About Short-Term Costs and Long-Term Costs

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Abstract. The paper treats the theory of short-term costs and long-term costs from an axiomatic perspective.

Keywords: cost, short-term, long-term

1 Introduction

Fixed costs (CF) are those costs that are independent of the value of production (rent, lighting costs, heating costs, interest etc.) and are paid whether or not production.

Quasi-fixed costs (CCF) are those costs which are also independent of the value of production, but are paid only if it exists production (e.g. advertising expenses).

Variable costs (CV) represents the total production costs that vary with output in the same direction (raw materials costs, wages, energy costs in the production process etc.).

The total cost (CT) of production is the sum of fixed and variable costs: CT=CF+CV.

Considering a level of output, we shall call average fixed cost (CFM) the value:

$$\text{CFM}=\frac{\text{CF}}{\text{Q}}$$

and represents the fixed cost per unit.

At a level Q of the output, the average variable cost (CVM) is the value:

$$\text{CVM}=\frac{\text{CV}}{\text{Q}}$$

and represents the variable cost per unit.

Similarly, the average total cost (CTM) is:

$$\text{CTM}=\frac{\text{CT}}{\text{Q}}$$

and represents the total cost of a unit of product.

From the expressions above, it follows:

$$\text{CTM}=\frac{\text{CT}}{\text{Q}}=\frac{\text{CF}}{\text{Q}}+\frac{\text{CV}}{\text{Q}}=\text{CFM}+\text{CVM}$$

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Considering a level of output Q, we call marginal cost (Cm) the value:

$$Cm=\frac{\partial CT}{\partial Q}$$

and represents the trend of variation of the total cost to a given production.

In terms of discrete, for two times t_1 and t_2 with values Q_1 and Q_2 of production, CT_1 and CT_2 - total costs at that times,we define:

$$Cm=\frac{CT_2-CT_1}{Q_2-Q_1}=\frac{\Delta CT}{\Delta Q}$$

meaning the variation of total cost required to change a unit of production.

2 Short-term costs and long-term costs

A production process is said to take place in a short term if at least one factor of production remains constant.

Typically, on short-term, we assume that the capital (all resources that contribute to the deployment of production without consumption record, but suffered a depreciation in time) is constant, as the land (geographical locations, mineral deposits etc.), the only significant variable being the labor. From the analysis of short-term production follows, in this case, that after a number of workers it manifests the law of decreasing marginal returns.

We shall note, to emphasize the special nature of these costs: CVS – the variable cost in the short term, CTS – the total cost of short-term, CVMS – the average variable cost in the short term, CTMS – the average total cost in the short term, CmS – the marginal cost on short-term, fixed costs fixed or quasi-fixed cost keeping the above notations: CF, CFM, respectively CCF because they can appear only in this situation.

We have therefore:

- $CTS=CF+CVS$;
- $CFM=\frac{CF}{Q}$;
- $CVMS=\frac{CVS}{Q}$;
- $CTMS=\frac{CTS}{Q}=CFM+CVMS$;
- $CmS=\frac{\partial CTS}{\partial Q}=\frac{\partial CVS}{\partial Q}$.

A production process is said to take place in the long term if all inputs are variable. Unlike the previous situation, in the long-term production, due to technological change or to improve the management, productivity may increase. On long-term, fixed costs are not recorded, so we consider CF=0, CFM=0 and CCF=0.

We note, in this case: CVL – the variable cost in the long term, CTL – the total cost in the long term, CVML – the average variable cost in the long term, CTML – the long-term average total cost and CmL – the long-term marginal cost.

We have now:

- CTL=CVL;
- $\text{CVML}=\frac{\text{CVL}}{Q}$;
- $\text{CTML}=\frac{\text{CTL}}{Q}=\text{CVML}$;
- $\text{CmL}=\frac{\partial \text{CTL}}{\partial Q}=\frac{\partial \text{CVL}}{\partial Q}$.

From the monotonicity property of the cost function with respect to the production, follows that CTL is increased relative to the output Q.

Relative to the two types of production we might make a few comments.

Let's consider for this question, two sets of inputs: fixed – 1,...,k and variable – k+1,...,n, $k\geq 1$ with prices $p_1,...,p_n$. It is obvious that this division into categories of factors makes sense only in the short term, because at long term they are all variable. We note also minimal consumption of fixed factors with $\overline{x}_1,...,\overline{x}_k$ which means that for any amount consumed by factor $x_i\leq\overline{x}_i$, $i=\overline{1,k}$, the fixed cost will be $p_i\ \overline{x}_i$ (for example the electricity bill relative to lighting for a production is the same regardless of the level of production achieved).

Fixed costs would be:

$$\text{CF}=\sum_{i=1}^{k} p_i\overline{x}_i$$

The total cost on short-term is the sum of fixed cost and the variable cost:

$$\text{CTS}=\text{CF}+\text{CVS}=\sum_{i=1}^{k} p_i\overline{x}_i+\sum_{j=k+1}^{n} p_j x_j$$

On long-term, fixed cost becomes variable, obtaining:

$$\text{CTL}=\text{CVL}=\sum_{i=1}^{k} p_i x_i+\sum_{j=k+1}^{n} p_j x_j$$

The difference between the two periods is therefore:

$$\text{CTL-CTS}=\sum_{i=1}^{k} p_i x_i+\sum_{j=k+1}^{n} p_j x_j-\sum_{i=1}^{k} p_i\overline{x}_i-\sum_{j=k+1}^{n} p_j x_j=\sum_{i=1}^{k} p_i\left(x_i-\overline{x}_i\right)$$

Therefore, if it is consumed the total amount of inputs 1,...,k then: $x_i\geq\overline{x}_i$, $i=\overline{1,k}$ so in this case: CTL≥CTS. If from each factor of production 1,...,k remain unused amounts, then: $x_i\leq\overline{x}_i$, $i=\overline{1,k}$ therefore: CTL≤CTS.

On the other hand, for a fixed period, because the needs of the production process, it is natural to consider that fixed inputs not consumed in larger quantities than they are currently available. The cost on long-term should not be mixed with some cost on short-term relative to a production which was done at a time when there were no changes in technology or other factors that contribute to reducing costs.

For this reason, we always have at a fixed time: $x_i\leq\overline{x}_i$, $i=\overline{1,k}$ therefore CTL≤CTS.

Absolutely natural, dividing costs at the production follows:

$$CTML=\frac{CTL}{Q}\leq\frac{CTS}{Q}=CTMS$$

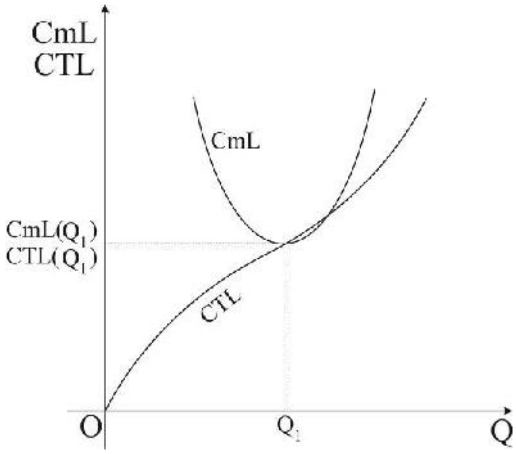
3 An axiomatic approach to cost

We shall impose to all these costs a number of axioms, namely:

C.1. The marginal cost CmL (CmS) is positive, convex and has a unique local minimum.

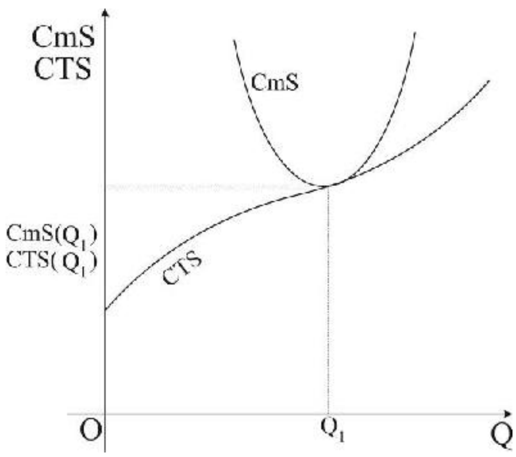
Be so Q_1 – the local minimum point of marginal cost and $CmL_m=CmL(Q_1)$. We therefore have: $CmL(Q)\geq CmL_m$, $CmL'(Q)<0\ \forall Q<Q_1$ and $CmL'(Q)>0\ \forall Q>Q_1$. Now $CTL''(Q)<0\ \forall Q<Q_1$ so the function CTL is concave for $Q<Q_1$ and $CTL''(Q)>0\ \forall Q>Q_1$ therefore the function CTL is convex for $Q>Q_1$. Due to the fact that in the case of the short-term marginal cost, the derivative of fixed cost is null (being constant) these statements remain valid for variable cost in the short term.

On the other hand, from the axiom C.1 the functions CmL and CmS are convex in the neighbourhood of Q_1. Also, the point of minimum of the marginal cost coincides with the inflection point of the curve CTL and that of CTS.



Marginal costs and total cost on long-term

Figure 1



Marginal costs and total cost on short-term

Figure 2

C.2. The average variable cost CVML (CVMS) is positive, convex and has a unique local minimum point.

Let Q_2 – the minimum point of CVML and $CVML_m = CVML(Q_2) = \frac{CVL(Q_2)}{Q_2}$.

The nature of the point Q_2 implies that: $CVML'(Q)<0\ \forall Q<Q_2$ and $CVML'(Q)>0\ \forall Q>Q_2$.

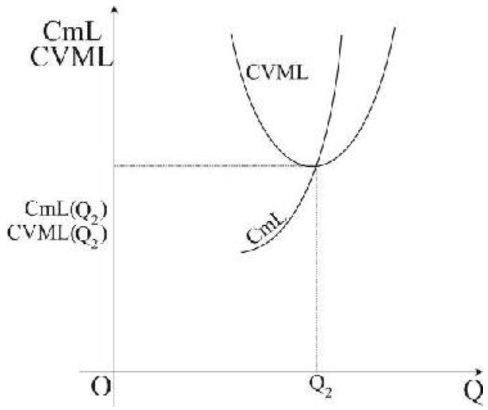
On the other hand:

$$CVML' = \left(\frac{CVL}{Q}\right)' = \frac{CVL' \cdot Q - CVL}{Q^2} = \frac{CVL' - CVML}{Q} = \frac{CmL - CVML}{Q} \text{ from where:}$$

$CmL<CVML\ \forall Q<Q_2$ and $CmL>CVML\ \forall Q>Q_2$. Also, $CmL(Q_2)=CVML(Q_2)$.

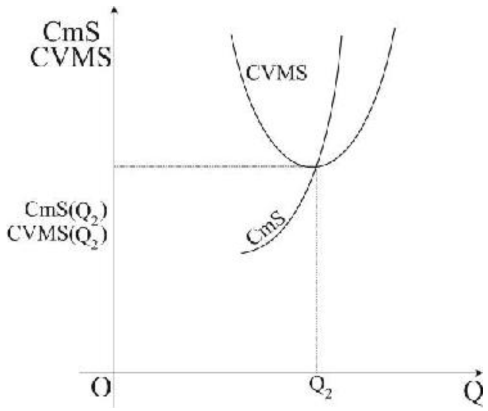
Following these considerations, the local minimum point of CVML coincide with the point of intersection of this curve with the long-term marginal cost.

The proof for short-term costs is the same. In this case, because $CFM = \frac{CF}{Q}$ follows that CFM is decreasing (*CF being constant*), therefore $CFM'<0\ \forall Q>0$. From CTMS=CFM+CVMS we have that $CTMS' = CFM' + CVMS' < 0\ \forall Q<Q_2$.



Marginal costs and average variable cost on long-term

Figure 3



Marginal costs and average variable cost on short-term

Figure 4

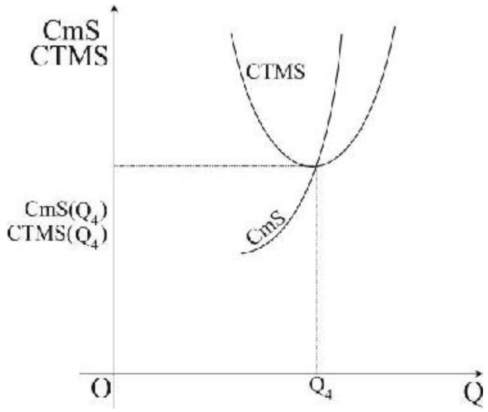
C.3. The average total cost on short-time CTMS is positive, convex and increasing for a value large enough of production.

From axiom statement $\exists Q_3$ such that: CTMS'>0 $\forall Q>Q_3$. How CTMS'<0 $\forall Q<Q_2$ follows that $Q_2<Q_3$. We have therefore $\exists Q_4$ such that: CTMS'(Q_4)=0 therefore Q_4 is a minimum point. We have therefore $Q_4 \in (Q_2,Q_3)$. Also:

$$\text{CTMS'}=\left(\frac{\text{CTS}}{Q}\right)'=\frac{\text{CTS}'\cdot Q-\text{CTS}}{Q^2}=\frac{\text{CTS}'-\text{CTMS}}{Q}=\frac{\text{CmS}-\text{CTMS}}{Q}$$

so in the point Q_4 we have: CmS(Q_4)=CTMS(Q_4), CmS<CTMS $\forall Q<Q_4$ and CmS>CTMS $\forall Q>Q_4$.

From these considerations, the local minimum point of CTMS coincide with the point of intersection of this curve with the short-term marginal cost.



Marginal cost and average total cost on short-term

Figure 5

Axioms C.1 – C.3 determined the existence of four points Q_1, Q_2, Q_3 and Q_4. The question is now the determination of the order of these points in order plotting the graphs of the above curves.

We have therefore:

$\text{CTMS'}=\frac{1}{Q}(\text{CmS}-\text{CTMS})$ from where:

$$\text{CTMS''}=\frac{(\text{CmS}'-\text{CTMS}')Q-(\text{CmS}-\text{CTMS})}{Q^2}=\frac{\left(\text{CmS}'-\frac{1}{Q}(\text{CmS}-\text{CTMS})\right)Q-(\text{CmS}-\text{CTMS})}{Q^2}=$$

$$\frac{\text{CmS}'Q-2(\text{CmS}-\text{CTMS})}{Q^2}.$$

We have now: $\text{CTMS''}(Q_1)=-\frac{2(\text{CmS}(Q_1)-\text{CTMS}(Q_1))}{Q_1^2}>0$ from where: CmS(Q_1)<CTMS(Q_1) therefore $Q_1<Q_4$.

Also, $\text{CVMS'}=\frac{\text{CmS}-\text{CVMS}}{Q}$ from where:

$$\text{CVMS''}=\frac{(\text{VCmS}'-\text{CTMS}')Q-(\text{CmS}-\text{CVMS})}{Q^2}=\frac{\left(\text{CmS}'-\frac{1}{Q}(\text{CmS}-\text{CVMS})\right)Q-(\text{CmS}-\text{CVMS})}{Q^2}=$$

$$\frac{\text{CmS}'Q-2(\text{CmS}-\text{CVMS})}{Q^2}.$$

In Q_1 we have: CVMS"(Q_1)= $\frac{-2(\text{CmS}(Q_1)-\text{CVMS}(Q_1))}{Q_1^2}$, and from the convexity of CVMS follows that $\text{CmS}(Q_1)<\text{CVMS}(Q_1)$ therefore $Q_1<Q_2$. From the fact that $Q_4\in(Q_2,Q_3)$ we have, finally:

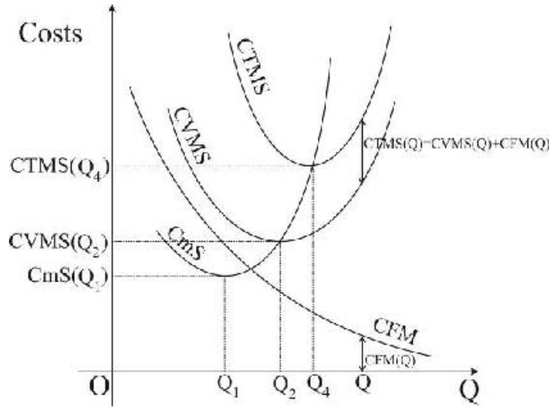
$$Q_1<Q_2<Q_4<Q_3$$

We now ask the question of determining the order of the curves corresponding to CTMS, CVMS, CFM, CmS.

Since CTMS=CVMS+CFM we have: CTMS>CVMS şi CTMS> CFM.

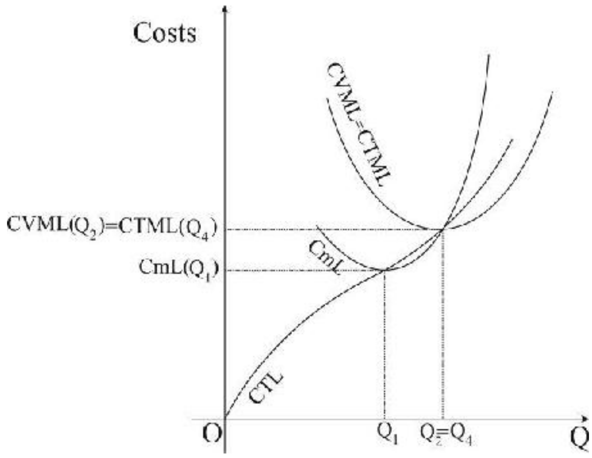
- $Q<Q_1$: CmS<CVMS<CTMS, CmS↓, CTMS↓, CVMS↓, CFM↓
- $Q=Q_1$: minimum for CmS
- $Q_1<Q<Q_2$: CmS<CVMS<CTMS, CmS↑, CTMS↓, CVMS↓, CFM↓
- $Q=Q_2$: minimum for CVMS
- $Q_2<Q<Q_4$: CVMS<CmS<CTMS, CmS↑, CTMS↓, CVMS↑, CFM↓
- $Q=Q_4$: minimum for CTMS
- $Q_4<Q$: CmS>CTMS>CVMS, CmS↑, CTMS↑, CVMS↑, CFM↓

In addition, CVMS and CTMS are convex curves and their intersections with CmS is performed in the local minimum points of these curves. Also, the graph of CFM=$\frac{CF}{Q}$ is an equilateral hyperbola (*CFM·Q=CF=constant*).



Costs on short-term

Figure 6



Costs on long-term

Figure 7

4 Conclusions

We approached the theory of short-term costs and the long-term costs from an axiomatic perspective because in the vast majority of the literature the graphs of costs are presented only after purely economic explanations. The authors have questioned the existence of a minimum cost axioms that generate their behavior.

5 References

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