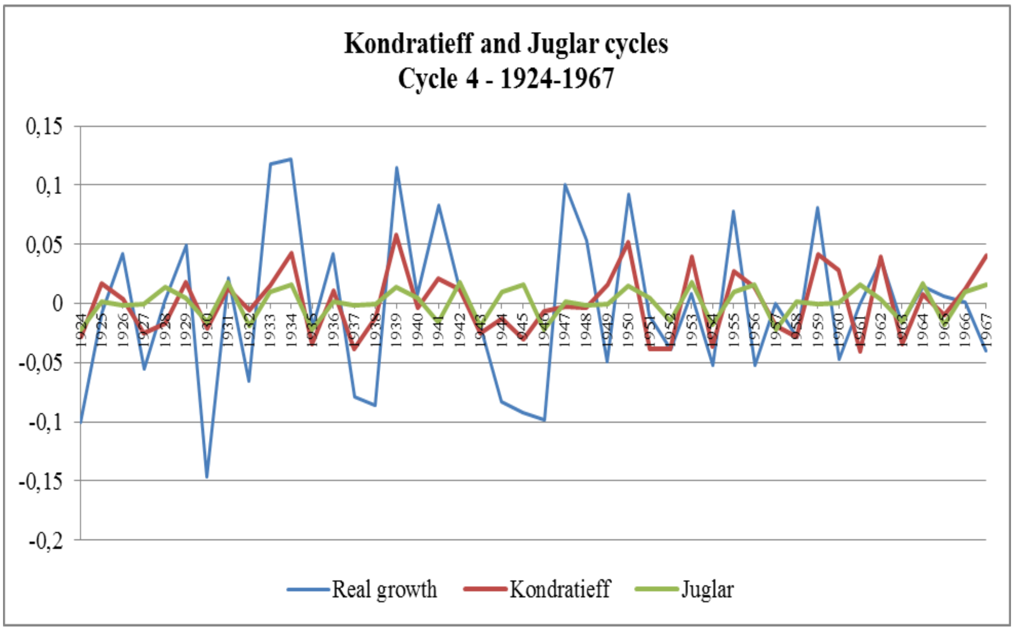

Figure 4

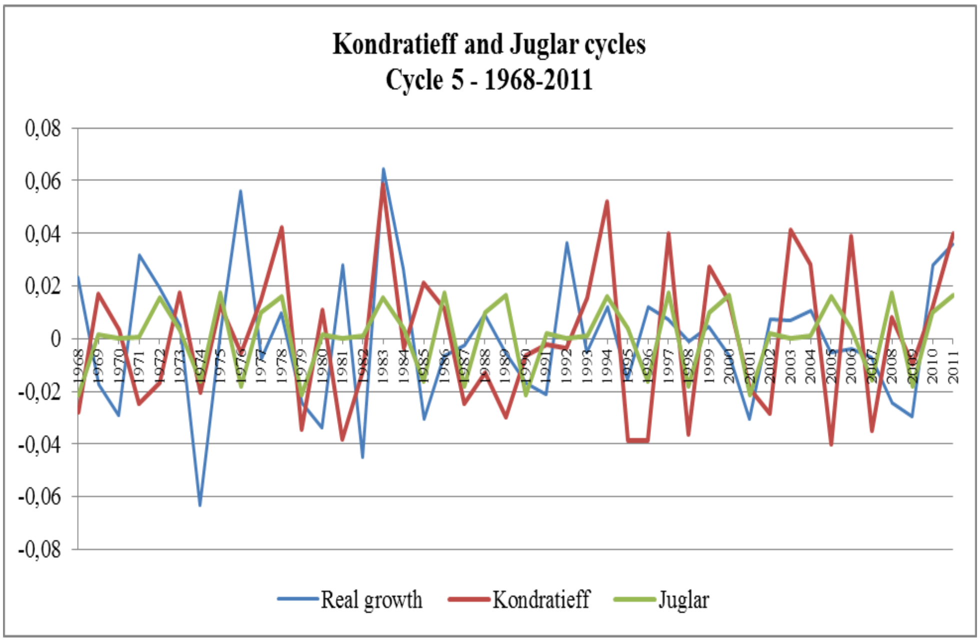

Figure 5

For Kuznet type cycles, we shall proceed similarly, resulting Fourier coefficients corresponding to a period of 22 years (the classical theory stating their duration as between 15 and 25 years) for a variable number of terms of the development ($n=\overline{1,20}$). The choose of this period was made simply to fit a whole number of Kuznet cycles in one of type Kondratieff, noting that between different periods (in the range 15-25 years) the mean square deviation being not significantly different.

Thus, the computer analysis revealed the error σ=0.04962 for the period T=22 and a number of 10 terms of development.

Finally, we obtain the new values (periodic, of period 22) of v_k.

Table 5

The absolute variation in the growth rate for the U.S. economy during 1792-2012

obtained from Fourier analysis (corresponding to Kuznet cycles)

Year	$v_k=\Delta r_k$ real	$v_k=\Delta r_k$ Fourier	Year	$v_k=\Delta r_k$ real	$v_k=\Delta r_k$ Fourier	Year	$v_k=\Delta r_k$ real	$v_k=\Delta r_k$ Fourier
1792	0	-0.0171	1866	-0.0745	-0.0193	1940	0.0069	0.0154
1793	0.0808	0.0074	1867	0.063	0.0202	1941	0.083	-0.0036
1794	0.0505	0.0004	1868	0.0219	0.0298	1942	0.0138	0.01
1795	-0.067	-0.0068	1869	-0.0118	-0.0276	1943	-0.0208	-0.0179
1796	-0.0324	0.0138	1870	0.0029	-0.0064	1944	-0.0829	-0.0018
1797	-0.0124	-0.0075	1871	0.0171	-0.0028	1945	-0.092	0.0013
1798	0.0236	-0.0285	1872	0.037	0.0057	1946	-0.0982	-0.0171
1799	0.0272	0.0247	1873	0.0005	0.0146	1947	0.1004	0.0074
1800	-0.0132	-0.0193	1874	-0.0666	0.0154	1948	0.053	0.0004
1801	-0.0085	0.0202	1875	-0.0195	-0.0036	1949	-0.0491	-0.0068
1802	-0.0177	0.0298	1876	0.0427	0.01	1950	0.0925	0.0138
1803	-0.0134	-0.0276	1877	0.0086	-0.0179	1951	-0.0101	-0.0075
1804	0.0206	-0.0064	1878	-0.018	-0.0018	1952	-0.039	-0.0285
1805	0.0152	-0.0028	1879	0.0847	0.0013	1953	0.0077	0.0247
1806	-0.0061	0.0057	1880	-0.0336	-0.0171	1954	-0.0523	-0.0193
1807	-0.0461	0.0146	1881	0.0421	0.0074	1955	0.0783	0.0202
1808	0.001	0.0154	1882	-0.0718	0.0004	1956	-0.0522	0.0298
1809	0.0749	-0.0036	1883	-0.026	-0.0068	1957	0.0004	-0.0276
1810	-0.0214	0.01	1884	-0.0436	0.0138	1958	-0.0292	-0.0064
1811	-0.0104	-0.0179	1885	0.0198	-0.0075	1959	0.0807	-0.0028
1812	-0.0056	-0.0018	1886	0.0776	-0.0285	1960	-0.0469	0.0057
1813	0.0175	0.0013	1887	-0.0085	0.0247	1961	-0.0015	0.0146
1814	-0.0153	-0.0171	1888	-0.015	-0.0193	1962	0.0373	0.0154
1815	-0.0339	0.0074	1889	-0.0289	0.0202	1963	-0.0169	-0.0036
1816	-0.0079	0.0004	1890	0.0686	0.0298	1964	0.0142	0.01
1817	0.0234	-0.0068	1891	-0.0857	-0.0276	1965	0.0063	-0.0179
1818	0.0132	0.0138	1892	0.0395	-0.0064	1966	0.001	-0.0018
1819	-0.0175	-0.0075	1893	-0.1092	-0.0028	1967	-0.0399	0.0013
1820	0.0206	-0.0285	1894	0.0109	0.0057	1968	0.0231	-0.0171
1821	0.0137	0.0247	1895	0.1612	0.0146	1969	-0.0173	0.0074

1822	-0.0152	-0.0193	1896	-0.1305	0.0154	1970	-0.0292	0.0004
1823	-0.002	0.0202	1897	0.0597	-0.0036	1971	0.0317	-0.0068
1824	0.0232	0.0298	1898	0.0663	0.01	1972	0.0195	0.0138
1825	-0.0149	-0.0276	1899	-0.0411	-0.0179	1973	0.0048	-0.0075
1826	-0.0091	-0.0064	1900	-0.0434	-0.0018	1974	-0.0634	-0.0285
1827	-0.0044	-0.0028	1901	0.0282	0.0013	1975	0.0034	0.0247
1828	-0.0175	0.0057	1902	-0.0018	-0.0171	1976	0.0558	-0.0193
1829	0.0249	0.0146	1903	-0.0224	0.0074	1977	-0.0077	0.0202
1830	0.0532	0.0154	1904	-0.0643	0.0004	1978	0.0098	0.0298
1831	-0.009	-0.0036	1905	0.148	-0.0068	1979	-0.0246	-0.0276
1832	-0.0151	0.01	1906	-0.0717	0.0138	1980	-0.0339	-0.0064
1833	-0.0367	-0.0179	1907	-0.0154	-0.0075	1981	0.0281	-0.0028
1834	-0.0138	-0.0018	1908	-0.1337	-0.0285	1982	-0.0448	0.0057
1835	0.0359	0.0013	1909	0.1804	0.0247	1983	0.0646	0.0146
1836	-0.0232	-0.0171	1910	-0.0615	-0.0193	1984	0.0267	0.0154
1837	-0.0208	0.0074	1911	0.0216	0.0202	1985	-0.0305	-0.0036
1838	0.0326	0.0004	1912	0.0144	0.0298	1986	-0.0068	0.01
1839	-0.016	-0.0068	1913	-0.0073	-0.0276	1987	-0.0026	-0.0179
1840	-0.0226	0.0138	1914	-0.1162	-0.0064	1988	0.0091	-0.0018
1841	0.0197	-0.0075	1915	0.104	-0.0028	1989	-0.0054	0.0013
1842	0.0091	-0.0285	1916	0.1114	0.0057	1990	-0.0169	-0.0171
1843	0.018	0.0247	1917	-0.1634	0.0146	1991	-0.0211	0.0074
1844	0.0071	-0.0193	1918	0.1149	0.0154	1992	0.0362	0.0004
1845	0.0065	0.0202	1919	-0.0822	-0.0036	1993	-0.0054	-0.0068
1846	0.0179	0.0298	1920	-0.0174	0.01	1994	0.0122	0.0138
1847	-0.0132	-0.0276	1921	-0.0136	-0.0179	1995	-0.0156	-0.0075
1848	-0.0344	-0.0064	1922	0.0787	-0.0018	1996	0.0123	-0.0285
1849	-0.0197	-0.0028	1923	0.0758	0.0013	1997	0.0072	0.0247
1850	0.0327	0.0057	1924	-0.1006	-0.0171	1998	-0.001	-0.0193
1851	0.0339	0.0146	1925	-0.0075	0.0074	1999	0.0047	0.0202
1852	0.0348	0.0154	1926	0.0419	0.0004	2000	-0.0069	0.0298
1853	-0.0335	-0.0036	1927	-0.0557	-0.0068	2001	-0.0306	-0.0276
1854	-0.0473	0.01	1928	0.0019	0.0138	2002	0.0073	-0.0064
1855	0.0072	-0.0179	1929	0.049	-0.0075	2003	0.0068	-0.0028
1856	-0.0015	-0.0018	1930	-0.1467	-0.0285	2004	0.0108	0.0057
1857	-0.0351	0.0013	1931	0.0213	0.0247	2005	-0.0052	0.0146
1858	0.0354	-0.0171	1932	-0.0658	-0.0193	2006	-0.0038	0.0154
1859	0.0319	0.0074	1933	0.1177	0.0202	2007	-0.0072	-0.0036
1860	-0.0622	0.0004	1934	0.1219	0.0298	2008	-0.0246	0.01

1861	0.0076	-0.0068	1935	-0.02	-0.0276	2009	-0.0298	-0.0179
1862	0.1064	0.0138	1936	0.0416	-0.0064	2010	0.028	-0.0018
1863	-0.0472	-0.0075	1937	-0.0793	-0.0028	2011	0.036	0.0013
1864	-0.0656	-0.0285	1938	-0.0856	0.0057	2012	-0.0007	-0.0171
1865	0.0172	0.0247	1939	0.1152	0.0146			

The evolution's graphs of the pair Kondratieff-Kuznet cycles corresponding to the 5 so determined are:

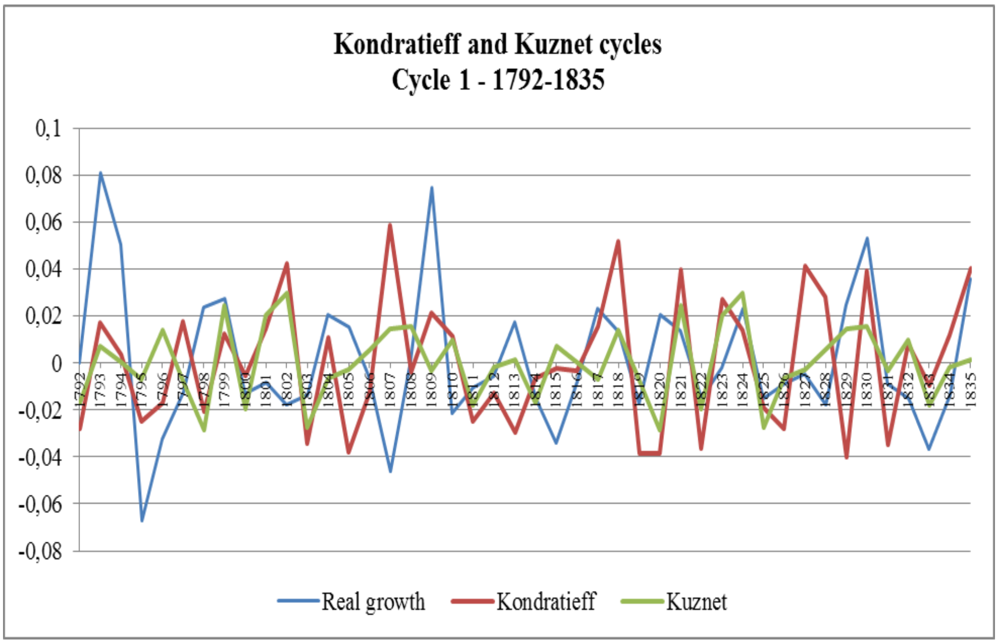

Figure 6

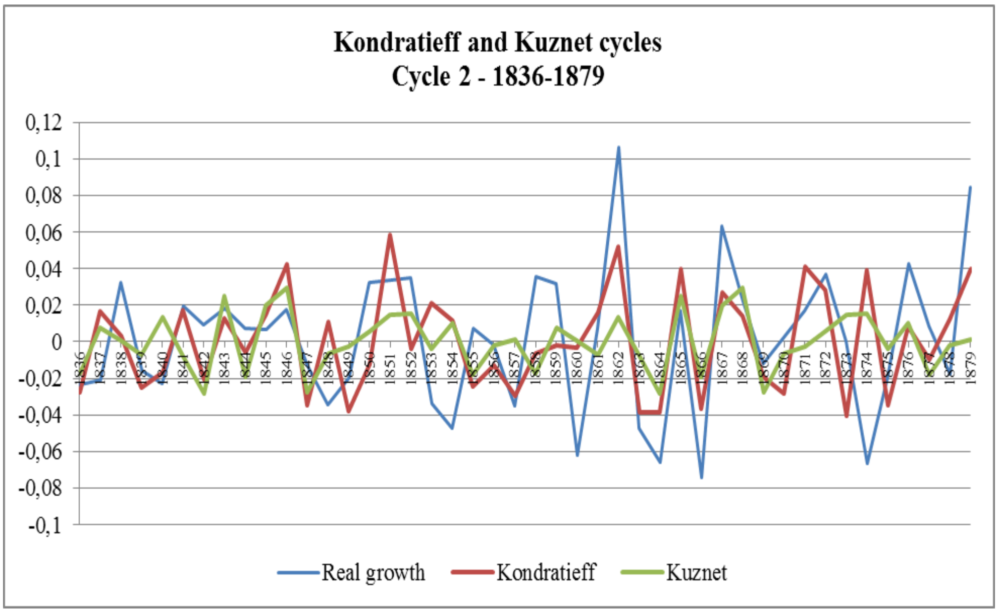

Figure 7

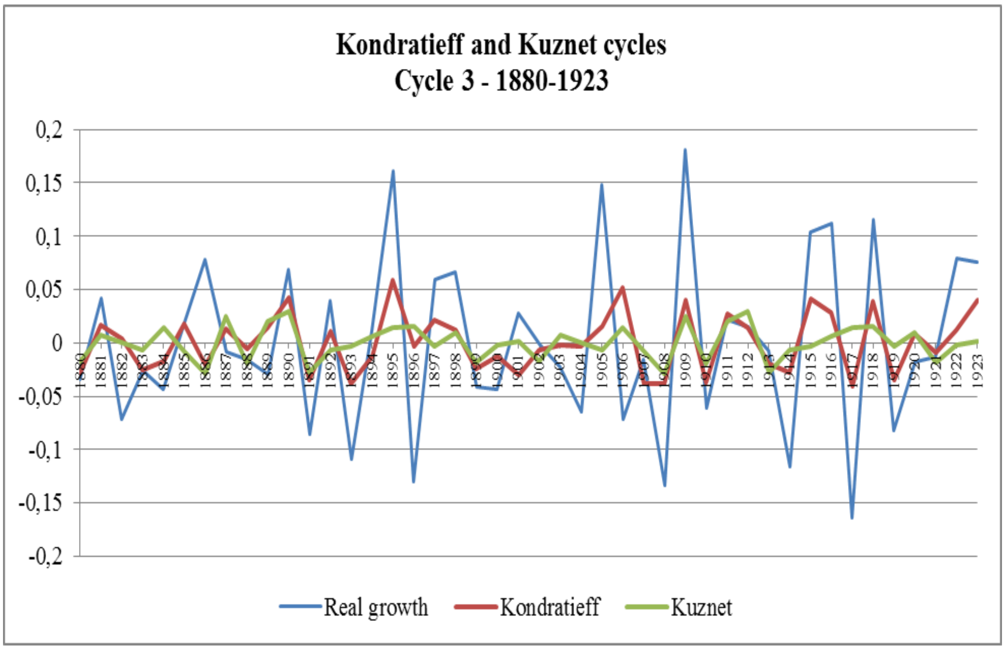

Figure 8

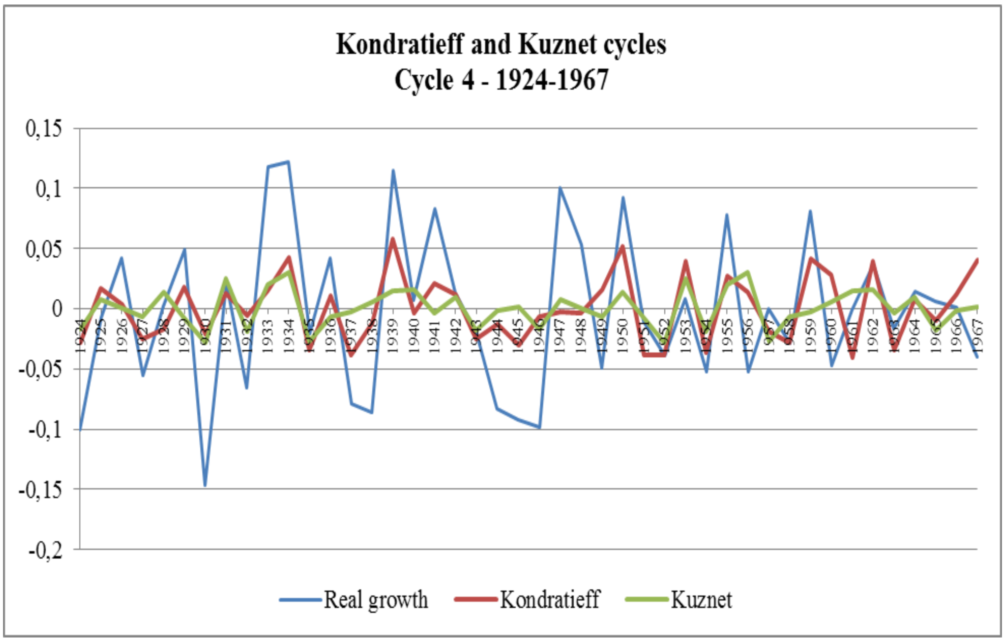

Figure 9

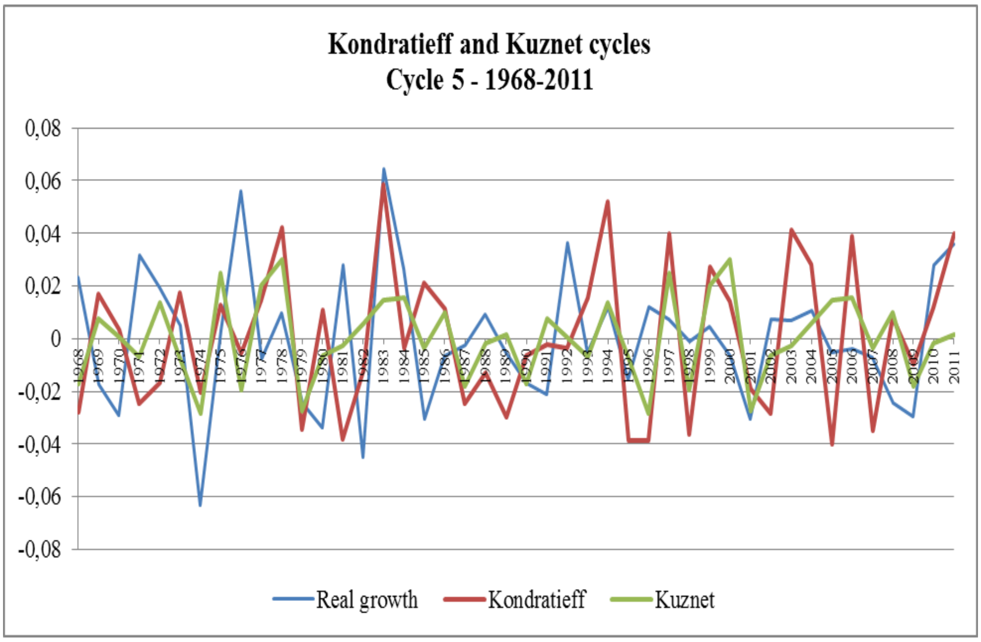

Figure 10

For the Kitchin cycles, we shall proceed similarly, determining the Fourier coefficients corresponding to a period of 4 years (classical theory stating their duration as 3 to 5 years) for a variable number of terms of development ($n=\overline{1,20}$). Choosing this period was made for the reason to fit a whole number of Kitchin cycles in one of type Kondratieff, noting that between different periods (in the range 3-5 years) the mean square deviation being not significantly different. Thus, the computer analysis revealed the error σ=0.05154 for the period T=4 and for 2 terms of development.

Finally, we obtain the new values (periodic of period 4) of v_k.

Table 6

The absolute variation in the growth rate for the U.S. economy during 1792-2012
obtained from Fourier analysis (corresponding to Kitchin cycles)

Year	$v_k=\Delta r_k$ real	$v_k=\Delta r_k$ Fourier	Year	$v_k=\Delta r_k$ real	$v_k=\Delta r_k$ Fourier	Year	$v_k=\Delta r_k$ real	$v_k=\Delta r_k$ Fourier
1792	0	-0.01	1866	-0.0745	-0.0019	1940	0.0069	-0.0113
1793	0.0808	0.0047	1867	0.063	0.0086	1941	0.083	0.007
1794	0.0505	-0.0007	1868	0.0219	-0.0105	1942	0.0138	-0.0041
1795	-0.067	0.0065	1869	-0.0118	0.0062	1943	-0.0208	0.0108
1796	-0.0324	-0.01	1870	0.0029	-0.0006	1944	-0.0829	-0.0101
1797	-0.0124	0.0047	1871	0.0171	0.0074	1945	-0.092	0.0057
1798	0.0236	-0.0008	1872	0.037	-0.0119	1946	-0.0982	-0.0028

1799	0.0272	0.0065	1873	0.0005	0.0076	1947	0.1004	0.0096
1800	-0.0132	-0.0101	1874	-0.0666	0.0007	1948	0.053	-0.0141
1801	-0.0085	0.0048	1875	-0.0195	0.0061	1949	-0.0491	0.0045
1802	-0.0177	-0.0011	1876	0.0427	-0.0106	1950	0.0925	-0.0015
1803	-0.0134	0.0066	1877	0.0086	0.0063	1951	-0.0101	0.0083
1804	0.0206	-0.0098	1878	-0.018	-0.0033	1952	-0.039	-0.0128
1805	0.0152	0.0048	1879	0.0847	0.0101	1953	0.0077	0.0085
1806	-0.0061	-0.0012	1880	-0.0336	-0.0093	1954	-0.0523	-0.0055
1807	-0.0461	0.0066	1881	0.0421	0.005	1955	0.0783	0.0123
1808	0.001	-0.0098	1882	-0.0718	-0.0021	1956	-0.0522	-0.0168
1809	0.0749	0.0048	1883	-0.026	0.0088	1957	0.0004	0.0019
1810	-0.0214	-0.0012	1884	-0.0436	-0.0133	1958	-0.0292	0.001
1811	-0.0104	0.0067	1885	0.0198	0.0037	1959	0.0807	0.0057
1812	-0.0056	-0.0099	1886	0.0776	-0.0008	1960	-0.0469	-0.0102
1813	0.0175	0.0056	1887	-0.0085	0.0076	1961	-0.0015	0.0059
1814	-0.0153	-0.0013	1888	-0.015	-0.0121	1962	0.0373	-0.003
1815	-0.0339	0.0067	1889	-0.0289	0.0077	1963	-0.0169	0.0097
1816	-0.0079	-0.0099	1890	0.0686	0.0005	1964	0.0142	-0.0143
1817	0.0234	0.0043	1891	-0.0857	0.0063	1965	0.0063	0.0099
1818	0.0132	-0.0013	1892	0.0395	-0.0108	1966	0.001	-0.007
1819	-0.0175	0.0068	1893	-0.1092	0.0065	1967	-0.0399	0.0138
1820	0.0206	-0.01	1894	0.0109	-0.0035	1968	0.0231	-0.0077
1821	0.0137	0.0056	1895	0.1612	0.005	1969	-0.0173	0.0034
1822	-0.0152	-0.0014	1896	-0.1305	-0.0095	1970	-0.0292	-0.0004
1823	-0.002	0.0068	1897	0.0597	0.0052	1971	0.0317	0.0072
1824	0.0232	-0.01	1898	0.0663	-0.0022	1972	0.0195	-0.0117
1825	-0.0149	0.0044	1899	-0.0411	0.009	1973	0.0048	0.0074
1826	-0.0091	-0.0014	1900	-0.0434	-0.0135	1974	-0.0634	-0.0044
1827	-0.0044	0.0069	1901	0.0282	0.0039	1975	0.0034	0.0112
1828	-0.0175	-0.0101	1902	-0.0018	-0.001	1976	0.0558	-0.0157
1829	0.0249	0.0057	1903	-0.0224	0.0077	1977	-0.0077	0.0114
1830	0.0532	-0.0015	1904	-0.0643	-0.0122	1978	0.0098	0.0021
1831	-0.009	0.0069	1905	0.148	0.0079	1979	-0.0246	0.0046
1832	-0.0151	-0.0114	1906	-0.0717	0.0003	1980	-0.0339	-0.0091
1833	-0.0367	0.0045	1907	-0.0154	0.0065	1981	0.0281	0.0048
1834	-0.0138	-0.0015	1908	-0.1337	-0.011	1982	-0.0448	-0.0019
1835	0.0359	0.0083	1909	0.1804	0.0066	1983	0.0646	0.0086
1836	-0.0232	-0.0101	1910	-0.0615	-0.0037	1984	0.0267	-0.0132
1837	-0.0208	0.0058	1911	0.0216	0.0105	1985	-0.0305	0.0088

1838	0.0326	-0.0002	1912	0.0144	-0.0097	1986	-0.0068	-0.0059
1839	-0.016	0.007	1913	-0.0073	0.0054	1987	-0.0026	0.0127
1840	-0.0226	-0.0115	1914	-0.1162	-0.0024	1988	0.0091	-0.0172
1841	0.0197	0.0045	1915	0.104	0.0092	1989	-0.0054	0.0023
1842	0.0091	-0.0016	1916	0.1114	-0.0137	1990	-0.0169	0.0007
1843	0.018	0.0057	1917	-0.1634	0.0041	1991	-0.0211	0.0061
1844	0.0071	-0.0102	1918	0.1149	-0.0011	1992	0.0362	-0.0106
1845	0.0065	0.0059	1919	-0.0822	0.0079	1993	-0.0054	0.0063
1846	0.0179	-0.0003	1920	-0.0174	-0.0124	1994	0.0122	-0.0033
1847	-0.0132	0.0071	1921	-0.0136	0.0081	1995	-0.0156	0.0101
1848	-0.0344	-0.0116	1922	0.0787	0.0001	1996	0.0123	-0.0146
1849	-0.0197	0.0046	1923	0.0758	0.0066	1997	0.0072	0.0103
1850	0.0327	-0.0017	1924	-0.1006	-0.0112	1998	-0.001	-0.0074
1851	0.0339	0.0085	1925	-0.0075	0.0068	1999	0.0047	0.0035
1852	0.0348	-0.0103	1926	0.0419	-0.0039	2000	-0.0069	-0.008
1853	-0.0335	0.006	1927	-0.0557	0.0054	2001	-0.0306	0.0037
1854	-0.0473	-0.0004	1928	0.0019	-0.0099	2002	0.0073	-0.0008
1855	0.0072	0.0072	1929	0.049	0.0056	2003	0.0068	0.0076
1856	-0.0015	-0.0117	1930	-0.1467	-0.0026	2004	0.0108	-0.0121
1857	-0.0351	0.0047	1931	0.0213	0.0094	2005	-0.0052	0.0077
1858	0.0354	-0.0018	1932	-0.0658	-0.0139	2006	-0.0038	-0.0048
1859	0.0319	0.0059	1933	0.1177	0.0043	2007	-0.0072	0.0116
1860	-0.0622	-0.0104	1934	0.1219	-0.0013	2008	-0.0246	-0.0161
1861	0.0076	0.0061	1935	-0.02	0.0081	2009	-0.0298	0.0118
1862	0.1064	-0.0005	1936	0.0416	-0.0126	2010	0.028	0.0018
1863	-0.0472	0.0073	1937	-0.0793	0.0083	2011	0.036	0.005
1864	-0.0656	-0.0118	1938	-0.0856	-0.0001	2012	-0.0007	-0.0095
1865	0.0172	0.0048	1939	0.1152	0.0068			

The evolution's graphs of the pair Kondratieff-Kitchin cycles corresponding to the 5 so determined are:

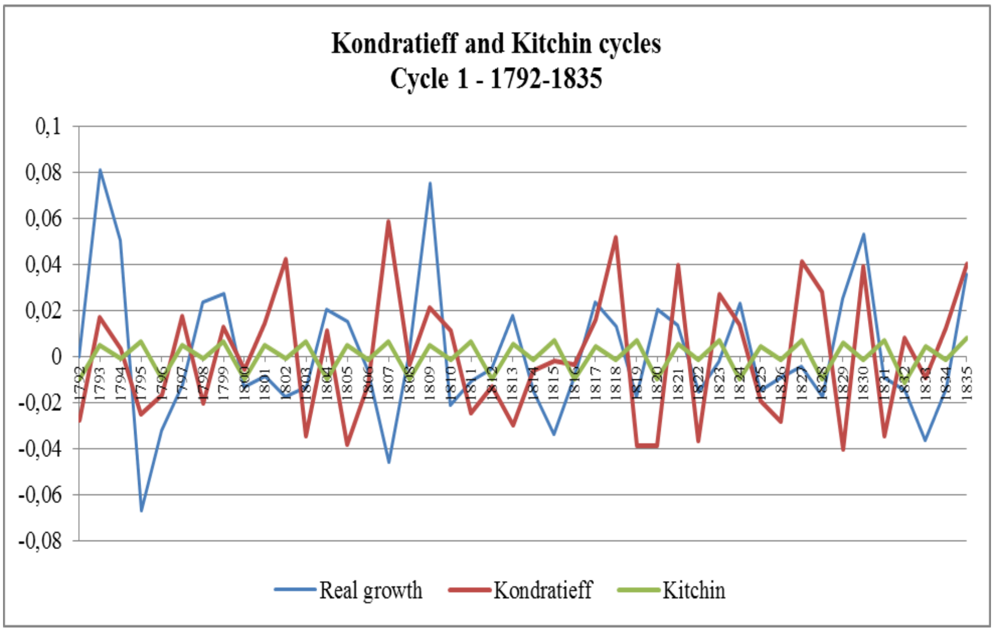

Figure 11

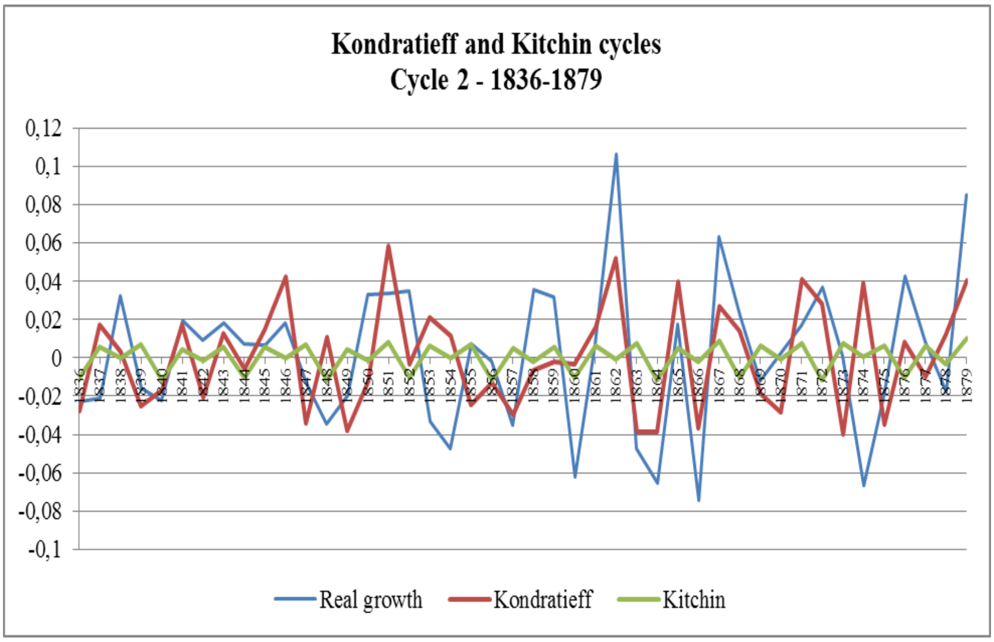

Figure 12

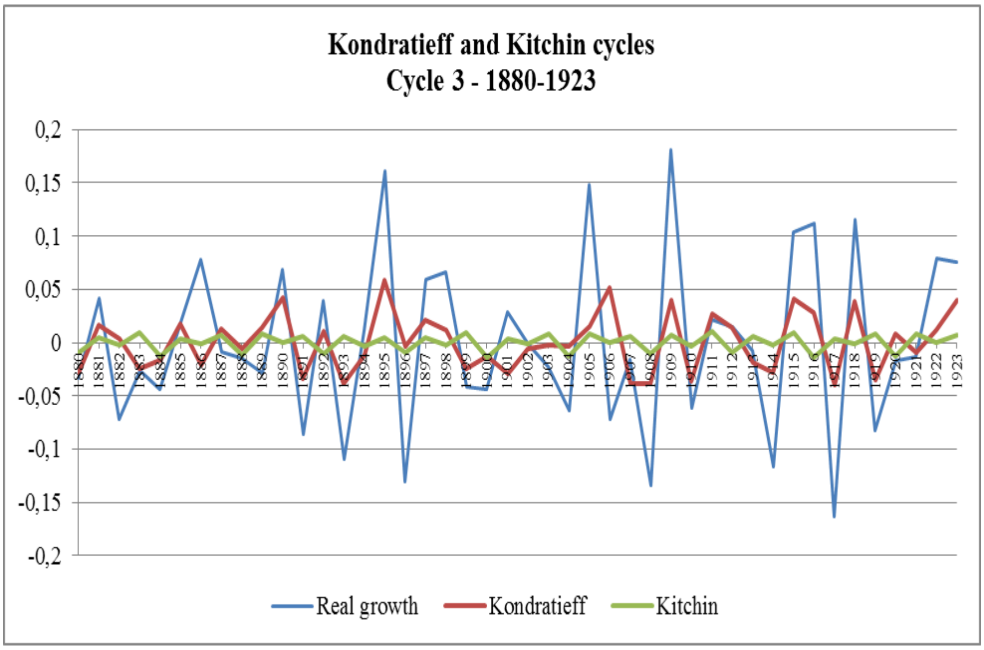

Figure 13

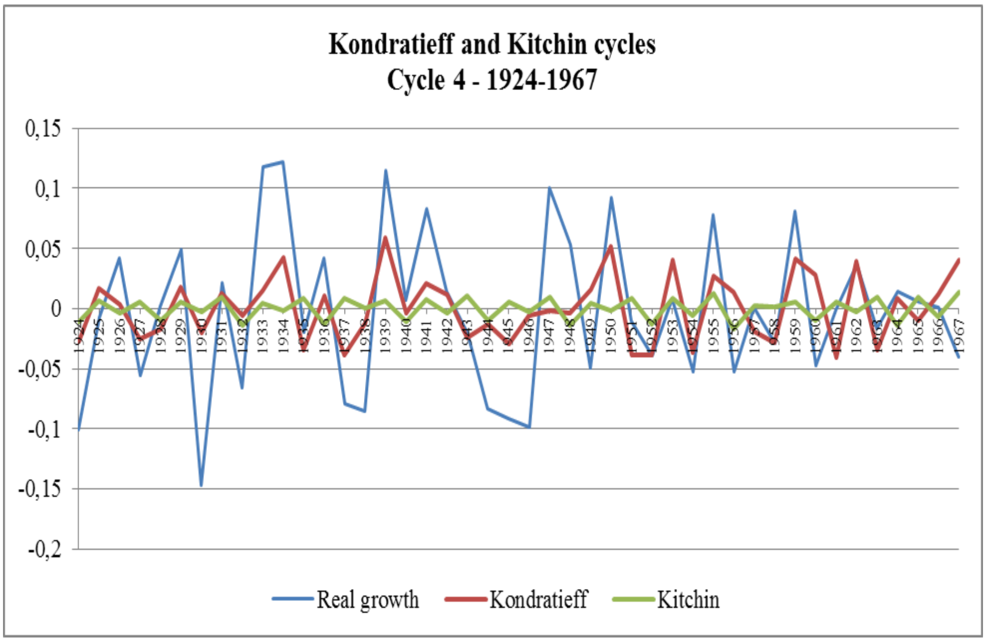

Figure 14

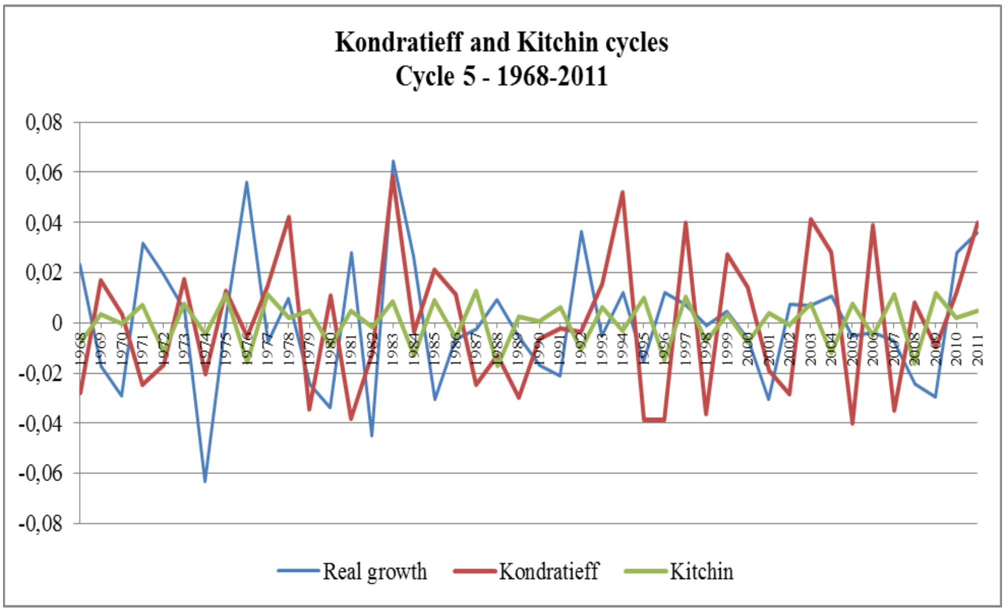

Figure 15

5 Conclusions

The above analysis, rigorously determine the positioning of the four types of cycles using Fourier analysis.

It thus appears that for a period of 44 years if Kondratieff cycle, Juglar cycle will extend over a period of 11 years, in the case of Kuznet to 22 years and for Kitchin - 4 years.

Clearly, from the above, it results that the allocation in different periods for the four types of cycles was performed from purely numerical reasons not revealing the causes.

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