

## **Mathematical and Quantitative Methods**

### **Aspects of Price Discrimination in the Monopoly**

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**Abstract:** The analysis allowed the determination in general of the consumer's surplus or of the manufacturer's surplus in the case of monopoly and the determination of the allocative inefficiency in relation to the situation of perfect competition. Also, we broached the price discrimination of third order, analyzing, in terms of goods elasticities, the opportunity to separate prices in the conditions of differences existing between groups of firms.

**Keywords** consumer; demand; monopoly; discrimination; manufacturer

**JEL Classification:** D01

#### **1. Introduction**

Monopoly is a market situation where there is a single bidder of an unsubstituted good and a sufficient number of consumers.

The existence of monopoly imply absence of competition between production companies, the only ones who can influence a lesser or greater price being the buyers.

We list some main categories of monopoly, namely:

- **natural monopoly** – as a result of the realization of inventions or possession of scarce resources or prohibitive for other potential competitors. Usually, such a monopoly has not a very long life because on the one hand, technological progress can give birth to new inventions to cancel the advantage of generating monopoly or, on the other hand, reallocation of resources for various reasons. In this type of monopoly, the long-term average cost is a decreasing function, contrary to the situation encountered in the perfect competition.

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- **public monopoly** – represented by state-owned companies, generated mostly by controlling prices or monitoring of hazardous activities. We meet such a monopoly, usually within companies providing essential services to society, such as railway companies and/or air, distribution companies (electric, gas, nuclear, wind etc.) , water distribution companies and/or heat etc.
- **monopoly as a final result of the competition** – when, one by one, competitors are removed from the market.

In the market monopoly, the price set will not follow the equilibrium law, the monopolist having the possibility of relatively unilaterally adjust to maximize its profits. On the other hand, an excessive increase in price implies a decline in production due to its sale entirely impossible.

The uninterchangeability of the product is essential, because if there is another product that could replace the original one, the buyers will move the request to this, the monopolistic company thus reducing the market outlet.

## 2. An Overview of Monopoly

Before beginning our analysis on the price of a good sold under monopoly situation, let recall the main costs of production, where we have noted  $C_m$  – the marginal cost,  $CT$  – the total cost,  $CTM$  – the average total cost.

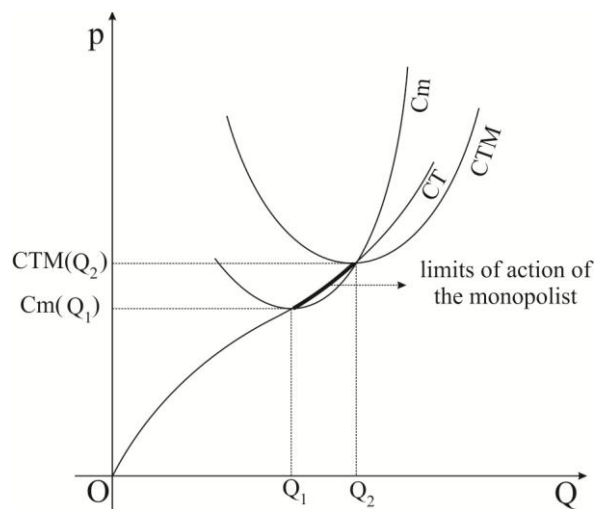


Figure 1. Long-term costs

The action of monopolist is so effective for  $Q \in [Q_1, Q_2]$  ( $Q_1$  – the minimum point of  $C_m$ ,  $Q_2$  – the CTM's minimum point) therefore on the part where the marginal cost is increasing, but the average total is decreasing.

We know that at a sale price of output  $Q$ , we have:  $\Pi(Q) = p(Q) \cdot Q - CT(Q)$  where, from the condition of extreme profit ( $\Pi'(Q) = 0$ ) implies:

$$Cm(Q) = p(Q) \left( 1 + \frac{1}{\varepsilon_{Q,p}} \right)$$

where  $\varepsilon_{Q,p} = \frac{dQ}{dp} \cdot \frac{p}{Q}$  is the coefficient of elasticity of demand in relation to price.

We will assume that the good is normal, so  $\varepsilon_{Q,p} < 0$ .

On the other hand, we know that the marginal income  $Vm(Q) = p(Q) \left( 1 + \frac{1}{\varepsilon_{Q,p}} \right)$ .

If  $1 + \frac{1}{\varepsilon_{Q,p}} < 0 \Leftrightarrow \varepsilon_{Q,p} > -1$  then  $Vm(Q) < 0$ . How any company production is at a positive marginal cost we have that  $Vm(Q) = Cm(Q) > 0$  so contradiction.

If  $\varepsilon_{Q,p} \leq -1$  then  $1 + \frac{1}{\varepsilon_{Q,p}} \geq 0$  having  $Vm(Q) \geq 0$ . If the equation  $Cm(Q) = Vm(Q)$  has the solution  $Q_0$  then  $Q = Q_0$  is the output at which the monopolist will maximize its profit, the price of production being  $p_0 = Cm(Q_0)$ . Therefore, to maximize its profit, the monopolist will run as long as the elasticity of production is less than -1, so the demand is elastic.

Also, as  $Vm = p'(Q) \cdot Q + p(Q) < p(Q)$  (for a normal good  $p'(Q) < 0$ ) we obtain that  $p^* = p(Q_0) > p_0$ , so the price that consumers are willing to offer is greater than those of production. As a result, the revenue of monopolist are  $V = p^* \cdot Q_0$ . At a such level of production, the average total cost  $CTM(Q_0)$  is greater than  $p_0$  because the production zone is under its minimum and less than  $p^*$ . The monopolist profit is therefore:

$$\Pi(Q_0) = (p^* - CTM(Q_0)) \cdot Q_0$$

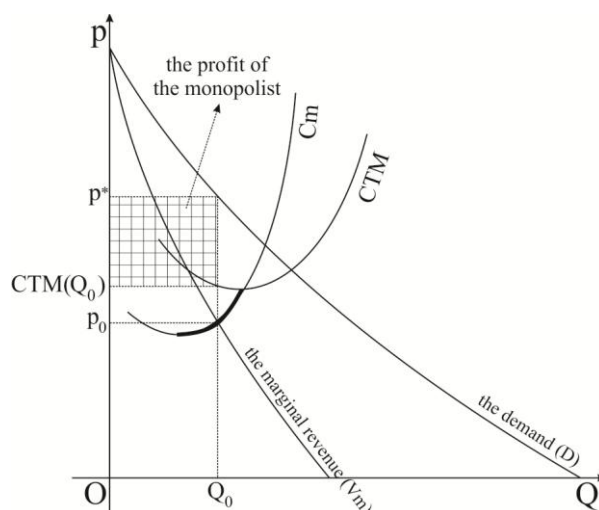


Figure 2. The profit of the monopolist

Let us note now  $Pp(Q) = \frac{p(Q)}{Cm(Q)}$  - the market power of the company corresponding to the production  $Q$ . We note that a value higher than one of  $Pp$  is a price that exceeds the marginal cost and thus is providing additional profit. But we have:  $Pp(Q) = \frac{\varepsilon_{Q,p}}{\varepsilon_{Q,p} + 1}$ .

Considering the function  $f: (-\infty, -1) \cup (-1, 0] \rightarrow \mathbf{R}$ ,  $f(x) = \frac{x}{x+1}$  we have  $f'(x) = \frac{1}{(x+1)^2} > 0$  therefore  $f$  is strictly increasing. As  $\lim_{x \rightarrow -\infty} f(x) = 1$ ,  $\lim_{\substack{x \rightarrow -1 \\ x < -1}} f(x) = \infty$ ,  $\lim_{\substack{x \rightarrow -1 \\ x > -1}} f(x) = -\infty$ ,  $f(0) = 0$ , follows that  $f(x) \in (1, \infty) \forall x \in (-\infty, -1)$  and  $f(x) \in (-\infty, 0)$

$\forall x \in (-1, 0]$ . In particular, for  $x = \varepsilon_{Q,p}$  we have that  $Pp(Q) \in (1, \infty) \forall \varepsilon_{Q,p} \in (-\infty, -1)$  and  $Pp(Q) \in (-\infty, 0) \forall \varepsilon_{Q,p} \in (-1, 0]$ .

Following these considerations, it result that for a demand increasingly more elastic, the monopolist can raise price increasing more than the marginal cost of production.

Similarly, we define the firm's pricing power (*the Lerner index*), corresponding to the production  $Q$ , as:  $L(Q) = \frac{p(Q) - Cm(Q)}{p(Q)}$ . We have:

$$L(Q) = \frac{p(Q) - Cm(Q)}{p(Q)} = 1 - \frac{Cm(Q)}{p(Q)} = 1 - \frac{1}{Pp(Q)} = -\frac{1}{\epsilon_{Q,p}}$$

The power price is the price relative deviation of the price monopoly in relation to the those coming from perfect competition ( $Cm(Q)$ ). Notice that at a demand becoming more elastic ( $\epsilon_{Q,p} < -1$ )  $L$  tends to zero.

In the monopolistic market, the production  $Q_0$  where the monopolist will maximize its profit, is thus a solution of the equation  $Cm(Q) = Vm(Q)$ , the output price being  $p_0 = Cm(Q_0)$ . The selling price of the product is  $p^* = p(Q_0)$ . As  $V(Q) = p(Q) \cdot Q$  we have that  $V_m(Q) = p'(Q) \cdot Q + p(Q)$  therefore  $Q_0$  satisfy the equality:  $Cm(Q_0) = p'(Q_0) \cdot Q_0 + p(Q_0)$ .

In this case, the consumer surplus (in the case of monopoly) is the curvilinear triangle area  $FAP^*$  i.e.:

$$S_{d,m} = \int_0^{Q_0} p(Q) dQ - p^* Q_0$$

Similarly, the excess of the monopolist (in the case of monopoly) is the curvilinear quadrilateral area  $p^*ACD$ :

$$S_{s,m} = p^* Q_0 - \int_0^{Q_0} Cm(Q) dQ$$

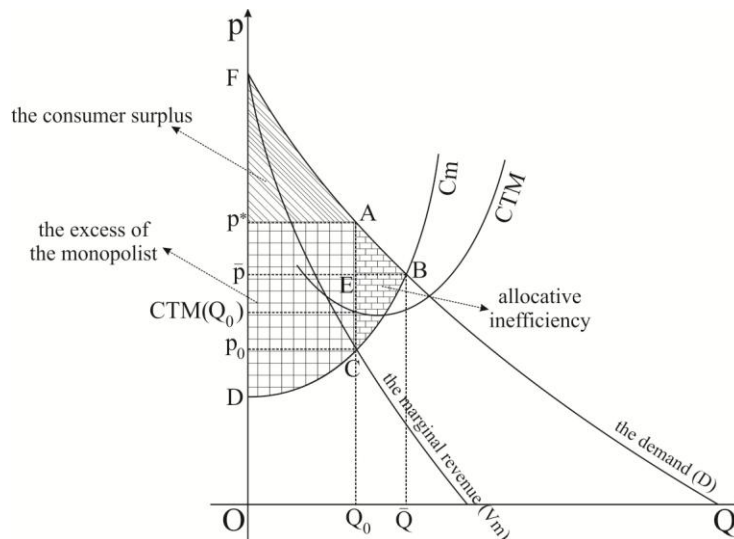


Figure 3. Allocative inefficiency

The total surplus (in the case of monopoly) is the sum of the two surpluses, namely:

$$S_m = S_{d,m} + S_{s,m} = \int_0^{Q_0} (p(Q) - Cm(Q)) dQ$$

As  $L(Q) = \frac{p(Q) - Cm(Q)}{p(Q)}$  follows that  $p(Q) - Cm(Q) = L(Q)p(Q) = -\frac{p(Q)}{\varepsilon_{Q,p}}$ .

The total surplus can be written thus:

$$S_m = - \int_0^{Q_0} \frac{p(Q)}{\varepsilon_{Q,p}} dQ$$

If the manufacturer would operate under perfect competition, the equilibrium production would satisfy the relationship  $Cm(Q) = p(Q)$ . Let  $\bar{Q}$  and  $\bar{p}$  - the production and the price equilibrium in this case.

The consumer surplus (in case of perfect competition) is the curvilinear triangle area  $FB\bar{p}$ , namely:

$$S_{d,c} = \int_0^{\bar{Q}} p(Q) dQ - \bar{p}\bar{Q}$$

Similarly, the excess monopolist (in case of perfect competition) is the curvilinear triangle area  $\bar{p}BD$ :

$$S_{s,c} = \bar{p}\bar{Q} - \int_0^{\bar{Q}} Cm(Q) dQ$$

The total surplus (in case of perfect competition) is the sum of the two surpluses, namely:

$$S_c = S_{d,c} + S_{s,c} = \int_0^{\bar{Q}} (p(Q) - Cm(Q)) dQ$$

Let notice now that since the optimum in monopoly conditions is performed on the descending curve of the CTM and in the perfect competition on the increase, resulting:  $Q_0 < \bar{Q}$ .

The area of the curvilinear triangle  $ABC$  is called allocative inefficiency and we have:

$$I_a = \int_{Q_0}^{\bar{Q}} (p(Q) - Cm(Q)) dQ = S_c - S_m$$

Because on the action area of the manufacturer:  $p(Q) \geq Cm(Q)$  we have that  $I_a \geq 0$  therefore:  $S_c \geq S_m$ . From these facts, the total surplus under monopoly is smaller or equal than in the case of perfect competition. Also, let note that:

$$\Delta S_s = S_{s,m} - S_{s,c} =$$

$$\left( p^* Q_0 - \int_0^{Q_0} Cm(Q) dQ \right) - \left( \bar{p} \bar{Q} - \int_0^{\bar{Q}} Cm(Q) dQ \right) = p^* Q_0 - \bar{p} \bar{Q} + \int_{Q_0}^{\bar{Q}} Cm(Q) dQ$$

is the difference of producer's surplus at the transition from perfect competition at the monopolistic competition.

On the other hand,  $p(\bar{Q}) = \bar{p} = Cm(\bar{Q})$ , and  $p^* = p(Q_0)$  where  $Q_0$  satisfy:  $Cm(Q_0) = p'(Q_0) \cdot Q_0 + p(Q_0)$ . We get so:

$$\Delta S_s = p(Q_0) Q_0 - p(\bar{Q}) \bar{Q} + \int_{Q_0}^{\bar{Q}} Cm(Q) dQ = p(Q_0) Q_0 - p(\bar{Q}) \bar{Q} + \int_{Q_0}^{\bar{Q}} CT'(Q) dQ =$$

$$p(Q_0) Q_0 - p(\bar{Q}) \bar{Q} + CT(\bar{Q}) - CT(Q_0) = (CT(\bar{Q}) - CT(Q_0)) \left( 1 - \frac{\bar{Q} p(\bar{Q}) - Q_0 p(Q_0)}{CT(\bar{Q}) - CT(Q_0)} \right)$$

From the Cauchy's theorem of finite increases, it follows that  $\exists Q^{**} \in (Q_0, \bar{Q})$  so that:

$$\frac{\bar{Q} p(\bar{Q}) - Q_0 p(Q_0)}{CT(\bar{Q}) - CT(Q_0)} = \frac{(Q \cdot p(Q))'}{CT'(Q)} \Big|_{Q=Q^{**}} = \frac{p(Q^{**}) + Q^{**} \cdot p'(Q^{**})}{Cm(Q^{**})} = \frac{Vm(Q^{**})}{Cm(Q^{**})}$$

Following these considerations, it follows:

$$\Delta S_s = (CT(\bar{Q}) - CT(Q_0)) \left( 1 - \frac{Vm(Q^{**})}{Cm(Q^{**})} \right) = \frac{(CT(\bar{Q}) - CT(Q_0)) (Cm(Q^{**}) - Vm(Q^{**}))}{Cm(Q^{**})} \geq 0$$

because the total cost is increasing,  $Cm \geq Vm$  on the range  $(Q_0, \bar{Q})$  and  $Cm$  is positive. Therefore, the shift to a monopoly, will increase the producer surplus.

Analogously:

$$\Delta S_d = S_{d,m} - S_{d,c} = \left( \int_0^{Q_0} p(Q) dQ - p^* Q_0 \right) - \left( \int_0^{\bar{Q}} p(Q) dQ - \bar{p} \bar{Q} \right) = \bar{p} \bar{Q} - p^* Q_0 - \int_{Q_0}^{\bar{Q}} p(Q) dQ$$

represents the difference of consumer's surplus is at the transition from perfect competition at those monopolistic.

We therefore have:

$$\Delta S_d = \bar{p} \bar{Q} - p^* Q_0 - \int_{Q_0}^{\bar{Q}} p(Q) dQ = p(\bar{Q}) \bar{Q} - p(Q_0) Q_0 - \int_{Q_0}^{\bar{Q}} p(Q) dQ$$

Let P be a primitive of p. We get:

$$\begin{aligned} \Delta S_d &= p(\bar{Q}) \bar{Q} - p(Q_0) Q_0 - \int_{Q_0}^{\bar{Q}} P'(Q) dQ = p(\bar{Q}) \bar{Q} - p(Q_0) Q_0 - P(\bar{Q}) + P(Q_0) = \\ &= (P(\bar{Q}) - P(Q_0)) \left( \frac{\bar{Q} p(\bar{Q}) - Q_0 p(Q_0)}{P(\bar{Q}) - P(Q_0)} - 1 \right). \end{aligned}$$

From Cauchy's theorem of finite increases, it follows that  $\exists Q^{***} \in (Q_0, \bar{Q})$  so that:

$$\frac{\bar{Q} p(\bar{Q}) - Q_0 p(Q_0)}{P(\bar{Q}) - P(Q_0)} = \frac{(Q \cdot p(Q))'|_{Q=Q^{***}}}{p(Q^{***})} = \frac{p(Q^{***}) + Q^{***} \cdot p'(Q^{***})}{p(Q^{***})} = \frac{Vm(Q^{***})}{p(Q^{***})}$$

Finally:

$$\Delta S_d = (P(\bar{Q}) - P(Q_0)) \left( \frac{Vm(Q^{***})}{p(Q^{***})} - 1 \right) = \frac{(P(\bar{Q}) - P(Q_0))(Vm(Q^{***}) - p(Q^{***}))}{p(Q^{***})} \leq 0$$

because the demand function being convex (*for normal goods*) it follows  $P' = p'' \geq 0$  therefore  $P(\bar{Q}) \geq P(Q_0)$  and  $p(Q^{***}) \geq Vm(Q^{***})$ .

Therefore, the shift to a monopoly will reduce consumer surplus.

### 3. Price Discriminations

In the monopoly situation, we saw that, unlike in the case of perfect competition, the monopoly does not produce at full capacity (marginal cost equal to the inverse function of the demand) phenomenon that leads to inefficiency allowance.

What happens when the sale takes place at different prices to different quantities? We call such a situation by price discrimination.



### 3.1. Discrimination of first order

In this case (also called perfect discrimination), the monopolist sells each buyer at the maximum price that he can bear it. Production will rise so that the marginal cost is equal to the opposite level of the demand.

If the sale price would be strictly higher than marginal cost, then (as it is increasing) the monopolist will produce until additional amount will satisfy the above condition. Following these findings, the monopolist will sell the last unit produced (those corresponding to the maximum marginal cost) to the buyer which offers the highest price, the penultimate to the second (in decreasing order of price offered) and so further.

### 3.2. Discrimination of second order

In this type of discrimination, the monopolist practice different prices depending on the amount requested. In this case, the price equals the marginal cost of the last units purchased (which has the highest marginal cost), on the grounds that if the next unit of product would be priced lower than the previous one, then the buyer will prefer to buy the additional unit increasing the surplus.

### 3.3. Discrimination of third order

In this type of discrimination, the monopolist use distinct prices for different groups of consumers.

Let consider the situation where a monopolist operate on a number of “n” consumer groups, selling the quantities  $Q_1, \dots, Q_n$  at prices  $p_1(Q_1), \dots, p_n(Q_n)$  where  $Q_1 + \dots + Q_n = Q$  is the total production achieved.

In this case, the monopolist profit is:

$$\Pi(Q_1, \dots, Q_n) = \sum_{i=1}^n p_i(Q_i) \cdot Q_i - CT(Q_1 + \dots + Q_n)$$

The extreme condition requires like necessary the cancellation of first order partial derivatives:  $\frac{\partial \Pi}{\partial Q_i} = 0 \quad \forall i = \overline{1, n}$  therefore:

$$p_i'(Q_i) \cdot Q_i + p_i(Q_i) - Cm(Q) = 0 \quad \forall i = \overline{1, n}$$

Considering now the coefficient of elasticity of demand for the group “i” in relation to the price  $p_i$ :  $\varepsilon_{Q_i, p_i} = \frac{dQ_i}{dp_i} \cdot \frac{p_i}{Q_i} = \frac{p_i(Q_i)}{p_i'(Q_i)Q_i}$  we get:

$$Cm(Q) = p_i(Q_i) \left( 1 + \frac{1}{\varepsilon_{Q_i, P_i}} \right) \quad \forall i = \overline{1, n}$$

Therefore, the necessary condition to maximize profit will be reduced to the condition of marginal revenue equal marginal cost for each group of the entire production. Also, from the above relationship, it provided compatibility groups namely:

$$p_1(Q_1) \left( 1 + \frac{1}{\varepsilon_{Q_1, P_1}} \right) = \dots = p_n(Q_n) \left( 1 + \frac{1}{\varepsilon_{Q_n, P_n}} \right)$$

Considering two arbitrary groups “i” and “j” we have first:

$$\frac{p_i(Q_i)}{p_j(Q_j)} = \frac{1 + \frac{1}{\varepsilon_{Q_j, P_j}}}{1 + \frac{1}{\varepsilon_{Q_i, P_i}}}$$

and after:

$$\frac{p_i(Q_i) - p_j(Q_j)}{p_j(Q_j)} = \frac{\varepsilon_{Q_i, P_i} - \varepsilon_{Q_j, P_j}}{\varepsilon_{Q_i, P_i} \varepsilon_{Q_j, P_j} \left( 1 + \frac{1}{\varepsilon_{Q_i, P_i}} \right)}$$

Because the monopolist acts only in the zone of elasticity of demand in relation to price, we have  $1 + \frac{1}{\varepsilon_{Q_i, P_i}}$  and  $\varepsilon_{Q_i, P_i}, \varepsilon_{Q_j, P_j} < 0$ .

We have therefore:  $(p_i(Q_i) - p_j(Q_j))(\varepsilon_{Q_i, P_i} - \varepsilon_{Q_j, P_j}) > 0$ . If (taking into account the fact that elasticities are negative)  $\varepsilon_{Q_i, P_i} > \varepsilon_{Q_j, P_j}$  then  $p_i(Q_i) > p_j(Q_j)$  so the monopolist must sell at a higher price to the group “i”. Conversely, if  $\varepsilon_{Q_i, P_i} < \varepsilon_{Q_j, P_j}$  then  $p_i(Q_i) < p_j(Q_j)$  so the monopolist must sell at a lower price to the group “i”. It is obvious that at equal elasticities will correspond to equal prices.

Following these considerations, it follows that if (after a possible renumbering)  $\varepsilon_{Q_1, P_1} \geq \varepsilon_{Q_2, P_2} \geq \dots \geq \varepsilon_{Q_n, P_n}$  then:  $p_1(Q_1) \geq p_2(Q_2) \geq \dots \geq p_n(Q_n)$ .

We intend now to study the decision of the monopolist to sell at the same price to the “n” groups compared with the application of differentiated pricing.

We have seen that:  $Cm(Q) = p(Q) \left( 1 + \frac{1}{\varepsilon_{Q,p}} \right)$  where  $Q = Q_1 + \dots + Q_n$  and for differentiated prices:  $Cm(Q) = p_i(Q_i) \left( 1 + \frac{1}{\varepsilon_{Q_i, p_i}} \right) \forall i = \overline{1, n}$ .

From above, we get:

$$p_i(Q_i) \left( 1 + \frac{1}{\varepsilon_{Q_i, p_i}} \right) = p(Q) \left( 1 + \frac{1}{\varepsilon_{Q,p}} \right)$$

from where:

$$p_i(Q_i) = p(Q) \frac{1 + \frac{1}{\varepsilon_{Q,p}}}{1 + \frac{1}{\varepsilon_{Q_i, p_i}}} \quad \forall i = \overline{1, n}$$

The monopolist profit is therefore:

$$\Pi_n = \Pi(Q_1, \dots, Q_n) = \sum_{i=1}^n p_i(Q_i) \cdot Q_i - CT(Q) = \sum_{i=1}^n p(Q) \frac{1 + \frac{1}{\varepsilon_{Q,p}}}{1 + \frac{1}{\varepsilon_{Q_i, p_i}}} \cdot Q_i - CT(Q)$$

For a single price, we have:

$$\Pi = \Pi(Q) = p(Q) \cdot Q - CT(Q)$$

Calculating the difference between the two profits, results:

$$\begin{aligned} \Pi_n - \Pi &= \sum_{i=1}^n p(Q) \frac{1 + \frac{1}{\varepsilon_{Q,p}}}{1 + \frac{1}{\varepsilon_{Q_i, p_i}}} \cdot Q_i - CT(Q) - (p(Q) \cdot Q - CT(Q)) = \\ &= \sum_{i=1}^n \left( p(Q) \frac{1 + \frac{1}{\varepsilon_{Q,p}}}{1 + \frac{1}{\varepsilon_{Q_i, p_i}}} \cdot Q_i - p(Q) \cdot Q_i \right) = p(Q) \sum_{i=1}^n Q_i \frac{\varepsilon_{Q_i, p_i} - \varepsilon_{Q,p}}{\varepsilon_{Q,p} (\varepsilon_{Q_i, p_i} + 1)}. \end{aligned}$$

Let  $f(x) = \frac{x - \varepsilon_{Q,p}}{\varepsilon_{Q,p}(x+1)}$ . We have  $f'(x) = \frac{\varepsilon_{Q,p} + 1}{\varepsilon_{Q,p}(x+1)^2} > 0$  so  $f$  is strictly increasing. We

have  $\lim_{x \rightarrow -\infty} f(x) = \frac{1}{\varepsilon_{Q,p}}$ ,  $\lim_{\substack{x \rightarrow -1 \\ x < -1}} f(x) = \infty$ ,  $f(x) = 0 \Rightarrow x = \varepsilon_{Q,p}$ ,  $\lim_{\substack{x \rightarrow -1 \\ x > -1}} f(x) = -\infty$ ,  $f(0) = -1$ .

For  $x = \varepsilon_{Q_i, p_i} < -1$ , it follows:  $f(\varepsilon_{Q_i, p_i}) < 0$  if  $\varepsilon_{Q_i, p_i} < \varepsilon_{Q,p}$  and  $f(\varepsilon_{Q_i, p_i}) > 0$  if  $\varepsilon_{Q_i, p_i} > \varepsilon_{Q,p}$ . If  $\varepsilon_{Q_i, p_i} = \varepsilon_{Q,p}$  then  $f(\varepsilon_{Q_i, p_i}) = 0$ .

Let  $I = \{i = \overline{1, n} \mid \varepsilon_{Q_i, p_i} < \varepsilon_{Q,p}\}$ ,  $J = \{j = \overline{1, n} \mid \varepsilon_{Q_j, p_j} > \varepsilon_{Q,p}\}$ . With these notations, we have:

$$\Pi_n - \Pi = p(Q) \sum_{i \in I} Q_i \frac{\varepsilon_{Q_i, p_i} - \varepsilon_{Q,p}}{\varepsilon_{Q,p}(\varepsilon_{Q_i, p_i} + 1)} + p(Q) \sum_{j \in J} Q_j \frac{\varepsilon_{Q_j, p_j} - \varepsilon_{Q,p}}{\varepsilon_{Q,p}(\varepsilon_{Q_j, p_j} + 1)}$$

From the above, it follows that the first sum is strictly negative and the second strictly positive.

We summarize those obtained as follows: if the loss of profit on groups where the elasticity is less than the overall is less than plus achieved by the groups where the elasticity is greater than the global then the monopolist will agree separate selling prices. Similarly, if the loss of profit on groups where the elasticity is less than the overall is greater than the plus achieved by the groups where the elasticity is greater than the overall, then the monopolist would prefer selling its product at a single price.

## 5. Conclusion

The above analysis allowed the determination in general of the consumer's surplus or of the manufacturer's surplus in the case of monopoly and the determination of the allocative inefficiency in relation to the situation of perfect competition.

In the second part, we broached the price discrimination of third order, analyzing, in terms of goods elasticities, the opportunity to separate prices in the conditions of differences existing between groups of firms.

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