An Equilibrium Model for an Open Economy. Romania's Case

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Abstract: The model presented in this article is an adaptation of the IS-LM model for an open economy in which both the static aspects and dynamic ones are approached. Also, based on the model built, it is determined the level of potential GDP and the natural unemployment rate. The determination of marginal main indicators of GDP and interest rates, allow to identify problems and the directions of action to achieve economic equilibrium.

Keywords: equilibrium; GDP; investments; interest rate; consumption

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1. Introduction

The economic equilibrium problem, whose origins and manifestations are lost in the mists of time, it is always new. After a number of approaches more or less rigorous, that have benchmarks the largest economic thinkers from different current and ideologies (François Quesnay, Léon Walras, Vilfredo Pareto, Alfred Marshall) John Maynard Keynes formulated a first economic equilibrium model for a closed economy without governmental sector.

The controversies on economic equilibrium get to the maturation and development of further researches, today being analyzed the fluctuations that accompany this process. Within theory of economic equilibrium, a synthetic analysis it is the IS-LM model consisting of simultaneous equilibrium in two markets, money market and the goods and services in an autarkic economy.

Based on Keynesian macroeconomic equilibrium, in 1937, Roy Harrod, James Meade and John Hicks tried to express mathematical majors relations of Keynes'

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theory, finally to elucidate the interrelationships between theory effective demand and liquidity preference theory. (Hahn, 1977)

John Hicks' IS-LL scheme (Hicks, 1937) is the predecessor IS-LM model, the author also trying to capture the real opposition between classical and Keynesian theory, much criticized by J.M. Keynes.

Subsequent developments of Alvin Hansen (based on LL-IS schema) of 1949 and 1953 play an important role in systematizing known IS-LM model and, also, its popularization. In his book (Hansen, 1959) in order to get the curve IS, Hansen calls the investment demand function of Keynes and the neoclassical paradigm and for the LL curve is the curve of points where supply and demand. (Beaud & Dostaler, 1996)

The IS-LM model (King, 1993; Lawn, 2003a; Lawn, 2003b; Romer, 1996; Romer, 2000; Smith & Zoega, 2009; Weerapana, 2003) was the basis for further researches and we refer both the theoretical and the empirical, the major aim being the theoretical reconstruction and development of the model and practical solutions to complex problems arising in the context of globalization. (Gali, 2000) Thus, Samuleson and Solow include the original model the Phillips curve (1960), Fleming Mundell and Fleming include balance of payments (1960 and 1962), Modigliani and Friedman use the consumption function (1954 and 1957), Tobin includes the demand for money (1958).

Until the mid 1990's, the most researches were focused on modeling a closed economy, then economic literature approached, with studies undertaken by Maurice Obstfeld and Kenneth Rogoff (1995), static and dynamic equilibrium in open economies.

Although economic literature that explores New Open Economy Macroeconomics (NOEM models) is not as rich as that of the closed economy model, it is a significant theoretical edifice for the current macroeconomic modeling: Bergin (2004), Schmitt-Grohe and Uribe (2002) Justiniano and Preston (2008 & 2010), Martínez-García and Vilán (2012). The new approach enables researchers to explain the new changes that have occurred in the international macroeconomic environment based on introspection but rather on empirical causal observations, and theory is empirically validated in these cases.

In this article we propose, based on ideological vision and studies of the most important researchers in the field to determine a model for an open economy, with applications on the Romanian case, with empirical arguments, meaning for each variable used in the model it is specified degree of influence.

2. The Model Equations

The first equation of the model is the formula of the aggregate demand:

(1) $D=C+G+I+NX$

where

- D – the aggregate demand;
- C – the actual final consumption of households;
- G – the actual final consumption of the government;
- I – the investments;
- NX – the net export.

A second equation relates the actual final consumption of households according to available income V:

(2) $C=c_V V+C_0$, $C_0\geq 0$, $c_Y\in(0,1)$

where c_V – the marginal propensity to consume, $c_V=\frac{dC}{dV}\in(0,1)$ and C_0 is the intrinsic achieved autonomous consumption of households.

We will assume below that G and NX are proportional to the GDP, denoted by Y, given that in the absence of GDP can not engage any government spending (excluding in this analysis foreign loans) and also can not conduct foreign trade.

(3) $G=g_Y Y$, $g_Y\in(0,1)$

(4) $NX=\nu_Y Y$, $|\nu_Y|\in(0,1)$

where:

- g_Y – the marginal government consumption;
- ν_Y – the marginal net exports

Relative to investments, we will consider a direct linear dependence of the GDP level and inverse from the interest rate:

(5) $I=in_Y Y+i_r r$, $in_Y\in(0,1)$, $i_r<0$, $I_0>0$

- in_Y –the rate of investments, $in_Y\in(0,1)$;
- i_r – a factor of influence on the investment rate, $i_r<0$;

The following equations express the dependence of the net income GDP, the government transfers (TR) and tax rate (TI):

(6) $V=Y+TR-TI,\ TR>0$

(7) $TR=\theta_Y Y,\ \theta_Y \in (0,1)$

(8) $TI=ri_Y Y+T_0,\ ri_Y \in (0,1),\ T_0 \geq 0$

In equation (7) we assumed the linear dependence of transfers of GDP, assuming in the case of the fees an affine dependence, T_0 being the independent taxes from the income (property taxes and so on). Let note that: θ_Y – the marginal government transfers and ri_Y – the tax rate, $ri_Y \in (0,1)$.

The static equilibrium equation is:

(9) $D=Y$

The following set of equations refers to monetary issues. We assume so:

(10) $MD=md_Y Y+m_r r,\ md_Y>0,\ m_r<0$

where:

- MD – the money demand in the economy;
- md_Y – the rate of money demand in the economy;
- m_r – a factor of influencing the demand for currency from the interest rate, $m_r<0$;
- r – the real interest rate.

The equilibrium equation being:

(11) $MD=M$

where M represents the money supply.

The dynamic equations of the model are:

(12) $$\frac{dY}{dt}=\alpha(D-Y),\ \alpha>0$$

(13) $$\frac{dr}{dt}=\beta(MD-M),\ \beta>0$$

3. The Static Equilibrium

From (1)-(8) we get:

(14) $D=c_V V+C_0+g_Y Y+in_Y Y+i_r r+\nu_Y Y=Y(c_V+c_V\theta_Y-c_V ri_Y+g_Y+in_Y+\nu_Y)-c_V T_0+C_0+i_r r$

Noting:

(15) $E = C_0 - c_V T_0$

(16) $\omega = 1 + \theta_Y - ri_Y$

(17) $\chi = 1 - c_V(1 + \theta_Y - ri_Y) - g_Y - in_Y - \nu_Y = 1 - c_V\omega - g_Y - in_Y - \nu_Y$

let note first that from (2), (6)-(8): $V=Y(1+\theta_Y-ri_Y)-T_0=\omega Y-T_0$, and:

(18) $C=c_V(\omega Y-T_0)+C_0=c_V\omega Y+E$

As in the absence of GDP (Y=0) the consumption must be positive, follows that $E\geq 0$.

From the fact that $ri_Y\in(0,1)$, $\theta_Y\in(0,1)$ we get that: $\omega = 1 + \theta_Y - ri_Y \in (0,2)$.

With the notations (15)-(17), equation (14) becomes:

(19) $D=Y(1-\chi)+i_r r+E$

The equilibrium condition D=Y in (9) implies: $Y(1-\chi)+i_r r+E=Y$ therefore:

(20) $Y = \frac{i_r}{\chi} r + \frac{E}{\chi}$

The natural condition that at the increase of r, Y must decrease required: $\frac{i_r}{\chi} < 0$ so $\chi > 0$.

From the fact that $c_V, g_Y, in_Y, |\nu_Y|, \theta_Y, ri_Y \in (0,1)$ we get that $\chi > 0$ if and only if:

(21) $c_V < \frac{1 - g_Y - in_Y - \nu_Y}{1 + \theta_Y - ri_Y}$

Similarly, from equations (10),(11): $MD=md_Y Y+m_r r=M$ therefore:

(22) $Y = -\frac{m_r}{md_Y} r + \frac{M}{md_Y}$

The condition of equilibrium on the two markets (goods & services and monetary):

(23) $$\begin{cases} Y = \frac{i_r}{\chi} r + \frac{E}{\chi} \\ Y = -\frac{m_r}{md_Y} r + \frac{M}{md_Y} \end{cases}$$

After solving the system we have:

(24) $$\begin{cases} Y = \dfrac{Mi_r + Em_r}{i_r md_Y + m_r\chi} \\ r = \dfrac{M\chi - Emd_Y}{i_r md_Y + m_r\chi} \end{cases}$$

The equations (24) give the static equilibrium model.

Noting now, for simplicity:

(25) $$\Lambda = \frac{1}{i_r md_Y + m_r\chi} < 0$$

(26) $$\Gamma = \left(Mi_r + m_r E\right)\Lambda^2 < 0$$

follows:

(27) $$md_Y E - \chi M = \frac{E\Lambda - \Gamma\chi}{i_r\Lambda^2} = \frac{md_Y\Gamma - M\Lambda}{m_r\Lambda^2}$$

(28) $$\begin{cases} Y = \dfrac{\Gamma}{\Lambda} \\ r = \dfrac{\Gamma\chi - E\Lambda}{i_r\Lambda} = \dfrac{M\Lambda - md_Y\Gamma}{m_r\Lambda} \end{cases}$$

From formulas (24) we have therefore:

(29) $$\frac{\partial Y}{\partial c_V} = m_r\left(\omega\Gamma - T_0\Lambda\right), \ \frac{\partial Y}{\partial g_Y} = \frac{\partial Y}{\partial \nu_Y} = \frac{\partial Y}{\partial in_Y} = m_r\Gamma, \ \frac{\partial Y}{\partial \theta_Y} = -\frac{\partial Y}{\partial ri_Y} = m_r c_V\Gamma,$$
$$\frac{\partial Y}{\partial i_r} = M\Lambda - md_Y\Gamma, \ \frac{\partial Y}{\partial md_Y} = -i_r\Gamma, \ \frac{\partial Y}{\partial m_r} = -i_r\frac{M\Lambda - md_Y\Gamma}{m_r}, \ \frac{\partial Y}{\partial M} = i_r\Lambda$$

(30) $$\frac{\partial r}{\partial c_V} = -md_Y\left(\omega\Gamma - T_0\Lambda\right), \ \frac{\partial r}{\partial g_Y} = \frac{\partial r}{\partial \nu_Y} = \frac{\partial r}{\partial in_Y} = -md_Y\Gamma,$$
$$\frac{\partial r}{\partial \theta_Y} = -\frac{\partial r}{\partial ri_Y} = -md_Y c_V\Gamma, \ \frac{\partial r}{\partial i_r} = -md_Y\frac{M\Lambda - md_Y\Gamma}{m_r}, \ \frac{\partial r}{\partial md_Y} = -\chi\Gamma,$$
$$\frac{\partial r}{\partial m_r} = -\chi\frac{M\Lambda - md_Y\Gamma}{m_r}, \ \frac{\partial r}{\partial M} = \chi\Lambda$$

(31) $\frac{\partial^2 Y}{\partial c_V^2} = 2(\omega\Gamma - T_0\Lambda)\omega m_r^2\Lambda$, $\frac{\partial^2 Y}{\partial g_Y^2} = \frac{\partial^2 Y}{\partial \nu_Y^2} = \frac{\partial^2 Y}{\partial in_Y^2} = 2m_r^2\Gamma\Lambda$,

$\frac{\partial^2 Y}{\partial \theta_Y^2} = -\frac{\partial^2 Y}{\partial ri_Y^2} = 2c_V m_r^2\Gamma\Lambda$, $\frac{\partial^2 Y}{\partial i_r^2} = -2(M\Lambda - md_Y\Gamma)md_Y\Lambda$, $\frac{\partial^2 Y}{\partial md_Y^2} = 2i_r^2\Gamma\Lambda$,

$\frac{\partial^2 Y}{\partial m_r^2} = 2\chi i_r\Lambda\frac{M\Lambda + md_Y\Gamma}{m_r}$, $\frac{\partial^2 Y}{\partial M^2} = 0$

(32) $\frac{\partial^2 r}{\partial c_V^2} = -2(\omega\Gamma - T_0\Lambda)\omega md_Y m_r\Lambda$, $\frac{\partial^2 r}{\partial g_Y^2} = \frac{\partial^2 r}{\partial \nu_Y^2} = \frac{\partial^2 r}{\partial in_Y^2} = -2m_r md_Y\Gamma\Lambda$,

$\frac{\partial^2 r}{\partial \theta_Y^2} = -\frac{\partial^2 r}{\partial ri_Y^2} = -2c_V m_r md_Y\Gamma\Lambda$, $\frac{\partial^2 r}{\partial i_r^2} = 2\frac{md_Y^2}{m_r}(M\Lambda - md_Y\Gamma)\Lambda$, $\frac{\partial^2 r}{\partial md_Y^2} = 2i_r\chi\Gamma\Lambda$

, $\frac{\partial^2 r}{\partial m_r^2} = 2\chi^2\Lambda\frac{M\Lambda + md_Y\Gamma}{m_r}$, $\frac{\partial^2 r}{\partial M^2} = 0$

To analyze the monotony of Y and of r, it is imperative to study the signs of $\omega\Gamma - T_0\Lambda$ and $M\Lambda - md_Y\Gamma$.

Noting:

(33) $\Phi_1 = \frac{c_V m_r\omega T_0^2 - Mi_r\omega T_0 + i_r md_Y + m_r\chi}{m_r\omega T_0}$, $\Phi_2 = \frac{M\chi + c_V md_Y T_0}{md_Y}$

we get that $\Phi_1 > \Phi_2$ if and only if $T_0 < \frac{md_Y}{\omega M}$.

On the other hand, since $E = C_0 - c_V T_0 \geq 0$ i.e. $C_0 \geq c_V T_0$ results:

$\Phi_2 - c_V T_0 = \frac{M\chi}{md_Y} > 0$ therefore: $\Phi_2 > c_V T_0 > 0$, $\Phi_1 - c_V T_0 = \frac{-Mi_r\omega T_0 + i_r md_Y + m_r\chi}{m_r\omega T_0} > 0$.

In conclusion, we get that:

$\Phi_1 > c_V T_0$ if $T_0 < \frac{md_Y}{M\omega} + \frac{m_r\chi}{Mi_r\omega}$ and $\Phi_1 \leq c_V T_0$ if $T_0 \geq \frac{md_Y}{M\omega} + \frac{m_r\chi}{Mi_r\omega}$

After these considerations, there are three main cases:

1. $T_0 < \frac{md_Y}{\omega M} \Rightarrow \Phi_1 > \Phi_2 > c_V T_0 > 0$

2. $\frac{md_Y}{M\omega} + \frac{m_r\chi}{Mi_r\omega} > T_0 \geq \frac{md_Y}{\omega M}$ $\Rightarrow \Phi_2 \geq \Phi_1 > c_V T_0 > 0$

3. $T_0 \geq \frac{md_Y}{M\omega} + \frac{m_r\chi}{Mi_r\omega}$ $\Rightarrow \Phi_2 > c_V T_0 \geq \Phi_1$.

On the other hand, the condition that $\omega\Gamma - T_0\Lambda > 0$ lead to $C_0 > \Phi_1$, and $M\Lambda - md_Y\Gamma > 0$ lead to $C_0 > \Phi_2$.

Regardless of the above, we have:

- Y is strictly increasing and strictly convex with respect to marginal government consumption g_Y, with respect to marginal net exports ν_Y, with the rate of investments in_Y and the marginal government transfers θ_Y. Y is strictly decreasing and strictly concave with respect to the tax rate ri_Y. Y is strictly decreasing and strictly convex in relation to the rate of money demand in the economy md_Y. Y is strictly increasing and affine in relation to the money supply M.

- r is strictly increasing and strictly convex with respect to the marginal government consumption g_Y, with respect to the marginal net exports ν_Y, with the rate of Investments in_Y and the marginal government transfers θ_Y. r is strictly decreasing and strictly concave with respect to the tax rate ri_Y. r is strictly increasing and strictly concave in relation to the rate of money demand in the economy md_Y. r is strictly decreasing and affine in relation to the money supply M.

We now have the following cases:

Case 1 $T_0 < \frac{md_Y}{\omega M}$ and $C_0 \in (c_V T_0, \Phi_2)$ implies: $\omega\Gamma - T_0\Lambda < 0$, $M\Lambda - md_Y\Gamma < 0$. In this case:

- Y is strictly increasing and strictly convex in relation to the marginal propensity to consume c_V and the factor of influencing the demand for currency from the interest rate m_r. Y is strictly decreasing and strictly convex in relation to the factor of influence on the investment rate i_r.

- r is strictly increasing and strictly convex in relation to the marginal propensity to consume c_V. r is strictly decreasing and strictly concave in relation to the factor of influence on the investment rate i_r and the factor of influencing the demand for currency from the interest rate m_r.

Case 2 $T_0 < \frac{md_Y}{\omega M}$ and $C_0 \in [\Phi_2, \Phi_1]$ implies: $\omega\Gamma - T_0\Lambda < 0$, $M\Lambda - md_Y\Gamma > 0$.

- Y is strictly increasing and strictly convex in relation to the marginal propensity to consume c_V and in relation to the factor of influence on the investment rate i_r. Y is strictly decreasing and strictly convex in relation to the factor of influencing the demand for currency from the interest rate m_r.

- r is strictly increasing and strictly convex in relation to the marginal propensity to consume c_V and the factor of influence on the investment rate i_r. r is strictly decreasing and strictly concave in relation to the factor of Influencing the demand for currency from the interest rate m_r.

Case 3 $T_0 < \frac{md_Y}{\omega M}$ and $C_0 \in (\Phi_1, \infty)$ implies: $\omega\Gamma - T_0\Lambda > 0$, $M\Lambda - md_Y\Gamma > 0$.

- Y is strictly decreasing and strictly concave in relation to the marginal propensity to consume c_V. Y is strictly decreasing and strictly convex in relation to the factor of influencing the demand for currency from the interest rate m_r. Y is strictly increasing and strictly convex in relation to the factor of Influence on the investment rate i_r.

- r is strictly decreasing and strictly concave in relation to the marginal propensity to consume c_V. r is strictly increasing and strictly convex in relation to the factor of influence on the investment rate i_r. r is strictly increasing and strictly concave in relation to the factor of influencing the demand for currency from the interest rate m_r.

Case 4 $\frac{md_Y}{M\omega} + \frac{m_r\chi}{Mi_r\omega} > T_0 \geq \frac{md_Y}{\omega M}$ and $C_0 \in (c_V T_0, \Phi_1)$ implies: $\omega\Gamma - T_0\Lambda < 0$, $M\Lambda - md_Y\Gamma < 0$.

- Y is strictly increasing and strictly convex in relation to the marginal propensity to consume c_V and the factor of influencing the demand for currency from the interest rate m_r. Y is strictly decreasing and strictly concave in relation to the factor of Influence on the investment rate i_r.

- r is strictly increasing and strictly convex in relation to the marginal propensity to consume c_V. r is strictly decreasing and strictly concave in relation to the factor of influence on the investment rate i_r and the factor of influencing the demand for currency from the interest rate m_r.

Case 5 $\frac{md_Y}{M\omega} + \frac{m_r\chi}{Mi_r\omega} > T_0 \geq \frac{md_Y}{\omega M}$ and $C_0 \in [\Phi_1, \Phi_2]$ implies: $\omega\Gamma - T_0\Lambda > 0$, $M\Lambda - md_Y\Gamma < 0$.

• Y is strictly decreasing and strictly concave in relation to the marginal propensity to consume c_V and the factor of influence on the investment rate i_r. Y is strictly increasing and strictly convex with respect to m_r.

• r is strictly decreasing and strictly concave in relation to the marginal propensity to consume c_V, the factor of influence on the investment rate i_r and the factor of influencing the demand for currency from the interest rate m_r.

Case 6 $\frac{md_Y}{M\omega}+\frac{m_r\chi}{Mi_r\omega}>T_0\geq\frac{md_Y}{\omega M}$ and $C_0\in(\Phi_2,\infty)$ implies: $\omega\Gamma-T_0\Lambda>0$, $M\Lambda-md_Y\Gamma>0$.

• Y is strictly decreasing and strictly concave in relation to the marginal propensity to consume c_V. Y is strictly increasing and strictly convex in relation to the factor of influence on the investment rate i_r. Y is strictly decreasing and strictly convex in relation to the factor of influencing the demand for currency from the interest rate m_r.

• r is strictly decreasing and strictly concave in relation to the marginal propensity to consume c_V. r is strictly increasing and strictly convex in relation to the factor of influence on the investment rate i_r. r is strictly increasing and strictly concave in relation to the factor of influencing the demand for currency from the interest rate m_r.

Case 7 $T_0\geq\frac{md_Y}{M\omega}+\frac{m_r\chi}{Mi_r\omega}$ and $C_0\in(c_VT_0,\Phi_2)$ implies: $\omega\Gamma-T_0\Lambda>0$, $M\Lambda-md_Y\Gamma<0$.

• Y is strictly decreasing and strictly concave in relation to the marginal propensity to consume c_V and the factor of Influence on the investment rate i_r. Y is strictly increasing and strictly convex in relation to the factor of Influencing the demand for currency from the interest rate m_r.

• r is strictly decreasing and strictly concave in relation to the marginal propensity to consume c_V, with the factor of influence on the investment rate i_r and the factor of influencing the demand for currency from the interest rate m_r.

Case 8 $T_0\geq\frac{md_Y}{M\omega}+\frac{m_r\chi}{Mi_r\omega}$ and $C_0\in[\Phi_2,\infty)$ implies: $\omega\Gamma-T_0\Lambda>0$, $M\Lambda-md_Y\Gamma>0$.

• Y is strictly decreasing and strictly concave in relation to the marginal propensity to consume c_V. Y is strictly increasing and strictly convex in relation to the factor of influence on the investment rate i_r. Y is strictly decreasing and strictly convex in relation to the factor of influencing the demand for currency from the interest rate m_r.

- r is strictly decreasing and strictly concave in relation to the marginal propensity to consume c_V. r is strictly increasing and strictly convex in relation to the factor of influence on the investment rate i_r. r is strictly increasing and strictly concave in relation to the factor of influencing the demand for currency from the interest rate m_r.

4. The Determination of the Potential GDP. Okun's Law

Considering the money supply constant in time, we can consider as potential GDP value, the static equilibrium value:

$$Y^* = \frac{Mi_r + Em_r}{i_r md_Y + m_r\chi} \quad (34)$$

Once determined the potential level of GDP, we naturally put the problem of determining the natural rate of unemployment. The known expression of Okun's law is:

$$\frac{Y^* - Y}{Y^*} = c(u - u^*) \quad (35)$$

where:

- Y – the actual GDP;
- Y^* – the potential GDP;
- u – the unemployment rate;
- u^* – the natural rate of unemployment;
- c – a factor of proportionality.

Due to the difficulties in the appliance of Okun's law (consisting in the impossibility to determine the potential GDP - made in conditions of full employment of labor) and also the natural rate of unemployment, is used in practice, a modified form of it, as follows:

$$\frac{\Delta Y}{Y} = a - c\Delta u \quad (36)$$

The advantage of this is to eliminate the explicit expressions of the potential GDP and the natural unemployment. On the other hand, in our analysis, we will determine the value of the constant c using the relation (36) and then inserting it into (35) which allows the determination of the natural rate of unemployment at a given time.

Being so determined the constant c, we have from (34), (35):

$$(37) \qquad u^* = u - \frac{Y^* - Y}{cY^*} = u - \frac{1}{c} + \frac{i_r m d_Y + m_r \chi}{c(Mi_r + Em_r)} Y$$

From equation (37) it is observed that u^* increases with Y with the factor $\frac{i_r m d_Y + m_r \chi}{c(Mi_r + Em_r)}$.

5. The Dynamic Equilibrium

The equations (12) and (13) is constituted as laws of dynamic equilibrium. Let so the system of first order differential equations:

$$(38) \qquad \begin{cases} \frac{dY}{dt} = \alpha(D - Y) \\ \frac{dr}{dt} = \beta(MD - M) \end{cases}, \ \alpha,\beta>0$$

From (10),(19) we can write (38) as:

$$(39) \qquad \begin{cases} \frac{dY}{dt} = -\alpha\chi Y + \alpha i_r r + \alpha E \\ \frac{dr}{dt} = \beta m d_Y Y + \beta m_r r - \beta M \end{cases}$$

Using the lemma from appendix A.1, it follows that: $\lim_{t\to\infty} Y(t) = \tilde{Y}$, $\lim_{t\to\infty} r(t) = \tilde{r}$, $\tilde{Y}, \tilde{r} \in \mathbf{R}_+$ if and only if:

1. $\Delta=(\alpha\chi+\beta m_r)^2+4\alpha\beta i_r m d_Y=0$ then:

$$(40) \qquad \begin{cases} Y = \left(-\frac{\alpha\chi + \beta m_r}{2} Y_0 + \alpha i_r r_0 + \alpha \frac{2 i_r \beta M + E(\alpha\chi + \beta m_r)}{\alpha\chi - \beta m_r}\right) t e^{\frac{-\alpha\chi+\beta m_r}{2} t} + \\ \left(Y_0 + 4\alpha\beta \frac{m_r E + i_r M}{(\alpha\chi - \beta m_r)^2}\right) e^{\frac{-\alpha\chi+\beta m_r}{2} t} + \frac{i_r M + m_r E}{\chi m_r + i_r m d_Y} \\ r = \left[r_0 - 4\alpha\beta \frac{-\chi M + m d_Y E}{(\alpha\chi - \beta m_r)^2}\right] e^{\frac{-\alpha\chi+\beta m_r}{2} t} + \\ \left(-\frac{\alpha\chi + \beta m_r}{2} Y_0 + \alpha i_r r_0 + \alpha \frac{2 i_r \beta M + E(\alpha\chi + \beta m_r)}{\alpha\chi - \beta m_r}\right) \frac{\alpha\chi + \beta m_r}{2\alpha i_r} t e^{\frac{-\alpha\chi+\beta m_r}{2} t} + \frac{\chi M - m d_Y E}{\chi m_r + i_r m d_Y} \end{cases}$$

and: $\begin{cases} \tilde{Y} = \dfrac{m_r E + i_r M}{\chi m_r + i_r m d_Y} \\ \tilde{r} = \dfrac{\chi M - m d_Y E}{\chi m_r + i_r m d_Y} \end{cases}$

2. $\Delta=(\alpha\chi+\beta m_r)^2+4\alpha\beta i_r m d_Y>0$ and $\lambda_1 \neq \lambda_2$ are roots of the equation: $\lambda^2+(\alpha\chi-\beta m_r)\lambda-\alpha\beta(\chi m_r+i_r m d_Y)=0$ then:

$$(41)\quad \begin{cases} Y = k_1 e^{\lambda_1 t} + k_2 e^{\lambda_2 t} + \dfrac{m_r E + i_r M}{\chi m_r + i_r m d_Y} \\ r = \dfrac{\lambda_1 + \alpha\chi}{\alpha i_r} k_1 e^{\lambda_1 t} + \dfrac{\lambda_2 + \alpha\chi}{\alpha i_r} k_2 e^{\lambda_2 t} - \dfrac{m d_Y E - \chi M}{\chi m_r + i_r m d_Y} \end{cases}$$

where:

$$k_1 = \frac{(\lambda_2 + \alpha\chi)Y_0 - (\lambda_2 + \alpha\chi)\dfrac{m_r E + i_r M}{\chi m_r + i_r m d_Y} - \alpha i_r r_0 - \alpha i_r \dfrac{m d_Y E - \chi M}{\chi m_r + i_r m d_Y}}{\lambda_2 - \lambda_1}$$

$$k_2 = \frac{\alpha i_r r_0 + \alpha i_r \dfrac{m d_Y E - \chi M}{\chi m_r + i_r m d_Y} - (\lambda_1 + \alpha\chi)Y_0 + (\lambda_1 + \alpha\chi)\dfrac{m_r E + i_r M}{\chi m_r + i_r m d_Y}}{\lambda_2 - \lambda_1}$$

and: $\begin{cases} \tilde{Y} = \dfrac{m_r E + i_r M}{\chi m_r + i_r m d_Y} \\ \tilde{r} = \dfrac{\chi M - m d_Y E}{\chi m_r + i_r m d_Y} \end{cases}$

3. $\Delta=(\alpha\chi+\beta m_r)^2+4\alpha\beta i_r m d_Y<0$ and $\lambda_1=\mu+i\nu$, $\lambda_2=\mu-i\nu$, $\nu\neq 0$ are roots of the equation: $\lambda^2+(\alpha\chi-\beta m_r)\lambda-\alpha\beta(\chi m_r+i_r m d_Y)=0$ then:

$$(42)\quad \begin{cases} Y=\left(Y_0-\dfrac{m_rE+i_rM}{\chi m_r+i_rmd_Y}\right)e^{\mu t}\cos\nu t+ \\ \dfrac{1}{\nu}\left(\alpha i_r r_0-\dfrac{\beta m_r+\alpha\chi}{2}Y_0+\dfrac{(\beta m_r+\alpha\chi)(i_rM+m_rE)+2\alpha i_r(-\chi M+md_YE)}{2(\chi m_r+i_rmd_Y)}\right)e^{\mu t}\sin\nu t+ \\ \dfrac{i_rM+m_rE}{\chi m_r+i_rmd_Y} \\ r=\left(r_0+\dfrac{-\chi M+md_YE}{\chi m_r+i_rmd_Y}\right)e^{\mu t}\cos\nu t+ \\ \dfrac{1}{\nu}\left(\beta md_YY_0+\dfrac{\beta m_r+\alpha\chi}{2}r_0+\dfrac{(\beta m_r+\alpha\chi)(-\chi M+md_YE)-2\beta md_Y(m_rE+i_rM)}{2(\chi m_r+i_rmd_Y)}\right)e^{\mu t}\sin\nu t+ \\ \dfrac{\chi M-md_YE}{\chi m_r+i_rmd_Y} \end{cases}$$

and: $$\begin{cases} \tilde{Y}=\dfrac{m_rE+i_rM}{\chi m_r+i_rmd_Y} \\ \tilde{r}=\dfrac{\chi M-md_YE}{\chi m_r+i_rmd_Y} \end{cases}$$

We will call $\tilde{Y}$ - the limit of the output and $\tilde{r}$ - the interest rate limit.

6. The Analysis of the Romanian Economy

Using the data table A.1 and the results of analyzes from the appendix A.2 there are obtained the corresponding regression equations for Romania during 2001-2011.

Table 1

The regression equation	The regression's coefficients	
C=0.59526V+18527.39699	c_V=0.59526	C_0=18527.39699
G=0.07703Y	g_Y=0.07703	
I=0.28077Y-79168.78775r	in_Y=0.28077	i_r=79168.78775
NX=-0.08858Y	ν_Y=-0.08858	
TR=0.09727Y	θ_Y=0.09727	
TI=0.06905Y+5117.37477	ri_Y=0.06905	T_0=5117.37477
MD=0.08850Y-59560.45339r	md_Y=0.08850	m_r=-59560.45339

Substituting in relations (24) we obtain the values of static equilibrium are, for 2011 (expressed in 2000-national currency) and M=11603.05: Y=130753.8 and r=-0.00053=-0.053%.

Considering the inflation rate from 2011 as i=5.79% we obtain using the formula: rn= $r(i+1)+i$ where rn is the nominal interest rate: rn=5.73%.

On the other hand, the potential level of GDP calculated by formula (32) in the period was:

Table 2

Anul	Y	Y^*	$Y-Y^*$	$\frac{Y-Y^*}{Y^*}$
2001	85841.1936	88910.28844	-3069.094844	-3.45%
2002	89658.25153	89721.72265	-63.47112168	-0.07%
2003	93904.04246	90443.68138	3460.361078	3.83%
2004	102310.9459	94351.2625	7959.683385	8.44%
2005	106421.2703	101420.1072	5001.163027	4.93%
2006	114561.3451	112366.9512	2194.393982	1.95%
2007	122371.7164	123753.4116	-1381.695222	-1.12%
2008	135665.8673	134831.2139	834.6533952	0.62%
2009	124029.6072	129248.2296	-5218.622443	-4.04%
2010	124837.2351	128197.8657	-3360.630587	-2.62%
2011	129050.5671	130753.8278	-1703.26069	-1.30%

It can therefore be seen that in 2011, the Romanian economy was close to the potential output level, the only disturbing factor being the rate averaged 6.25% higher than those of equilibrium.

Relative to Okun's law, the data in table A.2, gives us a value for c=1.707.

From formula (35) follows, for Romania:

$$(43)\qquad u^* = u - 0.5858 + \frac{14077.351}{135141121M + 1573970536} Y$$

Considering the monetary base for the reference period, we get:

Table 3

Year	The real unemployment rate (u)	The natural unemployment rate (u^*)	Difference $u-u^*$
2001	8.60%	6.59%	2.01%
2002	8.10%	8.06%	0.04%
2003	7.20%	9.42%	-2.22%
2004	6.20%	11.10%	-4.90%
2005	5.90%	8.77%	-2.87%
2006	5.20%	6.34%	-1.14%
2007	4.10%	3.45%	0.65%
2008	4.40%	4.76%	-0.36%
2009	7.80%	5.45%	2.35%
2010	6.87%	5.35%	1.52%
2011	5.12%	4.36%	0.76%

The corresponding data from the tables 2 and 3 show that in 2003-2006 and in 2008 the Romanian economy was overheated, Romania's GDP being in excess in comparison to the potential level. Thus, in 2004, the relative difference was 8.44% being explained and justified by an ill-founded relative increase in the monetary base of 15.67% from the previous period when the increase was ranging between 2.98% and 3.47%. Since 2009 the situation has changed radically, its level being 4.04% less than the potential, the difference becoming smaller over time.

Relative to the unemployment rate, the phenomenon has evolved almost identical. If in 2003-2006 and in 2008 was an over-hiring (with a maximum difference of -4.90% in the same year 2004), since 2009, the economic crisis set, the appropriate values over 1% (with a peak in 2009 of 2.35% above the natural level).

Relative to the rate evolution, we have:

Table 4

Year	The nominal interest rate (rn)	The equilibrium nominal interest rate (r)	rn-r
2001	38.80%	42.87%	-4.07%
2002	28.47%	29.97%	-1.50%
2003	18.84%	22.21%	-3.37%

2004	20.27%	17.95%	2.32%
2005	9.59%	13.74%	-4.15%
2006	8.44%	9.38%	-0.94%
2007	7.46%	5.89%	1.57%
2008	9.46%	7.13%	2.33%
2009	9.33%	5.77%	3.56%
2010	6.67%	6.44%	0.23%
2011	6.25%	5.73%	0.52%

It is noted that in the periods 2001-2003 and 2005-2006, the NBR's (the National Bank of Romania) interest rate was below the equilibrium level. During the crisis, since 2009, it has overwhelmed the equilibrium (even with 3.56% in 2009) which led to the deepening crisis by discouraging investments.

Considering now the dynamic evolution of GDP and the money demand are obtained average values $\alpha=3.183904003$ and $\beta=7.57236\cdot10^{-6}$ where $\Delta<0$.

The graphs of progression to equilibrium values are:

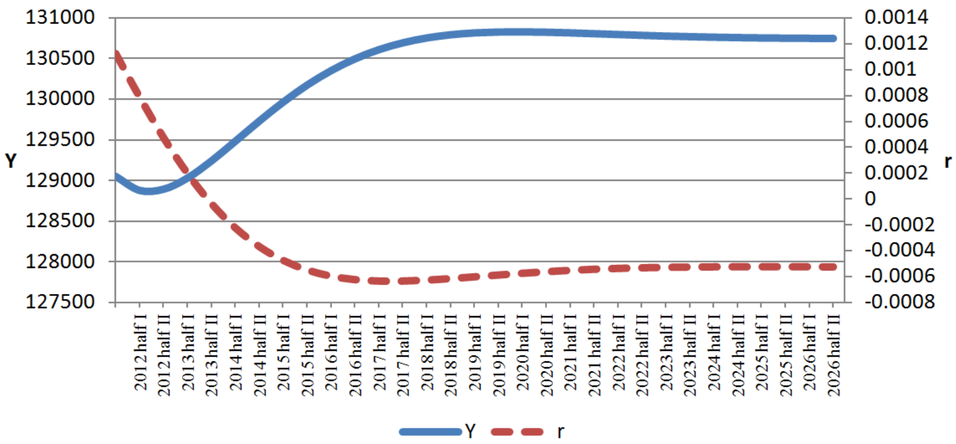

Figure 1. The evolution of GDP for Δ<0 (2000 national currency)

Considering now perturbed values $\alpha=3$ and $\beta=7,57236\cdot10^{-6}$ for which $\Delta=0$, we obtain graphs of evolution towards equilibrium values:

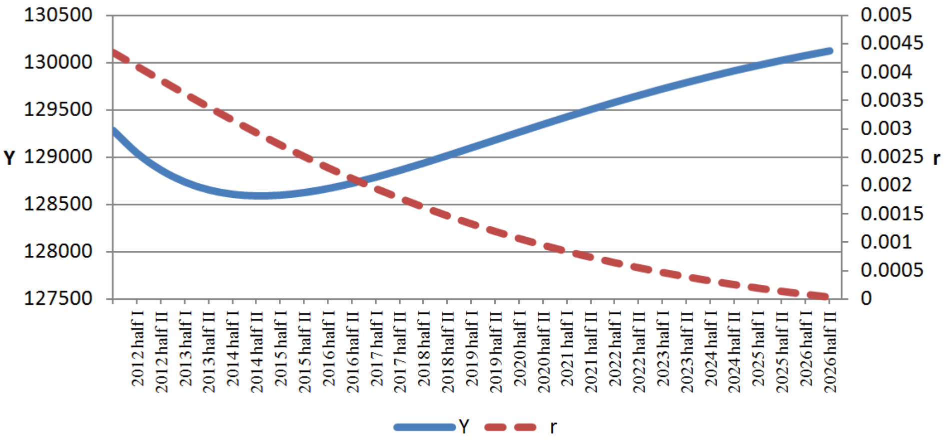

Figure 2. The evolution of GDP for Δ=0 (2000 national currency)

Finally, considering now new perturbed values $\alpha=3$ and $\beta=10^{-6}$ for which $\Delta>0$, we obtain graphs of evolution towards equilibrium values:

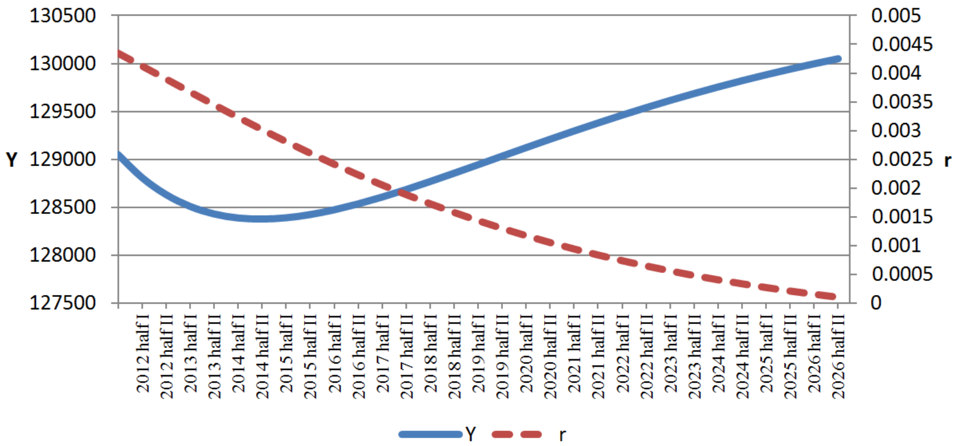

Figure 3. The evolution of GDP for Δ>0 (2000 national currency)

From the graphs above, it appears that the most favorable situation to achieve potential output in terms of a minimum interest is the corresponding value of $\Delta<0$ in which approximately seven years to obtain optimum.

Otherwise there is a very weak decrease of the real interest rate which is kept at high enough levels, accompanied by a reduction in GDP over a period of about three years, which is unacceptable. Therefore the condition that $\Delta<0 \Leftrightarrow (\alpha\chi+\beta m_r)^2+4\alpha\beta i_r md_Y<0$ is the most convenient.

Considering: $\chi^2\alpha^2 + 2(\chi m_r + 2i_r md_Y)\alpha\beta + m_r^2\beta^2 < 0$ we find that:

$$\frac{\alpha}{\beta} \in \left(\frac{-(\chi m_r + 2i_r md_Y) - 2\sqrt{i_r md_Y(\chi m_r + i_r md_Y)}}{\chi^2}, \frac{-(\chi m_r + 2i_r md_Y) + 2\sqrt{i_r md_Y(\chi m_r + i_r md_Y)}}{\chi^2} \right)$$

Computing the partial derivatives of Y for the existing monetary basis in 2011, we get to a 0.01 variation of parameters:

$\Delta Y\Big|_{\Delta c_V = 0,01} = 5472,$ $\Delta Y\Big|_{\Delta g_Y = 0,01} = \Delta Y\Big|_{\Delta \nu_Y = 0,01} = \Delta Y\Big|_{\Delta in_Y = 0,01} = 5532,$

$\Delta Y\Big|_{\Delta \theta_Y = 0,01} = \Delta Y\Big|_{\Delta ri_Y = 0,01} = 3293,$

$\Delta Y\Big|_{\Delta md_Y = 0,01} = -7353.$

In relation to the above indicators, it is noted that in the case of IS variables, the largest GDP's growth is due to the rate of investments, net exports and marginal government consumption. A similar increase can be achieved also by an increasing in the marginal propensity to consume.

7. Conclusions

The model presented above shows a more flexibility in macroeconomic modeling, because it removes the common assumptions of constancy of variables. Thus, net exports, government consumption and transfers are approached by their econometric dependence of GDP. After the analysis of static equilibrium there are obtained the value of potential GDP and the interest rate.

The dynamic analysis revealed three cases of economic development in which both GDP and interest rates converge to limit values, clearly identical with those in the static equilibrium. The three cases which are dependent on statistical parameters, push faster or slower the economy to the equilibrium. From predicted equilibrium values, we have defined the potential GDP, based on which we determined (with Okun's law) the natural rate of unemployment.

Romania's situation, presented in the case study, reveals a contradictory economic policy. Thus, although econometric indicators leading to optimal convergence ($\Delta<0$) of GDP to the potential, this is due to compensation data period. In fact, in 2003-2006 and 2008, the Romanian economy was overheated, with an overemployment of labor and a positive output gap. In the period of economic crisis, the unemployment has returned to a relatively normal situation, in turn the interest rate has increased unjustified (2008,2009) led to discouraging investments. Recent years (2010, 2011) approached the interest rate from equilibrium, which was reflected in an dynamic increased of investments. For Romania, the analysis of marginal indicators proposes as directions for growth, the increase of investments, net exports, government consumption marginal, but also the marginal propensity to consume (conditioned by the recovery of the trade balance which record a deficit and to stimulate the domestic production).

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Appendix A.1

A result on the stability of solutions of a system of differential equations of first order, linear, with constant coefficients satisfying some conditions

Lemma

Let the system of differential equations:

$$\begin{pmatrix} \frac{dX}{dt} \\ \frac{dY}{dt} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}, \ a,b,c,d,e,f\in\mathbf{R},\ a,b,d<0,\ c,e,f>0,\ X(0)=X_0,\ Y(0)=Y_0.$$

Then $\lim_{t\to\infty} X(t) = \tilde{X}$, $\lim_{t\to\infty} Y(t) = \tilde{Y}$, $\tilde{X}, \tilde{Y} \in \mathbf{R}$ if and only if:

1. $\Delta=(a-d)^2+4bc=0$ with the solution:

$$\begin{cases} X = \left(\frac{a-d}{2}X_0 + bY_0 + \frac{2bf + e(a-d)}{a+d}\right)te^{\frac{a+d}{2}t} + \left(X_0 + 4\frac{de-bf}{(a+d)^2}\right)e^{\frac{a+d}{2}t} + 4\frac{bf-de}{(a+d)^2} \\ Y = \left[Y_0 + 4\frac{af-ce}{(a+d)^2}\right]e^{\frac{a+d}{2}t} - \left(\frac{a-d}{2}X_0 + bY_0 + \frac{2bf + e(a-d)}{a+d}\right)\frac{a-d}{2b}te^{\frac{a+d}{2}t} + 4\frac{ce-af}{(a+d)^2} \end{cases}$$

2. $\Delta=(a-d)^2+4bc>0$ and $\lambda_1\neq\lambda_2$ are roots of the equation: $\lambda^2-(a+d)\lambda+(ad-bc)=0$: $e,f\in\mathbf{R}$ with the solution:

$$\begin{cases} X = k_1e^{\lambda_1 t} + k_2e^{\lambda_2 t} - \frac{de-bf}{ad-bc} \\ Y = \frac{\lambda_1 - a}{b}k_1e^{\lambda_1 t} + \frac{\lambda_2 - a}{b}k_2e^{\lambda_2 t} + \frac{ce-af}{ad-bc} \end{cases}$$

where:

$$k_1 = \frac{(\lambda_2 - a)X_0 + (\lambda_2 - a)\frac{de-bf}{ad-bc} - bY_0 + b\frac{ce-af}{ad-bc}}{\lambda_2 - \lambda_1}$$

$$k_2 = -\frac{(\lambda_1 - a)X_0 + (\lambda_1 - a)\frac{de-bf}{ad-bc} - bY_0 + b\frac{ce-af}{ad-bc}}{\lambda_2 - \lambda_1}$$

3. $\Delta=(a-d)^2+4bc<0$ and $\lambda_1=\alpha+i\beta$, $\lambda_2=\alpha-i\beta$, $\beta\neq0$ are the roots of the equation: $\lambda^2-(a+d)\lambda+(ad-bc)=0$: $e,f\in\mathbf{R}$ with the solution:

$$\begin{cases} X = \left(X_0 + \dfrac{de - bf}{ad - bc}\right)e^{\alpha t}\cos\beta t + \dfrac{1}{\beta}\left(bY_0 - \dfrac{d-a}{2}X_0 + \dfrac{(d-a)(bf-de)+2b(af-ce)}{2(ad-bc)}\right)e^{\alpha t}\sin\beta t + \dfrac{bf-de}{ad-bc} \\ Y = \left(Y_0 + \dfrac{af - ce}{ad - bc}\right)e^{\alpha t}\cos\beta t + \dfrac{1}{\beta}\left(cX_0 + \dfrac{d-a}{2}Y_0 + \dfrac{(d-a)(af-ce)+2c(de-bf)}{2(ad-bc)}\right)e^{\alpha t}\sin\beta t + \dfrac{ec-af}{ad-bc} \end{cases}$$

Appendix A.2

The linear regressions

Regression	**$C=c_VV+C_0$**	**$G=g_YY$**	**$I=in_YY+i_rr$**	**$NX=\nu_YY$**
Multiple R	0.943514561	0.993477251	0.993921138	0.937078818
R Square	0.890219727	0.986997047	0.987879229	0.878116712
Significance F	1.30505E-05	5.31E-10	1.35E-08	1.37507E-05
Intercept	18527.39699	-	-	-
*t Stat (*Intercept)	2.395137666	-	-	-
*P-value (*Intercept)	0.04021722	-	-	-
*Lower 95% (*Intercept)	1028.66011	-	-	-
*Upper 95% (*Intercept)	36026.13387	-	-	-
X Variable 1	0.595262357	0.077030115	0.280770328	-0.088582275
*t Stat (*X Variable 1)	8.542946909	27.5509728	19.29838414	-8.487973999
*P-value (*X Variable 1)	1.30505E-05	9.20E-11	1.2438E-08	6.98E-06
*Lower 95% (*X Variable 1)	0.437637942	0.070800433	0.25722237*	-0.111835598
*Upper 95% (*X Variable 1)	0.752886771	0.083259797	0.30431829*	-0.065328951
X Variable 2	-	-	-79168.78775	-
*t Stat (*X Variable 2)	-	-	-1.621662639	-
*P-value (*X Variable 2)	-	-	0.139325564	-
*Lower 95% (*X Variable 2)	-	-	-158185.0528*	-
*Upper 95% (*X Variable 2)	-	-	-152.52267*	-

* Lower 86.0%, Upper 86.0%

Regression	**$TR=\theta_YY$**	**$TI=ri_YY+T_0$**	**$MD=md_YY+m_rr$**
Multiple R	0.982295421	0.78282238	0.985492081
R Square	0.964904293	0.612810878	0.971194641
Significance F	4.71E-08	0.004388116	4.35363E-07
Intercept	-	5117.374767	-
*t Stat (*Intercept)	-	2.478080246	-
*P-value (*Intercept)	-	0.035101828	-
Lower 95%	-	445.8932831	-

(Intercept)			
Upper 95% (Intercept)	-	9788.856252	-
X Variable 1	0.097273692	0.069049932	0.088499489
t Stat (X Variable 1)	16.58116813	3.774182655	13.53248535
P-value (X Variable 1)	1.33E-08	0.004388116	2.7492E-07
Lower 95% (X Variable 1)	0.08420228	0.027663011	0.073705477
Upper 95% (X Variable 1)	0.11034511	0.110436852	0.10329351
X Variable 2	-	-	-59560.45339
t Stat (X Variable 2)	-	-	-2.714135812
P-value (X Variable 2)	-	-	0.023835548
Lower 95% (X Variable 2)	-	-	-109202.4447
Upper 95% (X Variable 2)	-	-	-9918.46212

Table A.1

Year	Actual final consumption of households (mil. lei 2000) - C -	Available income (mil. lei 2000) - V -	Actual final consumption of the government (mil. lei 2000) - G -	GDP (mil. lei 2000) - Y -	Investments (mil. lei 2000) - I -	Real interest rate (without inflation) - r -	Net export (mil. lei 2000) - NX -	Government transfers (mil. lei 2000) - TR -	Tax rates (mil. lei 2000) - TI -	Money demand –daily average (mil. lei 2000) - MD -
2001	67.086.8	85.192.1	6.225.9	85.841.2	19.058.37	0.0320	-6.529.9	10.038.8	10.687.9	4.162.7
2002	68.944.0	88.712.9	6.029.5	89.658.3	19.726.21	0.0487	-5.041.5	9.591.3	10.536.6	4.306.9
2003	71.058.3	91.729.8	9.238.3	93.904.0	20.628.51	0.0307	-7.021.0	9.804.4	11.978.7	4.435.3
2004	79.203.6	98.023.5	8.088.2	102.310.9	24.216.75	0.0748	-9.197.6	8.992.0	13.279.4	5.130.1
2005	83.577.3	103.294.1	8.879.4	106.421.3	24.781.49	0.0054	-10.816.9	10.326.8	13.454.0	6.387.1
2006	89.229.5	110.726.8	8.784.1	114.561.3	30.310.85	0.0182	-13.763.1	8.763.4	12.598.0	8.333.6
2007	92.137.1	117.981.3	9.328.8	122.371.7	37.904.81	0.0250	-16.999.1	8.795.0	13.185.4	10.358.3
2008	100.453.4	131.734.0	10.493.1	135.665.9	42.409.59	0.0149	-17.690.2	10.600.4	14.532.2	12.328.1
2009	89.197.8	125.597.9	10.858.5	124.029.6	31.465.90	0.0354	-7.492.5	13.485.9	11.917.6	11.335.3
2010	91.374.4	124.220.9	8.926.7	124.837.2	30.999.47	0.0055	-6.463.3	12.834.6	13.450.9	11.148.6
2011	90.396.4	130.417.6	8.118.7	129.050.6	37.178.33	0.0043	-6.642.9	16.875.8	15.508.7	11.603.0

Source: The Statistical Yearbook of Romania

Table A.2. The relative variation of GDP and the absolute variation of the unemployment rate during 2001-2011

	Relative variation of GDP (Y)	Absolute variation of the unemployment rate (u)
2001	5.7	-2.2
2002	5.1	1.2
2003	5.2	-2.6
2004	8.5	-0.8
2005	4.2	-1
2006	7.9	-0.4
2007	6.3	-1.1
2008	7.3	-0.3
2009	-6.6	2.3
2010	-1.6	1.3
2011	2.5	-2.2

Source: The Statistical Yearbook of Romania

Table A.3. The unemployment rate during 2000-2011

	Unemployment rate (u)
2001	8.60%
2002	8.10%
2003	7.20%
2004	6.20%
2005	5.90%
2006	5.20%
2007	4.10%
2008	4.40%
2009	7.80%
2010	6.87%
2011	5.12%

Source: The Statistical Yearbook of Romania