General Considerations on the Oligopoly

Catalin Angelo Ioan[1], Gina Ioan[2]

Abstract: In this paper we analyzed the main aspects of oligopoly, in the case of n firms. The analysis has made, as a rule, for arbitrary marginal costs, each time, however, by considering these costs constant recovering well known results of the models presented: the Stackelberg model, the case of more production leaders, the price leader, the Cournot equilibrium for duopoly, the Cournot equilibrium for oligopoly or in the case of perfect competition and cartels. We also treat the problems above for the general case of cost function, again customizing the overall results for linear functions and obtaining the corresponding classical relations.

Keywords oligopoly; duopoly; Cournot; Stackelberg; cartel

JEL Classification: D01

1. Introduction

The oligopoly is a market situation where there is a small number of suppliers (at least two) of a good unsubstituted and a sufficient number of consumers. The oligopoly composed of two producers called duopoly.

Considering below, two competitors A and B which produce the same normal good, we propose analyzing their activity in response to the work of each other company.

Each of them when it set the production level and the selling price will cover the production and price of other companies. If one of the two firms will set price or quantity produced first, the other adjusting for it, it will be called price leader or leader of production respectively, the second company called the satellite price, or satellite production respectively.

[1] Associate Professor, PhD, Danubius University of Galati, Faculty of Economic Sciences, Romania, Address: 3 Galati Blvd, Galati, Romania, tel: +40372 361 102, fax: +40372 361 290, Corresponding author: catalin_angelo_ioan@univ-danubius.ro.

[2] Assistant Professor, PhD in progress, Danubius University of Galati, Faculty of Economic Sciences, Romania, Address: 3 Galati Blvd, Galati, Romania, tel: +40372 361 102, fax: +40372 361 290, e-mail: gina_ioan@univ-danubius.ro

AUDŒ, Vol 8, no 3, pp. 113-135

2. The Production Leader (The Stackelberg Model)

Consider that the company is a leader of production. If it will produce Q_A units of a good, then the company B will adjust production after A, producing $Q_B=f(Q_A)$ units of good (f being called the reaction function). The selling price depends on the total quantity of products reached the market. Be so: $p=p(Q_A+Q_B)$ – the price per unit of good.

The company A must establish a level of production depending on the reaction of firm B, because it will determine through the production realized the selling price of the product. Similarly, the company B will adjust its production levels according to A, because at a higher or lower level, the price will change and therefore profit of the company.

Let therefore, the profit of the production leader:

$$\Pi_A(Q_A) = p(Q_A + Q_B)Q_A - CT_A(Q_A)$$

Since $Q_B=f(Q_A)$ we have:

$$\Pi_A(Q_A) = p(Q_A + f(Q_A))Q_A - CT_A(Q_A)$$

Consider also the profit of the satellite:

$$\Pi_B(Q_B) = p(Q_A + Q_B)Q_B - CT_B(Q_B)$$

The extreme condition for the profit of A is:

$$\frac{\partial \Pi_A(Q_A)}{\partial Q_A} = p'(Q_A + f(Q_A))(1 + f'(Q_A))Q_A + p(Q_A + f(Q_A)) - Cm_A(Q_A) = 0$$

and one for satellite B:

$$\frac{\partial \Pi_B(Q_B)}{\partial Q_B} = p'(Q_A + Q_B)Q_B + p(Q_A + Q_B) - Cm_B(Q_B) = 0$$

Considering the leader production Q_A as given, it follows that the satellite meets the condition:

$$p'(Q_A + Q_B)Q_B + p(Q_A + Q_B) - Cm_B(Q_B) = 0$$

Ranging the production Q_A we have: $Q_B=f(Q_A)$ therefore the problem of leader profit's maximizing becomes:

$$p'(Q_A + f(Q_A))(1 + f'(Q_A))Q_A + p(Q_A + f(Q_A)) - Cm_A(Q_A) = 0$$

with f determined above.

In particular, for a function of price (*the inverse function of the demand*) of the form: p(Q)=a-bQ, a,b>0 we obtain for the satellite company B:

$$-bQ_B + a - b(Q_A + Q_B) - Cm_B(Q_B) = 0$$

from where:

$$Q_B = \frac{a - bQ_A - Cm_B(Q_B)}{2b}$$

Let note that in this relationship, is purely formal Q_B's determination, requiring the knowledge of B's total cost and thus, implicitly, its marginal cost. Substituting the above expression of Q_B in the profit maximization condition of leader A, results for $f(Q_A)=\frac{a - bQ_A - Cm_B(Q_B)}{2b}$:

$$-b\left(1+\frac{-b-\frac{\partial Cm_B(Q_B)}{\partial Q_A}}{2b}\right)Q_A + a - b\left(Q_A + \frac{a - bQ_A - Cm_B(Q_B)}{2b}\right) - Cm_A(Q_A) = 0$$

from where:

$$\left(\frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_A} - b\right)Q_A = -\frac{a}{2} + Cm_A(Q_A) - \frac{Cm_B(Q_B)}{2}$$

or otherwise:

$$Q_A = \frac{a - 2Cm_A(Q_A) + Cm_B(Q_B)}{2b - \frac{\partial Cm_B(Q_B)}{\partial Q_A}}$$

Noting with Q_A^* - the solution of the above equation, we get:

$$Q_B^* = \frac{a - bQ_A^* - Cm_B(Q_B^*)}{2b} =$$

$$\frac{ab - a\frac{\partial Cm_B(Q_B)}{\partial Q_A} + 2bCm_A(Q_A^*) - 3bCm_B(Q_B^*) + \frac{\partial Cm_B(Q_B)}{\partial Q_A}Cm_B(Q_B^*)}{2b\left(2b - \frac{\partial Cm_B(Q_B)}{\partial Q_A}\right)}$$

where all partial derivatives are calculated in Q_A^* and Q_B^*.

The condition that the leader has a higher production than the satellite is: $Q_A^* > Q_B^*$ which is equivalent to:

$$\frac{ab + a\frac{\partial Cm_B(Q_B)}{\partial Q_A} - 6bCm_A(Q_A^*) + 5bCm_B(Q_B^*) - \frac{\partial Cm_B(Q_B)}{\partial Q_A}Cm_B(Q_B^*)}{2b\left(2b - \frac{\partial Cm_B(Q_B)}{\partial Q_A}\right)} > 0$$

With the additional assumption that the two marginal costs of A and B are constant (*on short term, marginal cost variations being very small, the assumption is not absurd*), we obtain for $Cm_A=\alpha$ and $Cm_B=\beta$:

$$Q_A^* = \frac{a - 2\alpha + \beta}{2b},\ Q_B^* = \frac{a + 2\alpha - 3\beta}{4b},\ Q_A^* - Q_B^* = \frac{a - 6\alpha + 5\beta}{4b}$$

If $a>6\alpha-5\beta$, it follows that the leader will have an output greater than that of the satellite. From the fact that $Cm_A=\alpha$ and $Cm_B=\beta$, results after a simple integration:

$$CT_A(Q)=\alpha Q+\gamma,\ CT_B(Q)=\beta Q+\delta,\ \alpha,\beta,\gamma,\delta\geq 0$$

Returning to the profits of both firms A and B we have:

$$\Pi_A(Q_A) = p(Q_A + Q_B)Q_A - CT_A(Q_A) = aQ_A - b(Q_A + Q_B)Q_A - \alpha Q_A - \gamma =$$

$$-bQ_A^2 - (bQ_B - a + \alpha)Q_A - \gamma$$

respectively:

$$\Pi_B(Q_B) = p(Q_A + Q_B)Q_B - CT_B(Q_B) = aQ_B - b(Q_A + Q_B)Q_B - \beta Q_B - \delta =$$

$$-bQ_B^2 - (bQ_A - a + \beta)Q_B - \delta.$$

Considering $\Pi_A(Q_A)=\pi_1$=constant, respectively $\Pi_B(Q_B)=\pi_2$=constant, we obtain the two isoprofit curves in the system axis Q_A-Q-Q_B:

$$-bQ_A^2 - (bQ_B - a + \alpha)Q_A - \gamma = \pi_A \text{ – for A}$$

$$-bQ_B^2 - (bQ_A - a + \beta)Q_B - \delta = \pi_B \text{ – for B}$$

For a graphical representation of the isoprofit curve of A, from the equation it follows:

$$Q_B = \frac{-bQ_A^2 + (a - \alpha)Q_A - \pi_A - \gamma}{bQ_A}$$

Let g(x)= $\frac{-bx^2+(a-\alpha)x-\pi_A-\gamma}{bx}$. We have:

$$g'(x)=\frac{-bx^2+\pi_A+\gamma}{bx^2}=0$$

hence, the stationary point of the function g is $x_d=\sqrt{\frac{\pi_A+\gamma}{b}}$. Therefore, $g'(x)>0$ $\forall x\in(0,x_d)$ and $g'(x)<0$ $\forall x\in(x_d,\infty)$. Also:

$$g(x_d)=\frac{(a-\alpha)-2\sqrt{b(\pi_A+\gamma)}}{b}$$

As g(x)=0 implies: $-bx^2+(a-\alpha)x-\pi_A-\gamma=0$ we get the two real roots:

$$x_{rad\ 1,rad\ 2}=\frac{(a-\alpha)\pm\sqrt{(a-\alpha)^2-4b(\pi_A+\gamma)}}{2b}$$

for π_A small enough so that: $(a-\alpha)^2-4b(\pi_A+\gamma)>0$.

With the observation that $\lim_{\substack{x\to 0\\ x>0}} g(x)=-\infty$, $\lim_{x\to\infty} g(x)=-\infty$, we obtain that the isoprofit curve will be assessed on the range:

$$D=\left[\frac{(a-\alpha)-\sqrt{(a-\alpha)^2-4b(\pi_A+\gamma)}}{2b},\frac{(a-\alpha)+\sqrt{(a-\alpha)^2-4b(\pi_A+\gamma)}}{2b}\right]$$

Considering now the reaction function of B to A: $Q_B=\frac{a-\beta-bQ_A}{2b}$, let the difference between this and the corresponding point of the isoprofit curve. We therefore:

$$h(Q_A,\pi_1)=\frac{bQ_A^2+(-a-\beta+2\alpha)Q_A+2(\pi_A+\gamma)}{2bQ_A}$$

The minimum difference will be obtain by canceling the first order partial derivative: $\frac{\partial h}{\partial Q_A}=\frac{bQ_A^2-2(\pi_A+\gamma)}{2bQ_A^2}=0$, from where:

$$\overline{Q}_A=\sqrt{\frac{2(\pi_A+\gamma)}{b}}$$

The minimum distance between the two curves is obtained for:

$$0=h(\overline{Q}_A,\pi_A)=\frac{(-a-\beta+2\alpha)+2\sqrt{b}\sqrt{2(\pi_A+\gamma)}}{2b}$$

from where:

$$\sqrt{2(\pi_A+\gamma)}=\frac{a+\beta-2\alpha}{2\sqrt{b}}$$

Substituting in the expression of $\overline{Q}_A$, we get:

$$\overline{Q}_A=\frac{a+\beta-2\alpha}{2b}=Q_A^*$$

Therefore, the steady production of the leader is the point of tangency of the reaction function of B relative to A at the family of isoprofit curves of A.

For the satellite company B will done analogously.

The general situation in which marginal costs are not constant, involves a series of additional assumptions. Thus, from the relation $Q_B=\frac{a-bQ_A-Cm_B(Q_B)}{2b}$ follows: $(Cm_B+2b\cdot 1_B)Q_B=a-bQ_A$ where 1_B is the identical function. If the function $Cm_B+2b\cdot 1_B$ is invertible, then:

$$Q_B=f(Q_A)=(Cm_B+2b\cdot 1_B)^{-1}(a-bQ_A)$$

With the expression of f thus obtained, from the equation:

$$-b(1+f'(Q_A))Q_A+a-b(Q_A+f(Q_A))-Cm_A(Q_A)=0$$

will be determine Q_A^* and after Q_B^*=f(Q_A^*).

3. The Case of more Production Leaders

Let consider that companies $A_1,...,A_n$ leaders of production. If they produce $Q_{A_1},...,Q_{A_n}$ units of good, then the company B will adjust its production as they, producing Q_B= $f(Q_{A_1},...,Q_{A_n})$ (*f being called the reaction function*). The selling price depends on the total quantity of products reached the market. Be so:

$$p=p\left(\sum_{k=1}^{n}Q_{A_k}+Q_B\right)$$

the price per unit of product. The firms $A_1,...,A_n$ must establish a level of production depending on the reaction of the firm B, because it will determine through the production realized, the selling price of the product. Similarly, the company B will adjust its production level, according to $A_1,...,A_n$, because at a higher or lower level, the price will change and therefore the profit of the company.

Let therefore the profit of the leader "i":

$$\Pi_{A_i}(Q_{A_i}) = p\left(\sum_{k=1}^{n} Q_{A_k} + Q_B\right)Q_{A_i} - CT_{A_i}(Q_{A_i})$$

Because $Q_B = f\left(Q_{A_1},...,Q_{A_n}\right)$ we have:

$$\Pi_{A_i}(Q_{A_i}) = p\left(\sum_{k=1}^{n} Q_{A_k} + f\left(Q_{A_1},...,Q_{A_n}\right)\right)Q_{A_i} - CT_{A_i}(Q_{A_i})$$

Consider also the profit of the satellite:

$$\Pi_B(Q_B) = p\left(\sum_{k=1}^{n} Q_{A_k} + Q_B\right)Q_B - CT_B(Q_B)$$

The extreme condition for the profit of A_i is:

$$\frac{\partial \Pi_{A_i}(Q_{A_i})}{\partial Q_{A_i}} = p'\left(\sum_{k=1}^{n} Q_{A_k} + f\left(Q_{A_1},...,Q_{A_n}\right)\right)\left(1 + \frac{\partial f}{\partial Q_{A_i}}\right)Q_{A_i} +$$
$$p\left(\sum_{k=1}^{n} Q_{A_k} + f\left(Q_{A_1},...,Q_{A_n}\right)\right) - Cm_{A_i}(Q_{A_i}) = 0$$

and the one for satellite B:

$$\frac{\partial \Pi_B(Q_B)}{\partial Q_B} = p'\left(\sum_{k=1}^{n} Q_{A_k} + Q_B\right)Q_B + p\left(\sum_{k=1}^{n} Q_{A_k} + Q_B\right) - Cm_B(Q_B) = 0$$

Considering the productions of the leaders $Q_{A_1},...,Q_{A_n}$ as given, it follows that the satellite will satisfy the condition:

$$p'\left(\sum_{k=1}^{n} Q_{A_k} + Q_B\right)Q_B + p\left(\sum_{k=1}^{n} Q_{A_k} + Q_B\right) - Cm_B(Q_B) = 0$$

Varying now the production Q_A we will have that $Q_B = f\left(Q_{A_1},...,Q_{A_n}\right)$ from where, the problem of maximizing the leader's profit becoming:

$$p'\left(\sum_{k=1}^{n} Q_{A_k} + f\left(Q_{A_1},...,Q_{A_n}\right)\right)\left(1+\frac{\partial f}{\partial Q_{A_i}}\right)Q_{A_i} + p\left(\sum_{k=1}^{n} Q_{A_k} + f\left(Q_{A_1},...,Q_{A_n}\right)\right) - Cm_{A_i}(Q_{A_i}) = 0$$

with f determined above.

In particular, for a function of price (*the inverse function of the demand*) of the form: p(Q)=a-bQ, a,b>0, we obtain for the satellite company B:

$$-bQ_B + a - b\left(\sum_{k=1}^{n} Q_{A_k} + Q_B\right) - Cm_B(Q_B) = 0$$

from where:

$$Q_B = \frac{a - b\sum_{k=1}^{n} Q_{A_k} - Cm_B(Q_B)}{2b}$$

Let note that, in this relationship, the determination of Q_B is purely formal, because it requires the knowledge of B's total cost and thus, implicitly, its marginal cost.

Substituting the above expression of Q_B in the condition of maximizing the leader's A profit, results for $f\left(Q_{A_1},...,Q_{A_n}\right) = \frac{a - b\sum_{k=1}^{n} Q_{A_k} - Cm_B(Q_B)}{2b}$:

$$-b\left(1+\frac{-b-\frac{\partial Cm_B(Q_B)}{\partial Q_{A_i}}}{2b}\right)Q_{A_i} + a - b\left(\sum_{k=1}^{n} Q_{A_k} + \frac{a - b\sum_{k=1}^{n} Q_{A_k} - Cm_B(Q_B)}{2b}\right) - Cm_{A_i}(Q_{A_i}) = 0$$

from where:

$$\left(\frac{-b+\frac{\partial Cm_B(Q_B)}{\partial Q_{A_i}}}{2}\right)Q_{A_i} - Cm_{A_i}(Q_{A_i}) = \frac{-a + b\sum_{k=1}^{n} Q_{A_k} - Cm_B(Q_B)}{2}$$

Because the right side does not depend on the amount of i explicit, it follows the condition of compatibility $\forall i,j=\overline{1,n}$:

$$\left(\frac{-b+\dfrac{\partial Cm_B(Q_B)}{\partial Q_{A_i}}}{2}\right)Q_{A_i}-Cm_{A_i}(Q_{A_i})=\left(\frac{-b+\dfrac{\partial Cm_B(Q_B)}{\partial Q_{A_j}}}{2}\right)Q_{A_j}-Cm_{A_j}(Q_{A_j})$$

Returning, we get:

$$\left(\frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_i}}-b\right)Q_{A_i}-\frac{b}{2}\sum_{\substack{k=1\\k\neq i}}^{n}Q_{A_k}=Cm_{A_i}(Q_{A_i})-\frac{a}{2}-\frac{Cm_B(Q_B)}{2}$$

In a matrix form, the system writes:

$$\begin{pmatrix}\frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_1}}-b & -\frac{b}{2} & \dots & -\frac{b}{2}\\ -\frac{b}{2} & \frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_2}}-b & \dots & -\frac{b}{2}\\ \dots & \dots & \dots & \dots\\ -\frac{b}{2} & -\frac{b}{2} & \dots & \frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_n}}-b\end{pmatrix}\begin{pmatrix}Q_{A_1}\\Q_{A_2}\\\dots\\Q_{A_n}\end{pmatrix}=\begin{pmatrix}Cm_{A_1}(Q_{A_1})-\frac{a}{2}-\frac{Cm_B(Q_B)}{2}\\Cm_{A_2}(Q_{A_2})-\frac{a}{2}-\frac{Cm_B(Q_B)}{2}\\\dots\\Cm_{A_n}(Q_{A_n})-\frac{a}{2}-\frac{Cm_B(Q_B)}{2}\end{pmatrix}$$

Let $M_2=\begin{pmatrix}\frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_1}}-b & -\frac{b}{2} & \dots & -\frac{b}{2}\\ -\frac{b}{2} & \frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_2}}-b & \dots & -\frac{b}{2}\\ \dots & \dots & \dots & \dots\\ -\frac{b}{2} & -\frac{b}{2} & \dots & \frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_n}}-b\end{pmatrix}$.

Now consider a matrix $M_1=(a_{ij})$ and $M_2=(b_{ij})$ where $b_{ij}=a_{ij}+\delta$. We have:

$$\det M_2=\det M_1+\delta\sum_{i,j=1}^{n}\Gamma_{ij}$$

where Γ_{ij} is the algebraic complement of a_{ij} in M_1. In this case, considering the matrix:

$$M_1=\begin{pmatrix} \frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_1}}-\frac{b}{2} & 0 & ... & 0 \\ 0 & \frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_2}}-\frac{b}{2} & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_n}}-\frac{b}{2} \end{pmatrix}$$

and $\delta=-\frac{b}{2}$ in the above relation, we get:

$$\det M_2=\prod_{k=1}^{n}\left(\frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_k}}-\frac{b}{2}\right)-\frac{b}{2}\sum_{i=1}^{n}\prod_{\substack{k=1\\k\neq i}}^{n}\left(\frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_k}}-\frac{b}{2}\right)=$$

$$\prod_{k=1}^{n}\left(\frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_k}}-\frac{b}{2}\right)-\frac{b}{2}\sum_{i=1}^{n}\frac{\prod_{k=1}^{n}\left(\frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_k}}-\frac{b}{2}\right)}{\frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_i}}-\frac{b}{2}}=$$

$$\left(1-\frac{b}{2}\sum_{i=1}^{n}\frac{1}{\frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_i}}-\frac{b}{2}}\right)\prod_{k=1}^{n}\left(\frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_k}}-\frac{b}{2}\right).$$

Also, for the matrix (*corresponding to column i*):

$$M_{4,i}=\begin{pmatrix} \frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_1}}-b & -\frac{b}{2} & ... & Cm_{A_1}(Q_{A_1})-\frac{a}{2}-\frac{Cm_B(Q_B)}{2} & ... & -\frac{b}{2} \\ -\frac{b}{2} & \frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_2}}-b & ... & Cm_{A_2}(Q_{A_2})-\frac{a}{2}-\frac{Cm_B(Q_B)}{2} & ... & -\frac{b}{2} \\ ... & ... & ... & ... & ... & ... \\ -\frac{b}{2} & -\frac{b}{2} & ... & Cm_{A_n}(Q_{A_n})-\frac{a}{2}-\frac{Cm_B(Q_B)}{2} & ... & \frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_n}}-b \end{pmatrix}$$

considering:

$$M_{3,i}=\begin{pmatrix} \frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_1}}-\frac{b}{2} & 0 & \dots & Cm_{A_1}(Q_{A_1})-\frac{Cm_B(Q_B)}{2}+\frac{b-a}{2} & \dots & 0 \\ 0 & \frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_2}}-\frac{b}{2} & \dots & Cm_{A_2}(Q_{A_2})-\frac{Cm_B(Q_B)}{2}+\frac{b-a}{2} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & Cm_{A_n}(Q_{A_n})-\frac{Cm_B(Q_B)}{2}+\frac{b-a}{2} & \dots & \frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_n}}-\frac{b}{2} \end{pmatrix}$$

we obtain:

$$\det M_{3,i}=\left(Cm_{A_i}(Q_{A_i})-\frac{Cm_B(Q_B)}{2}+\frac{b-a}{2}\right)\prod_{\substack{k=1\\k\neq i}}^{n}\left(\frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_k}}-\frac{b}{2}\right).$$

We have therefore:

$$\det M_{4,i}=\det M_{3,i}-$$
$$\frac{b}{2}\sum_{\substack{p=1\\p\neq i}}^{n}\left(Cm_{A_i}(Q_{A_i})-\frac{Cm_B(Q_B)}{2}+\frac{b-a}{2}\right)\prod_{\substack{k=1\\k\neq i,p}}^{n}\left(\frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_k}}-\frac{b}{2}\right)-$$
$$\frac{b}{2}\prod_{\substack{k=1\\k\neq i}}^{n}\left(\frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_k}}-\frac{b}{2}\right)=$$

$$\left(Cm_{A_i}(Q_{A_i})-\frac{Cm_B(Q_B)}{2}+\frac{b-a}{2}\right)\prod_{\substack{k=1\\k\neq i}}^{n}\left(\frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_k}}-\frac{b}{2}\right)-$$
$$\frac{b}{2}\sum_{\substack{p=1\\p\neq i}}^{n}\left(Cm_{A_i}(Q_{A_i})-\frac{Cm_B(Q_B)}{2}+\frac{b-a}{2}\right)\prod_{\substack{k=1\\k\neq i,p}}^{n}\left(\frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_k}}-\frac{b}{2}\right)-$$
$$\frac{b}{2}\prod_{\substack{k=1\\k\neq i}}^{n}\left(\frac{1}{2}\frac{\partial Cm_B(Q_B)}{\partial Q_{A_k}}-\frac{b}{2}\right)$$

Finally:

$$Q_{A_i}^{*}=\frac{\det M_{4,i}}{\det M_2},\ i=\overline{1,n}$$

Suppose now that all the marginal costs of A_i, $i=\overline{1,n}$, and B respectively, are constant (*on short term, the marginal cost variations are very small, the assumption being not absurd*). If therefore $Cm_{A_i}=\alpha_i$ and $Cm_B=\beta$ we obtain:

$$\det M_2=\left(1-\frac{b}{2}\sum_{i=1}^{n}\frac{1}{-\frac{b}{2}}\right)\prod_{k=1}^{n}\left(-\frac{b}{2}\right)=(-1)^n\frac{(n+1)b^n}{2^n};$$

$$\det M_{4,i}=\left(\alpha_i-\frac{\beta}{2}+\frac{b-a}{2}\right)\prod_{\substack{k=1\\k\neq i}}^{n}\left(-\frac{b}{2}\right)-\frac{b}{2}\sum_{\substack{p=1\\p\neq i}}^{n}\left(\alpha_i-\frac{\beta}{2}+\frac{b-a}{2}\right)\prod_{\substack{k=1\\k\neq i,p}}^{n}\left(-\frac{b}{2}\right)-$$

$$\frac{b}{2}\prod_{\substack{k=1\\k\neq i}}^{n}\left(-\frac{b}{2}\right)=(-1)^{n-1}\frac{b^{n-1}}{2^{n-1}}\left(n\alpha_i-\frac{n\beta}{2}-\frac{na}{2}+\frac{(n-1)b}{2}\right)$$

from where:

$$Q^*_{A_i}=\frac{\left(na+n\beta-2n\alpha_i-(n-1)b\right)}{(n+1)b};$$

$$Q^*_B=\frac{a-b\sum_{k=1}^{n}Q^*_{A_k}-\beta}{2b}=-\frac{(n^2-n-1)a}{2(n+1)b}-\frac{(n^2+n+1)\beta}{2(n+1)b}+\frac{n(n-1)}{2(n+1)}+\frac{n}{(n+1)b}\sum_{k=1}^{n}\alpha_k$$

On the other hand, the compatibility condition leads to:

$$\alpha_j-\alpha_i=\frac{b}{2}\left(Q^*_{A_i}-Q^*_{A_j}\right)\ \forall i,j=\overline{1,n}$$

therefore:

$$\alpha_j=\alpha_i\ \ \forall i,j=\overline{1,n}$$

Following these considerations, it follows that the problem has solution only if marginal costs are equal to leader firms. As a first conclusion, that detach, the n firms behave as leaders such as one that produces a common marginal cost.

Thus we obtain for $\alpha_i=\alpha$, $i=\overline{1,n}$:

$$Q^*_{A_i}=\frac{\left(na+n\beta-2n\alpha-(n-1)b\right)}{(n+1)b},$$

$$Q^*_B=-\frac{(n^2-n-1)a}{2(n+1)b}-\frac{(n^2+n+1)\beta}{2(n+1)b}+\frac{n(n-1)}{2(n+1)}+\frac{n^2\alpha}{(n+1)b}$$

The B's reaction function is:

$$Q_B = f\left(Q_{A_1},...,Q_{A_n}\right) = \frac{a-\beta}{2b} - \frac{1}{2}\sum_{k=1}^{n} Q_{A_k}$$

so in $\mathbf{R}^n$ ($O - Q_{A_1} - ... - Q_{A_n} - Q_B$) will be the equation of a hyperplane.

From the fact that $Cm_{A_i} = \alpha$ and $Cm_B = \beta$, we obtain after integration: $CT_{A_i}(Q) = \alpha Q + \gamma_i$, $i = \overline{1,n}$ and $CT_B(Q) = \beta Q + \delta$ respectively $\alpha, \beta, \gamma_i, \delta \geq 0$.

Considering now the cumulative profit of the n leader firms:

$$\Pi = \sum_{i=1}^{n} \Pi_{A_i}(Q_{A_i}) = p\left(\sum_{i=1}^{n} Q_{A_i} + Q_B\right)\sum_{i=1}^{n} Q_{A_i} - \sum_{i=1}^{n} CT_{A_i}(Q_{A_i}) = \pi \text{ =constant}$$

we obtain the equation of isoprofit hypersurfaces:

$$a - b\left(\sum_{i=1}^{n} Q_{A_i} + Q_B\right)\sum_{i=1}^{n} Q_{A_i} - n\alpha - \sum_{i=1}^{n} \gamma_i = \pi$$

or otherwise:

$$Q_B = \frac{a - \pi - n\alpha - \sum_{i=1}^{n} \gamma_i}{b\sum_{i=1}^{n} Q_{A_i}} - \sum_{i=1}^{n} Q_{A_i}$$

The condition of equilibrium will be reduced therefore to the tangent hyperplane of reaction of B to the n firms at the isoprofit hypersurface of the n firms.

4. The Price Leader

Consider now that the company A is a leader of price, in the sense that it sets the selling price. It is obvious that, regardless of the satellite firm behavior, the final sale price will be the same for the two companies, otherwise the demand being moving to the company with the lowest price.

Let Q_A – the production of the leader and Q_B – the satellite production, the price being $p>0$. We assume also the B's marginal cost as being an invertible function. The profit functions of the two companies are therefore:

$$\Pi_A(Q_A) = pQ_A - CT_A(Q_A)$$

$$\Pi_B(Q_B) = pQ_B - CT_B(Q_B)$$

The profit maximization condition of B is:

$$\frac{\partial \Pi_B}{\partial Q_B} = p - Cm_B(Q_B) = 0$$

from where: $p=Cm_B(Q_B)$. The production of B will therefore be:

$$Q_B = Cm_B^{-1}(p)$$

Meanwhile, the company leadership is aware that setting a selling price p will lead a production Q_B of the satellite firm, so in terms of a demand curve $Q=Q(p)$ its offer will be restricted to $Q_A=Q-Q_B=Q(p)-Cm_B^{-1}(p)$. Its profit function becomes:

$$\Pi_A(p) = pQ_A - CT_A(Q_A) = p\left(Q(p) - Cm_B^{-1}(p)\right) - CT_A\left(Q(p) - Cm_B^{-1}(p)\right)$$

The profit maximization condition of A is therefore:

$$\frac{\partial \Pi_A}{\partial p} = Cm_A\left(Q(p) - Cm_B^{-1}(p)\right)\left(Q'(p) - \left(Cm_B^{-1}\right)'(p)\right) = 0$$

from where:

$$Q(p) - Cm_B^{-1}(p) + p\left(Q'(p) - \frac{1}{Cm_B^{'}(Cm_B^{-1}(p))}\right) -$$
$$Cm_A\left(Q(p) - Cm_B^{-1}(p)\right)\left(Q'(p) - \frac{1}{Cm_B^{'}(Cm_B^{-1}(p))}\right) = 0$$

If the equation has the solution $p^*>0$ we obtain the allocation of production:

$$Q_A^* = Q(p^*) - Cm_B^{-1}(p^*),\ Q_B^* = Cm_B^{-1}(p^*)$$

In particular, for the demand and cost functions:

$$Q(p)=a-bp,\ a,b>0,\ CT_A(Q)=\alpha Q+\beta,\ CT_B(Q)=\gamma Q^2+\delta Q+\varepsilon,\ \alpha,\beta,\gamma,\delta,\varepsilon>0$$

we have: $Cm_A(Q)=\alpha$, $Cm_B=2\gamma Q+\delta$, $Cm_B^{'}(Cm_B^{-1}(p))=2\gamma$.

The above equation becomes:

$$a - bp - \frac{p-\delta}{2\gamma} + p\left(-b - \frac{1}{2\gamma}\right) - \alpha\left(-b - \frac{1}{2\gamma}\right) = 0$$

from where:

$$p^* = \frac{2\gamma(a+\alpha b) + \alpha + \delta}{2(2b\gamma + 1)}$$

$$Q_A^* = Q(p^*) - Cm_B^{-1}(p^*) = a - bp^* - \frac{p^* - \delta}{2\gamma} = \frac{2a\gamma + \delta - 2\gamma\alpha b - \alpha}{4\gamma}$$

$$Q_B^* = Cm_B^{-1}(p^*) = \frac{p^* - \delta}{2\gamma} = \frac{2\gamma(a + \alpha b) + \alpha - \delta - 4b\delta\gamma}{4\gamma(2b\gamma + 1)}$$

5. The Cournot Equilibrium for Duopoly

The Cournot duopoly model involves a successive adjustment yields the two companies by assuming leadership at a time.

Be so, at some time $t \in \mathbf{N}$, the production of the firm A based on the previous of the firm B:

$$Q_{A,t} = f(Q_{B,t-1}) \ \forall t \geq 1$$

and the production of the company B based on the previous firm A:

$$Q_{B,t} = g(Q_{A,t-1}) \ \forall t \geq 1$$

where f and g are continuous functions. The function $Q_A = f(Q_B)$ is called the A's firm reaction curve relative to B, and $Q_B = g(Q_A)$ – the B's firm reaction curve relative to A.

If $\exists \lim_{t\to\infty} Q_{A,t} = Q_A^*$, $\lim_{t\to\infty} Q_{B,t} = Q_B^*$ then from the above relations, it follows:

$$Q_A^* = f(Q_B^*), \ Q_B^* = g(Q_A^*)$$

The pair production $\left(Q_A^*, Q_B^*\right)$ is called Cournot equilibrium and it obtains like intersection of reaction curves of the two companies.

In the following we will consider a function of price of the form:

p(Q)=a-BQ, a,b>0 same for both companies.

Suppose now that at time t, firm A has a production $Q_{A,t}$. The company B is in a position of a satellite company and at time t+1 will have a production, in order that maximize its profit:

$$Q_{B,t+1} = \frac{a - bQ_{A,t} - Cm_B(Q_{B,t})}{2b}$$

Analogously, at the same time t, the firm A considers B as a leader and adjusts its output to:

$$Q_{A,t+1}=\frac{a-bQ_{B,t}-Cm_A(Q_{A,t})}{2b}$$

Suppose now that $Cm_A=\alpha$ and $Cm_B=\beta$. We obtain the recurrence relations:

$$Q_{B,t+1}=\frac{a-\beta-bQ_{A,t}}{2b}$$

$$Q_{A,t+1}=\frac{a-\alpha-bQ_{B,t}}{2b}$$

In a matrix writing, the relations become:

$$\begin{pmatrix} Q_{A,t+1} \\ Q_{B,t+1} \end{pmatrix}=\begin{pmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix}\begin{pmatrix} Q_{A,t} \\ Q_{B,t} \end{pmatrix}+\begin{pmatrix} \frac{a-\alpha}{2b} \\ \frac{a-\beta}{2b} \end{pmatrix}$$

If we note $Q_t=\begin{pmatrix} Q_{A,t} \\ Q_{B,t} \end{pmatrix}$, $C=\begin{pmatrix} \frac{a-\alpha}{2b} \\ \frac{a-\beta}{2b} \end{pmatrix}$ and $A=\begin{pmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix}$, we can write the above relations as:

$$Q_{t+1}=AQ_t+C$$

After induction, we obviously have:

$$Q_{t+n}=A^nQ_t+(A^{n-1}+...+A+I_2)C,\ n\geq1$$

where I_2 is the unit matrix.

In particular, for t=0, we obtain:

$$Q_n=A^nQ_0+(A^{n-1}+...+A+I_2)C$$

On the other hand, we can see (again induction) that:

$$A^{2k}=\frac{1}{2^{2k}}I_2 \text{ and } A^{2k+1}=\frac{1}{2^{2k}}A,\ \forall k\in\mathbf{N}$$

and also:

$$A^{n-1}+...+A+I_2=(A-I_2)^{-1}(A^n-I_2)$$

Because A-I$_2$=$\begin{pmatrix} -1 & -\frac{1}{2} \\ -\frac{1}{2} & -1 \end{pmatrix}$ it follows that: (A-I$_2$)$^{-1}$=$\begin{pmatrix} -\frac{4}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{4}{3} \end{pmatrix}$ and with the fact that lim $A^n=0_2$ we will have:

$$\lim (A^{n+1}+...+A+I_2)=\lim (A-I_2)^{-1}(A^n-I_2)=-(A-I_2)^{-1}=\begin{pmatrix} \frac{4}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{4}{3} \end{pmatrix}$$

From these facts, we obtain:

$$\lim Q_n=\begin{pmatrix} \frac{4}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{4}{3} \end{pmatrix}C=\begin{pmatrix} \frac{a-2\alpha+\beta}{3b} \\ \frac{a-2\beta+\alpha}{3b} \end{pmatrix}$$

The limit quantities of the two companies at equilibrium are therefore:

$$Q_A^* = \frac{a-2\alpha+\beta}{3b},\ Q_B^* = \frac{a-2\beta+\alpha}{3b}$$

the selling price being:

$$p^*=a-b(Q_A^* + Q_B^*) = \frac{a+\alpha+\beta}{3}$$

6. The Cournot Equilibrium for Oligopoly or in the Case of Perfect Competition

Let now consider a number of n firms whose productions are $Q_1,...,Q_n$. The selling price will be the same for all firms (*otherwise, the buyer choosing the lowest price*) and will depend on total production.

The corresponding profit function of the firm "k" is then:

$$\Pi_k(Q_k) = p\left(\sum_{i=1}^{n} Q_i\right)Q_k - CT_k(Q_k)$$

where CT_k is the total cost accordingly.

The profit maximization condition implies:

$$\frac{\partial \Pi_k}{\partial Q_k} = p'\left(\sum_{i=1}^{n} Q_i\right)Q_k + p\left(\sum_{i=1}^{n} Q_i\right) - Cm_k(Q_k) = 0$$

where Cm_k is the the marginal cost appropriate to the firm “k”.

Considering now the coefficient of elasticity of demand in relation to the price: $\varepsilon_{Q,p} = \frac{dQ}{dp} \cdot \frac{p}{Q}$ follows: $\frac{dp}{dQ} = \frac{p}{Q}\frac{1}{\varepsilon_{Q,p}}$ and for $Q = \sum_{i=1}^{n} Q_i$ we obtain:

$$p'\left(\sum_{i=1}^{n} Q_i\right) = \frac{p\left(\sum_{i=1}^{n} Q_i\right)}{\sum_{i=1}^{n} Q_i}\frac{1}{\varepsilon_{Q,p}}$$

Substituting in the profit maximization condition, results:

$$p\left(\sum_{i=1}^{n} Q_i\right)\left(1 + \frac{Q_k}{\sum_{i=1}^{n} Q_i}\frac{1}{\varepsilon_{Q,p}}\right) = Cm_k(Q_k)$$

Noting now $\nu_k = \frac{Q_k}{\sum_{i=1}^{n} Q_i}$, $k = \overline{1,n}$ - the share of “k” in all companies, we have:

$$p\left(\sum_{i=1}^{n} Q_i\right)\left(1 + \frac{\nu_k}{\varepsilon_{Q,p}}\right) = Cm_k(Q_k)$$

From this relationship, it follows that for n=1 that $\nu_1=1$ and we have:

$$p(Q_1)\left(1 + \frac{1}{\varepsilon_{Q,p}}\right) = Cm_1(Q_1)$$

so just state of monopoly.

On the other hand, if it exist a large number of companies on market whose share as a whole is negligible, we have: $\nu_k \approx 0$ $\forall k = \overline{1,n}$ and: $p\left(\sum_{i=1}^{n} Q_i\right) = Cm_k(Q_k)$ $\forall k = \overline{1,n}$ so the price equals the marginal cost of each firm, the market equilibrium being specific to the perfect competition.

7. Cartels

Considering any number of firms, the cartel is a situation where they collaborate to establish production that will maximize total profit, and then reverse them to divide it between them.

Let therefore be a number of n≥2 firms whose productions are $Q_1,...,Q_n$, the selling price depending on total production, the asset sold being normal.

The cartel's profit function has the following expression:

$$\Pi(Q_1,...,Q_n) = p\left(\sum_{i=1}^{n} Q_i\right)\sum_{i=1}^{n} Q_i - \sum_{i=1}^{n} CT_i(Q_i)$$

where CT_i is the total cost appropriate of the firm "i".

The profit maximization condition involves determining $Q_1,...,Q_n$ such that $\Pi(Q_1,...,Q_n)$ =maximum. We therefore have:

$$\frac{\partial \Pi}{\partial Q_k} = p'\left(\sum_{i=1}^{n} Q_i\right)\sum_{i=1}^{n} Q_i + p\left(\sum_{i=1}^{n} Q_i\right) - Cm_k(Q_k) = 0\text{ , } k=\overline{1,n}$$

where Cm_k is the marginal cost of the firm "k".

From the relationship above, it follows:

$$Cm_1(Q_1)=...=Cm_n(Q_n)$$

so at optimum, the marginal costs of the n firms must be equal. If one of the companies will have a higher marginal cost than the other, then their production will be increased to equal marginal costs at the dominant firm.

Consider now the optimal production of the n companies as: $Q_1^*,...,Q_n^*$. From the optimal relationship above, we have seen that:

$$p'\left(\sum_{i=1}^{n} Q_i^*\right)\sum_{i=1}^{n} Q_i^* + p\left(\sum_{i=1}^{n} Q_i^*\right) - Cm_k(Q_k^*) = 0\text{ , } k=\overline{1,n}$$

or:

$$p\left(\sum_{i=1}^{n} Q_i^*\right) - Cm_k(Q_k^*) = -p'\left(\sum_{i=1}^{n} Q_i^*\right)\sum_{i=1}^{n} Q_i^*\text{ , } k=\overline{1,n}$$

For some firm "j" the profit is:

$$\Pi_j(Q_1,...,Q_n) = p\left(\sum_{i=1}^{n} Q_i\right)Q_j - CT_j(Q_j)$$

from where:

$$\frac{\partial \Pi_j}{\partial Q_j} = p'\left(\sum_{i=1}^{n} Q_i\right)Q_j + p\left(\sum_{i=1}^{n} Q_i\right) - Cm_j(Q_j)$$

Adding the individual variations in profit for all companies involved in cartel result:

$$\sum_{j=1}^{n}\frac{\partial \Pi_j}{\partial Q_j} = \sum_{j=1}^{n} p'\left(\sum_{i=1}^{n} Q_i\right)Q_j + \sum_{j=1}^{n} p\left(\sum_{i=1}^{n} Q_i\right) - \sum_{j=1}^{n} Cm_j(Q_j) =$$

$$p'\left(\sum_{i=1}^{n} Q_i\right)\sum_{j=1}^{n} Q_j + np\left(\sum_{i=1}^{n} Q_i\right) - \sum_{j=1}^{n} Cm_j(Q_j)$$

In the optimum point:

$$\sum_{j=1}^{n}\frac{\partial \Pi_j}{\partial Q_j} = p'\left(\sum_{i=1}^{n} Q_i^*\right)\sum_{j=1}^{n} Q_j^* + np\left(\sum_{i=1}^{n} Q_i^*\right) - \sum_{j=1}^{n} Cm_j(Q_j^*) =$$

$$p'\left(\sum_{i=1}^{n} Q_i^*\right)\sum_{j=1}^{n} Q_j^* + \sum_{j=1}^{n}\left(p\left(\sum_{i=1}^{n} Q_i^*\right) - Cm_j(Q_j^*)\right) =$$

$$p'\left(\sum_{i=1}^{n} Q_i^*\right)\sum_{j=1}^{n} Q_j^* - \sum_{j=1}^{n}\left(p'\left(\sum_{i=1}^{n} Q_i^*\right)\sum_{i=1}^{n} Q_i^*\right) =$$

$$p'\left(\sum_{i=1}^{n} Q_i^*\right)\sum_{j=1}^{n} Q_j^* - np'\left(\sum_{i=1}^{n} Q_i^*\right)\sum_{j=1}^{n} Q_j^* = -(n-1)p'\left(\sum_{i=1}^{n} Q_i^*\right)\sum_{j=1}^{n} Q_j^* > 0$$

because good being normal: $p' < 0$.

If the firm "j" believes that all other firms will follow the terms of the cartel agreement and production will not change, so: $\frac{\partial \Pi_k}{\partial Q_k} = 0 \ \forall k = \overline{1,n}$, $k \neq j$ then from the above relationship follows: $\frac{\partial \Pi_j}{\partial Q_j} > 0$ so the firm "j" will be tempted to unilaterally increase its production to increase profit.

On the other hand, from the above relationship follows:

$$\frac{\partial \Pi_j}{\partial Q_j} = -(n-1)p'\left(\sum_{i=1}^{n} Q_i^*\right)\sum_{j=1}^{n} Q_j^* - \sum_{\substack{k=1 \\ k \neq j}}^{n}\frac{\partial \Pi_k}{\partial Q_k}$$

If the firm "j" believes that at least one of the companies do not comply with the cartel agreement and produce more, we have:

$$\frac{\partial \Pi_j}{\partial Q_j} = -(n-1)p'\left(\sum_{i=1}^{n} Q_i^*\right)\sum_{j=1}^{n} Q_j^* - \sum_{k\in I}\frac{\partial \Pi_k}{\partial Q_k} - \sum_{k\in J}\frac{\partial \Pi_k}{\partial Q_k} =$$

$$-(n-1)p'\left(\sum_{i=1}^{n} Q_i^*\right)\sum_{j=1}^{n} Q_j^* - \sum_{k\in I}\frac{\partial \Pi_k}{\partial Q_k}$$

where we denoted by I= $\left\{ k=\overline{1,n} \,\middle|\, \frac{\partial \Pi_k}{\partial Q_k} > 0 \right\}$ - the set of companies that violate the understanding and J= $\left\{ k=\overline{1,n} \,\middle|\, \frac{\partial \Pi_k}{\partial Q_k} = 0 \right\}$ - the companies that set the conditions cartel.

From optimal relationship, $\sum_{i=1}^{n} Q_i^*$ =constant thus $\frac{\partial \Pi_j}{\partial Q_j}$ varies in reverse with $\sum_{k\in I}\frac{\partial \Pi_k}{\partial Q_k}$ >0. Therefore at a breach of agreement by the other companies, the firm "j" will reduce the profit. As a result of this suspicion, the company will increase its production before this happens.

We see therefore that in the absence of strict regulations and a strict control, any firm in the cartel is tempted to increase production to achieve an increase in profit.

As a special case, let consider the case of two companies A and B that records constant marginal costs: $Cm_A=\alpha$ și $Cm_B=\beta$, the price function being of the form: $p(Q)=a-bQ$, $a>\alpha, b>0$ - the same for the two companies.

We have:

$$\Pi(Q_A, Q_B) = (a - b(Q_A + Q_B))(Q_A + Q_B) - CT_A(Q_A) - CT_B(Q_B)$$

and the profit maximizing conditions:

$$\frac{\partial \Pi}{\partial Q_A} = a - 2b(Q_A + Q_B) - \alpha = 0$$

$$\frac{\partial \Pi}{\partial Q_B} = a - 2b(Q_A + Q_B) - \beta = 0$$

We saw above that for the existence of optimal production, we have: $\alpha=\beta$ and from the above system:

$$Q_A^* + Q_B^* = \frac{a-\alpha}{2b}$$

the selling price being:

$$p^* = a - b\frac{a-\alpha}{2b} = \frac{a+\alpha}{2}$$

In the case of Cournot equilibrium, we have:

$$Q_{A,c}^* = \frac{a-2\alpha+\beta}{3b} = \frac{a-\alpha}{3b},\ Q_{B,c}^* = \frac{a-2\beta+\alpha}{3b} = \frac{a-\alpha}{3b}$$

from where:

$$Q_{A,c}^* + Q_{B,c}^* = \frac{2(a-\alpha)}{3b}$$

The selling price is:

$$p_c^* = a - b\,(Q_{A,c}^* + Q_{B,c}^*) = \frac{a+2\alpha}{3}$$

We now have:

$$\left(Q_A^* + Q_B^*\right) - \left(Q_{A,c}^* + Q_{B,c}^*\right) = \frac{a-\alpha}{2b} - \frac{2(a-\alpha)}{3b} = -\frac{(a-\alpha)}{6b} < 0$$

$$p^* - p_c^* = \frac{a-\alpha}{3} > 0$$

Following these considerations, it follows that if the cartel's total production is less than that resulting in oligopolistic competition, the selling price increases.

8. Conclusion

In this paper we analyzed the main aspects of oligopoly, in the case of n firms. The analysis has made, as a rule, for arbitrary marginal costs, each time, however, by considering these costs constant recovering well known results of the models presented.

We also treat the problems above for the general case of cost function, again customizing the overall results for linear functions and obtaining the corresponding classical relations.

9. References

Chiang, A.C. (1984). *Fundamental Methods of Mathematical Economics*. McGraw-Hill Inc.

Stancu, S. (2006). *Microeconomics*. Bucharest: Economica.

Varian, H. R. (2006). *Intermediate Microeconomics*. W.W.Norton & Co.