Estimating SME's risk of bankruptcy using stochastic methods: Romanian Development Regions

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Abstract: Nowadays, Romania has a functional market economy. Retrospective financial analysis is used to increase the profitability and value of an economic entity searching for sources of capital and an efficient allocation. This analysis requires certain financial methods and techniques to assess firm's financial performance at a given moment. Economic theory has always been interested in developing methods to predict the risk of bankruptcy. One of the methods commonly used in current financial analysis is to determine the areas where the risk of bankruptcy is higher using the Score function. This paper presents the theory of the Score function method for financial analysis of an economic entity and then applies it for SMEs in Romanian development regions. The bankruptcy risk for some SMEs is approximated and the strengths and weaknesses of financial management in order to design new strategies are identified. Changes induced by the current economic and financial crisis may involve, when the probability is a priori, an exchange of the bankruptcy rate. The techniques presented and applied in this paper, allowed us, for each considered region, to obtain a general picture of the economic and financial situation of SMEs and allowed for a global diagnosis and a synthesis of their economic and financial situation.

Keywords: Score function; bankruptcy risk; scale analysis; small and medium enterprises (SMEs)

JEL Classification: C12, C16, C44, C53, P42

1 Introduction

Financial diagnosis allows, through the system of return and financial balance rates, highlighting the strengths and weaknesses in financial management, even the operational risks, financial risks, indebtedness risk, etc., but does not allow a global assessment of the bankruptcy risk, this being determined by a complex of factors.

In economic theory, there was and still is interest for the development of bankruptcy risk prediction methods starting from rates related to the status of "health" or "weakness" of economic entities. One of the financial analysis methods commonly used today is the Score function. Comparing *a priori* and *a posteriori* probabilities allows one to identify the Score areas where the risk of bankruptcy is higher.

The first major model for the analysis of the risk of bankruptcy, both in the literature and in financial practice, belonged to E. I. Altman and was published in its original form in 1968 and it is known as the Z-Score function. Later, in 1979, economists J. Conan and M. Holder created a model for classifying economic entities similar to Altman's model.

This paper presents, in the first part, the theoretical formulation of the Score function method for the financial analysis of an economic entity and, in the second part, the results obtained by applying this methodology to small and medium enterprises in the four Romanian development regions. This methodology allowed to approximate the risk of bankruptcy for some SMEs and to identify the strengths and weaknesses in financial management in order to design new strategies for maintaining and developing the economic entity.

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2. Theoretical formulation of the problem

Let us assume K economic entities are considered as groups: $N_1, N_2, ...N_L$ are "non-bankrupt (viable)" entities and $B_1, B_2, ...B_M$ economic entities are "bankrupt".

The Score function "Z" proposed by Altman is, in fact, a form of multiple regression:

$$Z = a_0 + a_1 R_1 + a_2 R_2 + ... + a_n R_n \qquad (1)$$

where

$R_1, R_2, ..., R_n$ are the ratios considered in the drafting of the model of economic entities;

$a_1, a_2, ..., a_n$ are the coefficients of the considered financial rates;

a_0 is the open term (the intercept) of the classification function.

The least-squares method is used to calculate parameters $a_0, a_1, a_2, ..., a_n$ under the condition of a minimum sum of squared deviations of empirical terms from the regression line.

For each of the considered groups ("non-bankrupt" and "bankrupt" entities, but for the entire assembly, as well) the Score function is determined.

The analysis of the distribution curve of each of the determined Score functions allows establishing the corresponding *a posteriori* probabilities, knowing that the Score for each of them belongs to one of the unfavourable, neutral or favourable areas. For each of the determined Scores, the average density is calculated (Stancu, 1995):

$$f(Z, \overline{Z}, \sigma_Z) = \frac{1}{\sigma_Z \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{Z-\overline{Z}}{\sigma_Z}\right)^2} \qquad (2)$$

where

σ_Z = the average standard deviation

$\overline{Z}$ = random variable Z average

with the inflection points:

$$\left(\overline{Z} - \sigma_Z \; ; \frac{1}{\sigma_Z \sqrt{2\pi}} e^{-\frac{1}{2}}\right) \text{ and } \left(\overline{Z} + \sigma_Z \; ; \frac{1}{\sigma_Z \sqrt{2\pi}} e^{-\frac{1}{2}}\right) \qquad (3)$$

So, for each group of considered variables, the average density function becomes:

$$f(Z_N, \overline{Z}_N, \sigma_N) = \frac{1}{\sigma_N \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{Z-\overline{Z}_N}{\sigma_N}\right)^2} \quad \text{for "non-bankrupt (viable)" units (4)}$$

$$f(Z_B, \overline{Z}_B, \sigma_{Z_B}) = \frac{1}{\sigma_B \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{Z-\overline{Z}_B}{\sigma_B}\right)^2} \quad \text{for "bankrupt" units (5)}$$

If the two distributions are represented in the same coordinate system, the intersection points of the two graphs are determined as follows:

If: $\begin{cases} \sigma_N = \sigma_B \\ \overline{Z}_N = \overline{Z}_B \end{cases}$ then their intersection lies on the axis z = 0.

If: $\begin{cases} \sigma_N \neq \sigma_B \\ \overline{Z}_N = \overline{Z}_B \end{cases}$ then the coordinates of the intersection points are the solutions of the equation:

$$z^2 \frac{\sigma_N^2 - \sigma_B^2}{(\sigma_N \sigma_B)} - 2z\left(\frac{\overline{Z}_B}{\sigma_B^2} - \frac{\overline{Z}_N}{\sigma_N^2}\right) + \left(\frac{\overline{Z}_B^2}{\sigma_B^2} - \frac{\overline{Z}_N^2}{\sigma_N^2}\right) - \ln\left(\frac{\sigma_B}{\sigma_N}\right)^2 = 0 \qquad (6)$$

As the calculated values are higher or lower than the positive solution of equation (6), the economic units belong to one or the other of the considered groups.

The *scale method* is another way to analyze the bankruptcy risk for an economic entity (Stancu, 1995). Applying this method requires i) *a priori* establishing the values for the Score function and ii) calculating, using the following equation, the probabilities for the unit in the considered group to be bankrupt:

$$P(\text{bankruptcy}|z = z_0) = \frac{P(z = z_0|\text{bankruptcy})}{P(z = z_0|\text{bankruptcy}) + P(z = z_0|\text{non-bankruptcy})} \qquad (7)$$

with $z = z_0$ an arbitrary value, set by user.

If the probability density of the random variable Z is considered defined by:

$$\frac{\Delta F(z_i)}{\Delta z_i} = \varphi(z_i) \qquad (8)$$

with

Δz_i = amplitude of the interval on which Z values are defined,

$\Delta F(z)$ = the allocation function

and we consider that $P(Z < z) = F(z) = \int_a^z \varphi(t)dt$ and $f(z) = P(Z = z)$,

then equation (7) might be re-written for each one of the considered groups:

$$P(z = z_0 | \text{bankruptcy}) = D(z)$$
$$P(z = z_0 | \text{non-bankruptcy}) = S(z) \qquad (9)$$

Under these hypotheses, equation (6) becomes:

$$P(\text{bankruptcy } | z = z_0) = \frac{D(z)}{D(z) + S(z)} \qquad (10)$$

with

$$D(z) = \frac{1}{\sigma_B \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{Z - \bar{Z}_B}{\sigma_B}\right)^2} \text{ and } S(z) = \frac{1}{\sigma_N \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{Z - \bar{Z}_N}{\sigma_N}\right)^2} \qquad (11)$$

For the proposed study it is useful to build as well the reduced centered variables:

$$t_{\text{bankruptcy}} = \frac{z_0 - \bar{Z}_B}{\sigma_B} \text{ and } t_{\text{non-bankruptcy}} = \frac{z_0 - \bar{Z}_N}{\sigma_N} \qquad (12)$$

For an economic unit to be considered bankrupt, the calculated probability should be greater than 95%. If low probabilities result, then the values around Z giving this probability are calculated and thus the uncertainty area is determined.

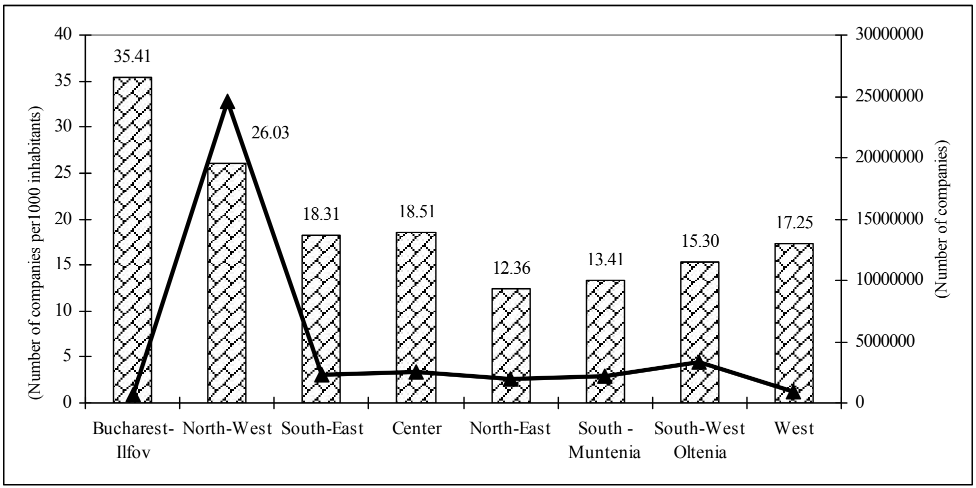
3. The analysis of bankruptcy risk using Score function

In order to differentiate using the Score function between bankrupt and non-bankrupt companies in Romania, it is imperative to present, the situation and economic performance of small and medium enterprises by development region.

3.1. Condition and economic performance of SMEs by development regions in Romania

In Romania, in 2011, were 404526 small and medium enterprises, with less then 250 employees (Romanian Yearbook). By development regions, Bucharest-Ilfov concentrates 20% of the total number of SMEs in Romania, followed by North-West and South-East with 13.65% and 13.28%, respectively, of the total number of firms. South-West Oltenia and West host less than 10% of all firms (9.08% and 8.70% respectively).

Figure 1 SMEs' density by development region of Romania (companies to 1000 people)



Source: Cartea Albă a IMM-urilor din România, 2011, www.cnipmmr.ro

With regards to the number of inhabitants, Romanian development regions are unequal. Therefore, the indicator "number of SMEs per 1000 inhabitants" (Figure 1) offers a better picture of the 'privatization through free enterprise' degree of penetration. In this context, the region with the highest average density of SMEs (companies per 1000 inhabitants) is Bucharest-Ilfov with 35.41 companies per 1000 inhabitants. More, only North-West, South-East and Centre regions are above the national average for this indicator (18.03 companies per 1000 inhabitants).

Out of the total of 404526 firms, 27% of the companies with a turnover of over 2000 thousand lei are in Bucharest-Ilfov Region, compared to 8.7% in the West Region.

Regarding the distribution of SMEs by CAEN activities and development regions, Bucharest-Ilfov holds majority shares for a maximal number of activities. Statistical data show a concentration of retail firms: 43.8% at the country level, over 50% in North-East, South-East, South- Muntenia and South-West Oltenia.

At the country level, profitable firms outpaced by 7.2% the firms with losses, with the most significant difference for Bucharest-Ilfov (+32.8%).

For the 404526 SMEs in Romania, Table 1 presents a comparison of economic performance rates at national level to those at development region level.

Table 1 Rates of economic performance of SMEs for Romanian regions and national level

Region	Number of firms	Commercial profitability rate	Financial profitability rate	Degree of covering of debt	Labour productivity
		(%)	(%)	(%)	(lei / employee)
North-East	47275	4.87	31.86	32.91	480.14
South-East	53723	5.19	68.69	15.79	550.28
South - Muntenia	46468	5.66	42.26	29.86	562.62
South-West Oltenia	36728	4.58	33.66	29.39	509.35
West	35204	5.85	45.71	24.79	501.35
North-West	55299	5.6	45.66	27.69	517.43
Centre	48921	5.31	37.82	30.05	518.7
Bucharest-Ilfov	80908	6.78	79.69	13.83	1129.95
Total	404526	5.84	52.11	21.6	639.7

Source: Cartea Albă a IMM-urilor din România, 2011, www.cnipmmr.ro

3.2. Differentiating between "non-bankrupt" and "bankrupt" companies using Score function

The database consists of records of the financial indicators in the balance sheets of 479 companies operating in "Mining and quarrying" and having a relatively uniform distribution in the eight development regions of Romania.

The primary data used are economic indicators for enterprises in the considered sector, namely: gross income profitability (R_1), gross operating margin ratio (R_2), net economic return on capital (R_3), return on assets (R_4), net operating margin ratio (R_5), the share of personnel costs in total operating expenses (R_6), depreciation share of total operating expenses (R_7), financial return on permanent capital (R_8), overall solvency ratio (R_9), self-financing ratio (R_{10}), the overall debt ratio (R_{11}), financial debt ratio (R_{12}), own working capital ratio (R_{13}), debt repayment period (R_{14}).

For operators in the "Mining and quarrying" sector, Score function estimators, according to equation (1), were obtained using the least squares method and solving the system for model parameters with the Gauss method; the result was:

$$\begin{aligned}\hat{Z} &= 0.019 \cdot R_1 + 0.525 \cdot R_2 + 0.011 \cdot R_3 + 0.295 \cdot R_4 + 0.038 \cdot R_5 - 0.021 \cdot R_6 - \\ &- 0.031 \cdot R_7 + 0.102 \cdot R_8 - 0.831 \cdot R_9 + 0.178 \cdot R_{10} - 0.0871 \cdot R_{11} - 0.376 \cdot R_{12} + \\ &+ 0.041 \cdot R_{13} - 0.076 \cdot R_{14}\end{aligned} \tag{13}$$

After calculating the Score function for each of the considered economic entities, the average value $\overline{Z} = -1.712$ was obtained. "Bankruptcy" rating, assimilated with 0, was assigned for firms with $Z < -1.712$ and "non-bankruptcy" rating, assimilated with 1, was assigned for firms with $Z > -1.712$.

Thus, when comparing the ratings awarded using this methodology with those obtained through discriminant analysis, no significant differences were revealed.

Further, a statistical-probabilistic analysis of the correctness of Score function firm separation was imposed. First, the Score function for each of the two groups of firms was determined. Then, in order to analyze the Score function distribution, using equations (4) and (5), its average density function was calculated for the two groups of economic entities.

The intersection of the two curves, according to equation (6), is the point with abscissa $Z = -1.782$ thereby achieving the critical value $Z_{critical} = -1.782$ located between $\overline{Z}_B = -2.19$ and $\overline{Z}_N = -1.08$. Therefore, it can be said that if $Z > -1.782$ the company can be considered "non-bankrupt" and if $Z < -1.782$, the company can be considered "bankrupt".

Comparing the results obtained, it is obvious the fact that the Score function values obtained through the two methods presented are quite close. Therefore, it can be concluded that the decisions to assign the firms to the two groups are correct.

The analysis of the contribution of each exogenous variable to the Score function brings comprehensive information on the health of the company and helps draw attention to its further development.

4. Conclusion

In conclusion, given the present situation, with the concept of risk increasingly present, it is natural that economic actors would use the necessary risk indicators and employ methods that allow updating the sample data to capture the effects of the context.

Scores were constructed by robust statistical methods and were tested on several samples of economic entities.

Using the Score function may highlight the weaknesses and the strengths of economic entities.

Comparison between the *a priori* and *a posteriori* probabilities allows identifying the Score areas where the risk of default is higher.

Applying this methodology for SMEs, for their specific indicators, enables the adoption of decisions that lead to the improvement of their activities and to avoid failures in management.

Score function is very sensitive to any marked change in the economic situation and signals the alarming financial-economic status, while offering the possibility of prediction of the highest quality.

A change of context or a particular context, may involve a change in the bankruptcy rate when the probability is *a priori*. Using the Score distributions for each category, updated values of *a priori* probabilities, adapting *a posteriori* probabilities to a new context and following the techniques presented in this paper could give a bird-eye view of the economic and financial situation of the economic entity and, in the same time, a global diagnosis can be made, which would synthesize its situation.

Also, progress of discriminant analysis methods and the need to better control the risks for bankruptcy imposed that several financial institutions develop credit-scoring formulas and encourage their use.

5. References

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