# Determination of underground economy with Schneider model trend in Romania with analytical methods

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**Abstract:** Trend determination of underground economy with Schneider model in Romania with analytical methods requires least squares method. Comparing the results of the absolute deviations between empirical and theoretical values for the linear, curvilinear and modified exponential regression, will choose the best trend equation for the smallest variation. The best trend model for underground economy with Schneider model in Romania is modelled using curvilinear regression equation equation.

**Keywords:** underground economy, Schneider model, least squares method, trend

## 1 Introduction

Given data on the evolution of underground economy with Schneider model in Romania between 2004 and 2010, we will adjust the series by least squares method.

We will calculate the linear, curvilinear and exponential modified regression, with the method of least squares for determining the trend of underground economy - Schneider model in Romania.

Then, on the base of the coefficients of variation we will analyze the smallest variation $\sum(y_i - y_t)^2$ or for $\sum|y_i - y_t|$ and after we can choose the best trend.

## 2 Determining the linear trend for the underground economy with Schneider model

For a linear regression, least squares method involves solving the following system of equations:

$$\begin{cases} \sum y_i = n * a + b\sum t_i \\ \sum t_i y_i = a\sum t_i + b * \sum t_i^2 \end{cases}$$

We will consider origin of the time variable the centre of the series such that $\sum t_i = 0$, because the terms of the series are consecutive numbers and the anterior system of equations becomes:

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$$\begin{cases} \sum y_i = n * a \\ \sum t_i\, y_i = b * \sum t_i^2 \end{cases}$$

**Table no. 1**

| Years | $y_i$ | $t_i$ | $t_i\, y_i$ | $t_i^2$ | $y_t = 37{,}257 - 0{,}082 t_i$ | $\lvert y_i - y_t \rvert$ |
|---|---|---|---|---|---|---|
| 2004 | 37,0 | -3 | -111 | 9 | 37,50 | 0,5 |
| 2005 | 37,3 | -2 | -74,6 | 4 | 37,42 | 0,1 |
| 2006 | 38,3 | -1 | -38,3 | 1 | 37,34 | 1,0 |
| 2007 | 37,5 | 0 | 0 | 0 | 37,26 | 0,2 |
| 2008 | 36,8 | 1 | 36,8 | 1 | 37,18 | 0,4 |
| 2009 | 36,9 | 2 | 73,8 | 4 | 37,09 | 0,2 |
| 2010 | 37 | 3 | 111 | 9 | 37,01 | 0,0 |
| Total | 260,8 | 0 | -2,3 | 28 | 260,80 | **2,4** |
| 2011 forecast | | 4 | | 16 | 36,93 | |
| 2012 forecast | | 5 | | 25 | 36,85 | |
| 2013 forecast | | 6 | | 36 | 36,77 | |
| 2014 forecast | | 7 | | 49 | 36,68 | |
| 2015 forecast | | 8 | | 64 | 36,60 | |
| 2016 forecast | | 9 | | 81 | 36,52 | |
| 2017 forecast | | 10 | | 100 | 36,44 | |

So, on the data in Table no. 1 the system of equations becomes:

$$\begin{cases} 260{,}8 = 7a \\ -2{,}3 = 28b \end{cases}$$

We will obtain:

$$\begin{cases} a = \dfrac{260,8}{7} \\ b = \dfrac{-2,3}{28} \end{cases}$$

$$\begin{cases} a = 37,257 \\ b = -0,082 \end{cases}$$

The linear regression equation of underground economy with Schneider model is:

$$y_t = a + bt_i = 37,257 - 0,082t_i$$

It can be observed that the linear regression equation for the underground economy with Schneider model is $y_t = 37,257 - 0,082t_i$ in the Table no.1.

## 3 Determining the curvilinear regression equation for underground economy with Schneider model

For a curvilinear regression, least squares method involves solving the following system of equations:

$$\begin{cases} \sum y_i = n * a + b\sum t_i + c\sum t_i^2 \\ \sum t_i y_i = a\sum t_i + b * \sum t_i^2 + c * \sum t_i^3 \\ \sum t_i^2 y_i = a\sum t_i^2 + b * \sum t_i^3 + c * \sum t_i^4 \end{cases}$$

We will consider origin of the time variable the centre of the series such that $\sum t_i = 0$, because the terms of the series are consecutive numbers and the anterior system of equations becomes:

$$\begin{cases} \sum y_i = n * a + c\sum t_i^2 \\ \sum t_i y_i = b * \sum t_i^2 \\ \sum t_i^2 y_i = a\sum t_i^2 + c * \sum t_i^4 \end{cases}$$

**Table no. 2**

| Years | $y_i$ | $t_i$ | $t_i y_i$ | $t_i^2$ | $t_i^3$ | $t_i^4$ | $y_i t_i^2$ | $y_t = a + bt_i + ct_i^2$ | $\lvert y_i - y_t \rvert$ |
|---|---|---|---|---|---|---|---|---|---|
| 2004 | 37,0 | -3 | -111 | 9 | -27 | 81 | 333 | 37,19 | 0,19 |
| 2005 | 37,3 | -2 | -74,6 | 4 | -8 | 16 | 149,2 | 37,42 | 0,12 |
| 2006 | 38,3 | -1 | -38,3 | 1 | -1 | 1 | 38,3 | 37,53 | 0,77 |
| 2007 | 37,5 | 0 | 0 | 0 | 0 | 0 | 0 | 37,51 | 0,01 |
| 2008 | 36,8 | 1 | 36,8 | 1 | 1 | 1 | 36,8 | 37,36 | 0,56 |
| 2009 | 36,9 | 2 | 73,8 | 4 | 8 | 16 | 147,6 | 37,09 | 0,19 |
| 2010 | 37 | 3 | 111 | 9 | 27 | 81 | 333 | 36,70 | 0,30 |
| | 260,8 | 0 | -2,3 | 28 | 0 | 196 | 1037,9 | 260,80 | **2,15** |
| 2011 forecast | | 4 | | 16 | | | | 36,17 | |
| 2012 forecast | | 5 | | 25 | | | | 35,52 | |
| 2013 forecast | | 6 | | 36 | | | | 34,75 | |
| 2014 forecast | | 7 | | 49 | | | | 33,85 | |
| 2015 forecast | | 8 | | 64 | | | | 32,82 | |
| 2016 forecast | | 9 | | 81 | | | | 31,67 | |
| 2017 forecast | | 10 | | 100 | | | | 30,39 | |

So, on the data in the Table no. 2 the system of equations becomes:

$$\begin{cases} 260{,}8 = 7a + 28c \\ -2{,}3 = 28b \\ 1037{,}9 = 28a + 196c \end{cases}$$

We will obtain:

$$\begin{cases} b = \frac{-2,3}{28} \\ 260,8 = 7a + 28c \\ 1037,9 = 28a + 196c \end{cases} \quad |(-4) \quad \rightarrow \quad \begin{array}{c} b = -0,082 \\ -1043,2 = -28a - 112c \\ 1037,9 = 28a + 196c \end{array}$$

$$-5,3 = 0 + 84c$$

$$c = \frac{-5,3}{84} = -0,063$$

$$\begin{cases} b = -0,082 \\ c = -0,063 \\ 1037,9 = 28a - 196 * 0,063 \end{cases} \rightarrow$$

$$\begin{cases} b = -0,082 \\ c = -0,063 \\ 1037,9 = 28a - 196 * 0,063 \end{cases}$$

$$\rightarrow \begin{cases} b = -0,082 \\ c = -0,063 \\ a = 37,509 \end{cases}$$

The curvilinear regression equation of underground economy with Schneider model is:

$$y_t = a + bt_i + ct_i^2 = 37,509 - 0,082t_i - 0,063t_i^2$$

It can be observed that the curvilinear regression equation for the underground economy with Schneider model is determined in column 9 of Table no.2.

## 4 Determining the regression equation type modified exponential $y_t = a * b^{ti}$ for underground economy with Schneider model

For a type modified exponential regression of the type $y_t = a * b^{ti}$, least squares method involves solving the following system of equations:

$$\begin{cases} \log a = \frac{\sum \log y_i}{n} \\ \log b = \frac{\sum t_i \log y_i}{\sum t_i^2} \end{cases}$$

We will consider origin of the time variable the centre of the series such that $\sum t_i = 0$, because the terms of the series are consecutive numbers.

**Table no. 3**

| Years | $y_i$ | $t_i$ | $t_i y_i$ | $t_i^2$ | log $y_i$ | $t_i$ log $y_i$ | **log $y_t$ = log a + $t_i$ log b** | **$y_t = a*b^{ti}$** | $\|y_i - y_t\|$ |
|---|---|---|---|---|---|---|---|---|---|
| 2004 | 37,0 | -3 | -111 | 9 | 1,5682 | -4,7046 | 1,57385 | 39,77 | 2,77 |
| 2005 | 37,3 | -2 | -74,6 | 4 | 1,57171 | -3,1434 | 1,5729 | 38,90 | 1,60 |
| 2006 | 38,3 | -1 | -38,3 | 1 | 1,5832 | -1,5832 | 1,57195 | 38,06 | 0,24 |
| 2007 | 37,5 | 0 | 0 | 0 | 1,57403 | 0 | 1,571 | 37,24 | 0,26 |
| 2008 | 36,8 | 1 | 36,8 | 1 | 1,56585 | 1,56585 | 1,57005 | 36,43 | 0,37 |
| 2009 | 36,9 | 2 | 73,8 | 4 | 1,56703 | 3,13405 | 1,5691 | 35,64 | 1,26 |
| 2010 | 37 | 3 | 111 | 9 | 1,5682 | 4,70461 | 1,56815 | 34,87 | 2,13 |
| **Total** | 260,8 | 0 | -2,3 | 28 | 10,9982 | -0,0267 | 10,997 | 260,92 | **8,62** |
| 2011 forecast | | 4 | | 16 | | | | 34,12 | |
| 2012 forecast | | 5 | | 25 | | | | 33,38 | |
| 2013 forecast | | 6 | | 36 | | | | 32,66 | |
| 2014 forecast | | 7 | | 49 | | | | 31,95 | |
| 2015 forecast | | 8 | | 64 | | | | 31,26 | |
| 2016 forecast | | 9 | | 81 | | | | 30,58 | |
| 2017 forecast | | 10 | | 100 | | | | 29,92 | |

So, on the data in the Table no. 3 the system of equations becomes:

$$\begin{cases} \log a = \dfrac{\sum \log y_i}{n} = \dfrac{10{,}9982}{7} = 1{,}571 \\ \log b = \dfrac{\sum t_i \log y_i}{\sum t_i^2} = \dfrac{-0{,}0267}{28} = -0{,}00095 \end{cases} \rightarrow \begin{cases} a = 37{,}239 \\ b = 0{,}97836 \end{cases} \rightarrow$$

Results that the exponential trend equation is:

$\log y_t = \log a + t_i \log b = 1{,}571 - 0{,}00095 t_i$, or

$y_t = 37{,}239 * 0{,}97836^{ti}$

Therefore, the modified exponential regression equation for the underground economy with Schneider model is determined in column 9 of Table no.3

## 5 Conclusions

Therefore**,** the best trend with the method of least squares for underground economy with Schneider model in Romania for the period 2004 - 2010 is what leads to minimum value for $\sum(y_i - y_t)^2$ or for $\sum|y_i - y_t|$.

The data obtained in previous calculations we can summarize in the following table, no. 4 thus:

**Table no. 4**

| Years | Linear regression equation | | Curvilinear regression equation | | Modified exponential regression equation | |
|---|---|---|---|---|---|---|
| | $y_t = a + bt_i$ | I $y_i$ - $y_t$I | $y_t = a + bt_i + ct_i^2$ | I $y_i$ - $y_t$I | **$y_t$=a*b$^{ti}$** | I $y_i$ - $y_t$I |
| 2004 | 37,50 | 0,5 | 37,19 | 0,19 | 39,77 | 2,77 |
| 2005 | 37,42 | 0,1 | 37,42 | 0,12 | 38,90 | 1,60 |
| 2006 | 37,34 | 1,0 | 37,53 | 0,77 | 38,06 | 0,24 |
| 2007 | 37,26 | 0,2 | 37,51 | 0,01 | 37,24 | 0,26 |
| 2008 | 37,18 | 0,4 | 37,36 | 0,56 | 36,43 | 0,37 |
| 2009 | 37,09 | 0,2 | 37,09 | 0,19 | 35,64 | 1,26 |
| 2010 | 37,01 | 0,0 | 36,70 | 0,30 | 34,87 | 2,13 |
| Total | 260,80 | **2,4** | 260,80 | **2,15** | 260,92 | **8,62** |

On the base of analyze the results synthesized in Table no. 4 the values for the linear and modified exponential equation regression are so close, but the lowest value is for the curvilinear regression equation, and this is the best trend with the method of least squares for underground economy with Schneider model in Romania in the analyzed period

# 6 References

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