Macroeconomics and Monetary Economics

The Evolution of GDP in USA Using Polynomial Regression Analysis

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Abstract: The paper deals with the problem of statistical forecasts in terms of polynomial regression. Thus, it compares actual results with predicted variables using data sets sequentially go through all the set initially.

Keywords: polynomial regression; standard deviation; GDP

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1. Introduction

Consider, in the following, two sets of data $X=(x_i)_{i\in I}$ and $Y=(y_i)_{i\in I}$ where $I=\{1,...,n\}$. To make a choice, we assume that X is the exogenous variable and Y - endogenous. In addition, X is non-constant. We shall seek a polynomial regression function $f(x)=a_mx^m+...+a_1x+a_0$, $x\in\mathbf{R}$ with unknown coefficients a_i, $i=0,...,m$ to be determined from the condition that:

$$\sum_{i=1}^{n}(a_m x_i^m+...+a_1x_i+a_0-y_i)^2 = \text{minimum}$$

Let $F(a_m,...,a_0)=\sum_{i=1}^{n}(a_m x_i^m+...+a_1x_i+a_0-y_i)^2$.

The minimum necessary condition is: $\frac{\partial F}{\partial a_k}=0$, $k=\overline{0,m}$.

We have then:

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$$2\sum_{i=1}^{n} x_i^k (a_m x_i^m + ... + a_1 x_i + a_0 - y_i) = 0,\ k=\overline{0,m}$$

from where:

$$\mathbf{a_m}\sum_{i=1}^{n} x_i^{m+k} \mathbf{+...+a_1}\sum_{i=1}^{n} x_i^{1+k} \mathbf{+a_0}\sum_{i=1}^{n} x_i^{k} \mathbf{=} \sum_{i=1}^{n} y_i x_i^k\mathbf{,\ k=}\overline{0,m}$$

The resulting system has the solutions:

$$a_k = \frac{\overset{\text{col.}m-k+1}{\begin{vmatrix} \sum_{i=1}^{n} x_i^m & \sum_{i=1}^{n} x_i^{m-1} & \cdots & \sum_{i=1}^{n} y_i & \cdots & n \\ \sum_{i=1}^{n} x_i^{m+1} & \sum_{i=1}^{n} x_i^{m} & \cdots & \sum_{i=1}^{n} x_i y_i & \cdots & \sum_{i=1}^{n} x_i \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \sum_{i=1}^{n} x_i^{2m} & \sum_{i=1}^{n} x_i^{2m-1} & \cdots & \sum_{i=1}^{n} x_i^m y_i & \cdots & \sum_{i=1}^{n} x_i^m \end{vmatrix}}}{\begin{vmatrix} \sum_{i=1}^{n} x_i^m & \sum_{i=1}^{n} x_i^{m-1} & \cdots & \sum_{i=1}^{n} x_i^k & \cdots & n \\ \sum_{i=1}^{n} x_i^{m+1} & \sum_{i=1}^{n} x_i^{m} & \cdots & \sum_{i=1}^{n} x_i^{k+1} & \cdots & \sum_{i=1}^{n} x_i \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \sum_{i=1}^{n} x_i^{2m} & \sum_{i=1}^{n} x_i^{2m-1} & \cdots & \sum_{i=1}^{n} x_i^{k+m} & \cdots & \sum_{i=1}^{n} x_i^m \end{vmatrix}},\ k=\overline{0,m}$$

2. The Analysis

Let consider now, the values GDP_k, $k=\overline{1,m}$ corresponding to a period of m consecutive years and the growth rate: $r_k=\frac{GDP_k - GDP_{k-1}}{GDP_{k-1}}$, $k=\overline{2,m}$.

The cyclical analysis (through the theory above) of U.S. GDP for the period 1792-2012 taking into account the growth rate does not provide acceptable results, especially in terms of the current period. For this reason, we shall consider the absolute variation of it: $v_k=\Delta r_k=r_k-r_{k-1}$, $k=\overline{3,m}$. This indicator provides more direct information on the phenomenon of crisis, meaning that $v_k<0$ indicates a decrease in the growth rate, while $v_k>0$ is equivalent to leave the crisis phenomenon.

For our analysis, we consider therefore value pairs: (k,v_k), $\overline{3,m}$ for the period 1792-2012.

Table 1. The Evolution of GDP, Growth Rate and absolute change in Growth Rate for U.S. Economy During 1792-2012

Year	**GDP$_k$**	**r$_k$**	**$v_k=\Delta r_k$**	**Year**	**GDP$_k$**	**r$_k$**	**$v_k=\Delta r_k$**
1792	4.58	0	0	1903	481.8	0.029	-0.0224
1793	4.95	0.0808	0.0808	1904	464.8	-0.0353	-0.0643
1794	5.6	0.1313	0.0505	1905	517.2	0.1127	0.148
1795	5.96	0.0643	-0.067	1906	538.4	0.041	-0.0717
1796	6.15	0.0319	-0.0324	1907	552.2	0.0256	-0.0154
1797	6.27	0.0195	-0.0124	1908	492.5	-0.1081	-0.1337
1798	6.54	0.0431	0.0236	1909	528.1	0.0723	0.1804
1799	7	0.0703	0.0272	1910	533.8	0.0108	-0.0615
1800	7.4	0.0571	-0.0132	1911	551.1	0.0324	0.0216
1801	7.76	0.0486	-0.0085	1912	576.9	0.0468	0.0144
1802	8	0.0309	-0.0177	1913	599.7	0.0395	-0.0073
1803	8.14	0.0175	-0.0134	1914	553.7	-0.0767	-0.1162
1804	8.45	0.0381	0.0206	1915	568.8	0.0273	0.104
1805	8.9	0.0533	0.0152	1916	647.7	0.1387	0.1114
1806	9.32	0.0472	-0.0061	1917	631.7	-0.0247	-0.1634
1807	9.33	0.0011	-0.0461	1918	688.7	0.0902	0.1149
1808	9.35	0.0021	0.001	1919	694.2	0.008	-0.0822
1809	10.07	0.077	0.0749	1920	687.7	-0.0094	-0.0174
1810	10.63	0.0556	-0.0214	1921	671.9	-0.023	-0.0136
1811	11.11	0.0452	-0.0104	1922	709.3	0.0557	0.0787
1812	11.55	0.0396	-0.0056	1923	802.6	0.1315	0.0758
1813	12.21	0.0571	0.0175	1924	827.4	0.0309	-0.1006
1814	12.72	0.0418	-0.0153	1925	846.8	0.0234	-0.0075
1815	12.82	0.0079	-0.0339	1926	902.1	0.0653	0.0419
1816	12.82	0	-0.0079	1927	910.8	0.0096	-0.0557
1817	13.12	0.0234	0.0234	1928	921.3	0.0115	0.0019
1818	13.6	0.0366	0.0132	1929	977	0.0605	0.049
1819	13.86	0.0191	-0.0175	1930	892.8	-0.0862	-0.1467
1820	14.41	0.0397	0.0206	1931	834.9	-0.0649	0.0213
1821	15.18	0.0534	0.0137	1932	725.8	-0.1307	-0.0658
1822	15.76	0.0382	-0.0152	1933	716.4	-0.013	0.1177
1823	16.33	0.0362	-0.002	1934	794.4	0.1089	0.1219

1824	17.3	0.0594	0.0232	1935	865	0.0889	-0.02
1825	18.07	0.0445	-0.0149	1936	977.9	0.1305	0.0416
1826	18.71	0.0354	-0.0091	1937	1028	0.0512	-0.0793
1827	19.29	0.031	-0.0044	1938	992.6	-0.0344	-0.0856
1828	19.55	0.0135	-0.0175	1939	1072.8	0.0808	0.1152
1829	20.3	0.0384	0.0249	1940	1166.9	0.0877	0.0069
1830	22.16	0.0916	0.0532	1941	1366.1	0.1707	0.083
1831	23.99	0.0826	-0.009	1942	1618.2	0.1845	0.0138
1832	25.61	0.0675	-0.0151	1943	1883.1	0.1637	-0.0208
1833	26.4	0.0308	-0.0367	1944	2035.2	0.0808	-0.0829
1834	26.85	0.017	-0.0138	1945	2012.4	-0.0112	-0.092
1835	28.27	0.0529	0.0359	1946	1792.2	-0.1094	-0.0982
1836	29.11	0.0297	-0.0232	1947	1776.1	-0.009	0.1004
1837	29.37	0.0089	-0.0208	1948	1854.2	0.044	0.053
1838	30.59	0.0415	0.0326	1949	1844.7	-0.0051	-0.0491
1839	31.37	0.0255	-0.016	1950	2006	0.0874	0.0925
1840	31.46	0.0029	-0.0226	1951	2161.1	0.0773	-0.0101
1841	32.17	0.0226	0.0197	1952	2243.9	0.0383	-0.039
1842	33.19	0.0317	0.0091	1953	2347.2	0.046	0.0077
1843	34.84	0.0497	0.018	1954	2332.4	-0.0063	-0.0523
1844	36.82	0.0568	0.0071	1955	2500.3	0.072	0.0783
1845	39.15	0.0633	0.0065	1956	2549.7	0.0198	-0.0522
1846	42.33	0.0812	0.0179	1957	2601.1	0.0202	0.0004
1847	45.21	0.068	-0.0132	1958	2577.6	-0.009	-0.0292
1848	46.73	0.0336	-0.0344	1959	2762.5	0.0717	0.0807
1849	47.38	0.0139	-0.0197	1960	2830.9	0.0248	-0.0469
1850	49.59	0.0466	0.0327	1961	2896.9	0.0233	-0.0015
1851	53.58	0.0805	0.0339	1962	3072.4	0.0606	0.0373
1852	59.76	0.1153	0.0348	1963	3206.7	0.0437	-0.0169
1853	64.65	0.0818	-0.0335	1964	3392.3	0.0579	0.0142
1854	66.88	0.0345	-0.0473	1965	3610.1	0.0642	0.0063
1855	69.67	0.0417	0.0072	1966	3845.3	0.0652	0.001
1856	72.47	0.0402	-0.0015	1967	3942.5	0.0253	-0.0399
1857	72.84	0.0051	-0.0351	1968	4133.4	0.0484	0.0231

1858	75.79	0.0405	0.0354	1969	4261.8	0.0311	-0.0173
1859	81.28	0.0724	0.0319	1970	4269.9	0.0019	-0.0292
1860	82.11	0.0102	-0.0622	1971	4413.3	0.0336	0.0317
1861	83.57	0.0178	0.0076	1972	4647.7	0.0531	0.0195
1862	93.95	0.1242	0.1064	1973	4917	0.0579	0.0048
1863	101.18	0.077	-0.0472	1974	4889.9	-0.0055	-0.0634
1864	102.33	0.0114	-0.0656	1975	4879.5	-0.0021	0.0034
1865	105.26	0.0286	0.0172	1976	5141.3	0.0537	0.0558
1866	100.43	-0.0459	-0.0745	1977	5377.7	0.046	-0.0077
1867	102.15	0.0171	0.063	1978	5677.6	0.0558	0.0098
1868	106.13	0.039	0.0219	1979	5855	0.0312	-0.0246
1869	109.02	0.0272	-0.0118	1980	5839	-0.0027	-0.0339
1870	112.3	0.0301	0.0029	1981	5987.2	0.0254	0.0281
1871	117.6	0.0472	0.0171	1982	5870.9	-0.0194	-0.0448
1872	127.5	0.0842	0.037	1983	6136.2	0.0452	0.0646
1873	138.3	0.0847	0.0005	1984	6577.1	0.0719	0.0267
1874	140.8	0.0181	-0.0666	1985	6849.3	0.0414	-0.0305
1875	140.6	-0.0014	-0.0195	1986	7086.5	0.0346	-0.0068
1876	146.4	0.0413	0.0427	1987	7313.3	0.032	-0.0026
1877	153.7	0.0499	0.0086	1988	7613.9	0.0411	0.0091
1878	158.6	0.0319	-0.018	1989	7885.9	0.0357	-0.0054
1879	177.1	0.1166	0.0847	1990	8033.9	0.0188	-0.0169
1880	191.8	0.083	-0.0336	1991	8015.1	-0.0023	-0.0211
1881	215.8	0.1251	0.0421	1992	8287.1	0.0339	0.0362
1882	227.3	0.0533	-0.0718	1993	8523.4	0.0285	-0.0054
1883	233.5	0.0273	-0.026	1994	8870.7	0.0407	0.0122
1884	229.7	-0.0163	-0.0436	1995	9093.7	0.0251	-0.0156
1885	230.5	0.0035	0.0198	1996	9433.9	0.0374	0.0123
1886	249.2	0.0811	0.0776	1997	9854.3	0.0446	0.0072
1887	267.3	0.0726	-0.0085	1998	10283.5	0.0436	-0.001
1888	282.7	0.0576	-0.015	1999	10779.8	0.0483	0.0047
1889	290.8	0.0287	-0.0289	2000	11226	0.0414	-0.0069
1890	319.1	0.0973	0.0686	2001	11347.2	0.0108	-0.0306
1891	322.8	0.0116	-0.0857	2002	11553	0.0181	0.0073

1892	339.3	0.0511	0.0395	2003	11840.7	0.0249	0.0068
1893	319.6	-0.0581	-0.1092	2004	12263.8	0.0357	0.0108
1894	304.5	-0.0472	0.0109	2005	12638.4	0.0305	-0.0052
1895	339.2	0.114	0.1612	2006	12976.2	0.0267	-0.0038
1896	333.6	-0.0165	-0.1305	2007	13228.9	0.0195	-0.0072
1897	348	0.0432	0.0597	2008	13161.9	-0.0051	-0.0246
1898	386.1	0.1095	0.0663	2009	12703.1	-0.0349	-0.0298
1899	412.5	0.0684	-0.0411	2010	12615	-0.0069	0.028
1900	422.8	0.025	-0.0434	2011	12982	0.0291	0.036
1901	445.3	0.0532	0.0282	2012	13351	0.0284	-0.0007
1902	468.2	0.0514	-0.0018				

* GDP-billion $-2005 US

Source: http://www.usgovernmentrevenue.com

The analysis that we perform, try to determine the best polynomial regression in forecasting purposes. Thus, for the whole data set presented, we consider, in turn, polynomial regressions of degrees 1,2 etc. determined by a variable number of data. Thus, for example, in the case of regression of order 1 (linear), we consider the first two data from the above set (v_k values in Table 1), we perform regression and we predicted for the third. We then determine regression for data 2 and 3 and we predicted for the fourth. Proceeding similarly, until finally, we then determine the standard deviation between data values (v_k) and predicted (z_k): $\sigma = \sqrt{\frac{\sum_{k=1}^{m}(z_k - v_k)^2}{m}}$ (m being the number of pairs of values).

We then proceed to determine the linear regression using sets of three, four, etc. values. Finally, we hold that type of linear regression, dependent on a specified number of previous values which lead to the smallest square deviation.

We proceed analogously with the regressions of degrees 2,3 etc.

Finally, we select the type of regression (its order) that leads to the smallest square deviation.

A problem that needs to be solved is about the exogenous variable. Because the regressions are time dependent (the year), at the high degree of regression polynomials, the prediction introduces enormous errors due to the floating point data representation in computers. For example, $2013^8 \approx 2{,}7 \cdot 10^{26}$ leading to the wrong final results (in terms of square deviation).

For this reason, we proceed as follows: if for determining a regression they are need n exogenous variable values, we shall note as follows: $-\left[\frac{n}{2}\right], -\left[\frac{n}{2}\right]+1, ..., -1, 0, 1, ..., \left[\frac{n}{2}\right]-1$ where [a] is the integer part of a number (the largest integer less than or equal to a). Prediction will be realized for x=$\left[\frac{n}{2}\right]$.

The computer analysis performed to regression of degree 7 (for higher degrees, the errors were, even with the above simplification, significant) based on a number of consecutive data between the polynomial degree and 90 revealed the following results:

Table 2

Degree	**The optimal number of values considered for the regression**	**The mean square deviation**	**The maximum absolute deviation**
1	21	0.054472841322422	0.18900
2	23	0.0557802505791187	0.20034
3	39	0.0569753758609295	0.17026
4	**76**	**0.0591226518154144**	**0.16713**
5	89	0.0661482363939285	0.19579
6	78	0.0701805725693703	0.23218
7	14	0.103008173406124	0.40049

From the data presented in Table 2, we see that with increasing the degree of polynomial regression, the mean square deviation increases. On the other hand, for a correct prediction is self-evident that at close values of the mean square deviation will be the preferred that method which will record the lowest absolute deviation (absolute difference between actual and predicted data).

From the data of Table 2, we retain therefore the polynomial regression of degree 4 which will record a maximum absolute deviation of 16.713%.

The obtained data are as follows:

Table 3

Year	v_k	Value after regression	Absolute error	Year	v_k	Value after regression	Absolute error
1868	0.021835954	-0.029389457	0.051225411	1941	0.082994323	-0.001472498	0.084466821
1869	-0.011731556	-0.039176642	0.027445087	1942	0.013831216	-0.001969281	0.015800497
1870	0.002855468	-0.00296291	0.005818378	1943	-0.020839523	-0.032963051	0.012123527
1871	0.017108791	0.024987801	0.00787901	1944	-0.082929339	-0.000478246	0.082451093
1872	0.03698866	-0.000745548	0.037734208	1945	-0.091973899	0.023027589	0.115001488
1873	0.000522209	-0.017559978	0.018082187	1946	-0.098218756	0.026328998	0.124547754
1874	-0.066629237	-0.020969539	0.045659698	1947	0.100438214	0.029595718	0.070842496
1875	-0.0194971	0.000108244	0.019605344	1948	0.052956122	0.008928651	0.044027471
1876	0.042672233	0.014805807	0.027866425	1949	-0.049096253	0.014692162	0.063788415
1877	0.00861161	0.003725049	0.004886561	1950	0.092563195	0.023861857	0.068701339
1878	-0.017983102	0.001456578	0.019439679	1951	-0.010121646	-0.021436777	0.011315131
1879	0.084765363	-0.00230045	0.087065813	1952	-0.039004224	-0.031089061	0.007915163
1880	-0.033641697	-0.020288131	0.013353565	1953	0.007722098	-0.009820952	0.01754305
1881	0.042126392	-0.002065507	0.044191899	1954	-0.052341305	-0.00984206	0.042499245
1882	-0.071840261	0.003124222	0.074964483	1955	0.078291322	-0.014006704	0.092298026
1883	-0.026013357	0.016498131	0.042511487	1956	-0.052228308	0.011129234	0.063357542
1884	-0.043550817	-0.002295812	0.041255005	1957	0.000401605	0.009680883	0.009279277
1885	0.019756894	-0.002628916	0.02238581	1958	-0.029193874	0.03253312	0.061726994
1886	0.077645179	0.023009577	0.054635602	1959	0.080768035	0.008049067	0.072718968
1887	-0.008495559	0.003923716	0.012419275	1960	-0.046973214	-0.016925268	0.030047946
1888	-0.015019255	0.0048068	0.019826055	1961	-0.001446041	-0.031496375	0.030050334
1889	-0.028960887	0.006954371	0.035915258	1962	0.037267861	-0.029118989	0.06638685
1890	0.068665463	0.020584739	0.048080723	1963	-0.016870245	-0.000667397	0.016202848
1891	-0.085722633	0.005498997	0.09122163	1964	0.01416706	0.003437747	0.010729313
1892	0.03952013	0.003881525	0.035638605	1965	0.006325411	-0.001466444	0.007791855
1893	-0.109175955	-0.011065684	0.098110271	1966	0.000946323	-0.01278169	0.013728012
1894	0.010814155	0.010209944	0.000604211	1967	-0.039872938	0.022627612	0.06250055
1895	0.161203865	0.006066574	0.155137291	1968	0.023143441	-0.005876652	0.029020093
1896	-0.130466741	-0.029273567	0.101193174	1969	-0.017357038	0.007767687	0.025124724
1897	0.059674902	0.001058407	0.058616495	1970	-0.02916341	-0.041607971	0.012444562
1898	0.066317291	-0.003537867	0.069855158	1971	0.031683319	-	0.075552289

						0.043868969	
1899	-0.04110669	-0.017185586	0.023921104	1972	0.019528259	0.020059873	0.000531614
1900	-0.043406371	-0.007911641	0.035494731	1973	0.004830455	-0.039194412	0.044024867
1901	0.028246954	0.010486345	0.017760609	1974	-0.063454129	-0.014985302	0.048468827
1902	-0.001790646	0.000250008	0.002040654	1975	0.003384658	0.028272521	0.024887863
1903	-0.022378589	-0.002813195	0.019565394	1976	0.055779871	0.013083313	0.042696558
1904	-0.064331766	-0.002285833	0.062045933	1977	-0.00767245	-0.01328864	0.005616191
1905	0.148021011	-0.005328688	0.153349699	1978	0.009786747	0.003693754	0.006092993
1906	-0.071746715	-0.020583042	0.051163673	1979	-0.024521739	0.006225027	0.030746766
1907	-0.015358445	0.019853508	0.035211953	1980	-0.033978304	0.003485774	0.037464078
1908	-0.133744503	0.022787926	0.156532429	1981	0.028113765	-0.023459044	0.05157281
1909	0.180397266	0.036114604	0.144282662	1982	-0.044805831	0.038136282	0.082942113
1910	-0.061490854	-0.011537719	0.049953135	1983	0.064613756	0.018697466	0.04591629
1911	0.021615732	-0.007513813	0.029129544	1984	0.026663303	0.002912055	0.023751248
1912	0.014406318	0.004431493	0.009974825	1985	-0.030466265	-0.064282533	0.033816267
1913	-0.007293879	-0.006478486	0.000815393	1986	-0.006754745	0.018444128	0.025198873
1914	-0.1162266	-0.015247254	0.100979346	1987	-0.002626761	-0.004950432	0.002323671
1915	0.103976105	0.013123695	0.09085241	1988	0.00909868	0.00516744	0.00393124
1916	0.111441995	-0.015228237	0.126670232	1989	-0.005379059	0.012460079	0.017839139
1917	-0.163415875	-0.041695241	0.121720634	1990	-0.016956462	0.013164594	0.030121056
1918	0.1149355	-0.007647242	0.122582742	1991	-0.021107758	-0.040544853	0.019437095
1919	-0.082246645	-0.024593016	0.057653629	1992	0.03627603	0.001195181	0.035080849
1920	-0.017349357	-0.002922589	0.014426768	1993	-0.005421749	0.049745409	0.055167158
1921	-0.013611839	0.000801309	0.014413147	1994	0.012232454	-0.017772354	0.030004808
1922	0.07863818	0.003621436	0.075016743	1995	-0.01560771	0.032739086	0.048346796
1923	0.075875091	0.000886806	0.074988285	1996	0.012271574	0.002259354	0.010012221
1924	-0.10063856	-0.008812022	0.091826538	1997	0.007152179	-0.009919979	0.017072158
1925	-0.007452634	-0.003192149	0.004260485	1998	-0.001008104	-0.021209445	0.020201341
1926	0.041857734	-0.013999625	0.055857359	1999	0.004707188	0.012977006	0.008269818
1927	-0.055660513	-0.007870621	0.047789892	2000	-0.006869545	0.05392647	0.060796015
1928	0.001884163	0.017742509	0.015858346	2001	-0.030595868	0.017637887	0.048233755
1929	0.048929722	0.036676308	0.012253413	2002	0.007340267	0.019134697	0.011794429
1930	-0.146640239	0.020491786	0.167132025	2003	0.00676599	0.037436515	0.030670525

1931	0.02133004	0.022373108	0.001043068	2004	0.010830062	0.012660172	0.00183011
1932	-0.065822182	0.014613475	0.080435657	2005	-0.005187503	0.010279317	0.01546682
1933	0.117723106	0.016915213	0.100807893	2006	-0.003817115	0.034260352	0.038077467
1934	0.121828948	-0.024700048	0.146528996	2007	-0.007253953	-0.034776413	0.02752246
1935	-0.020005617	-0.026319802	0.006314185	2008	-0.024538783	-0.037417431	0.012878648
1936	0.041648126	0.000778406	0.040869721	2009	-0.02979352	-0.081155704	0.051362184
1937	-0.079287999	-0.025648493	0.053639506	2010	0.027922874	-0.043412036	0.07133491
1938	-0.08566803	-0.007070236	0.078597794	2011	0.036027665	-0.000290385	0.036318051
1939	0.115233702	0.055907721	0.059325981	2012	-0.000668379	-0.01257802	0.011909642
1940	0.006916488	0.023154258	0.01623777				

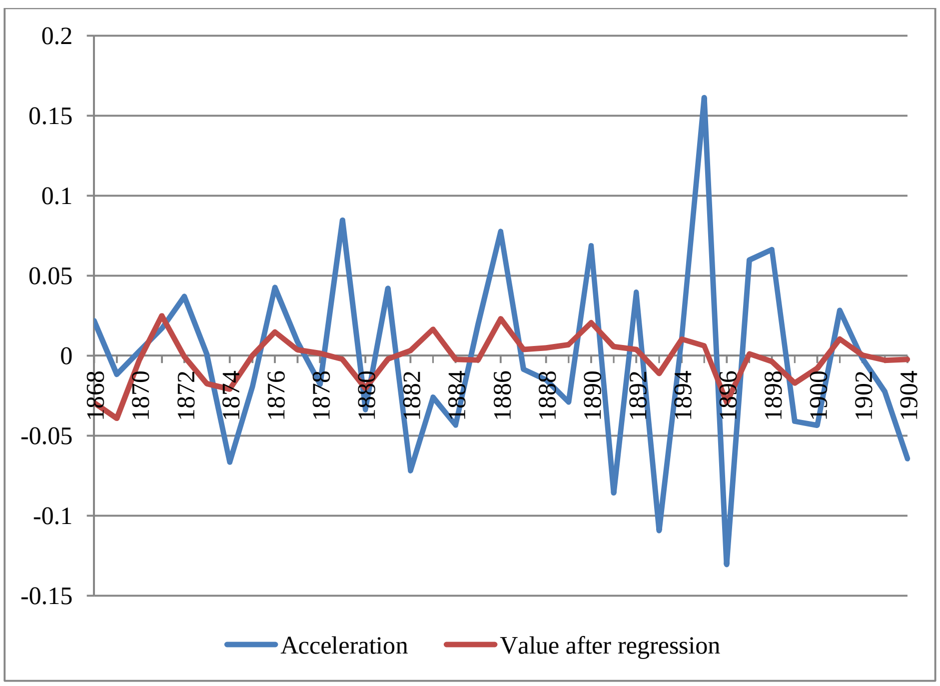

Figure 1. Forecasting the growth acceleration using polynomial regressions of degree 4 during 1868-1904

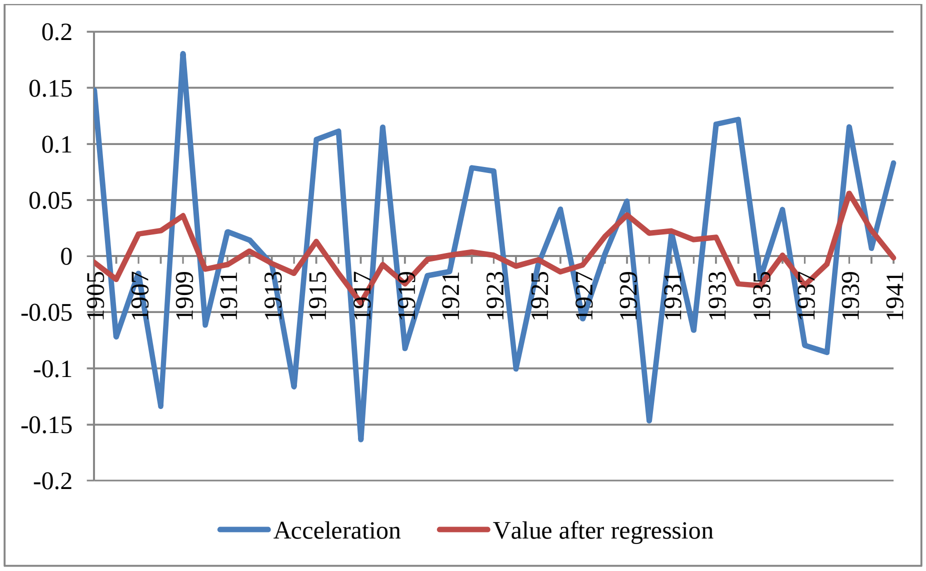

Figure 2. Forecasting the growth acceleration using polynomial regressions of degree 4 during 1905-1941

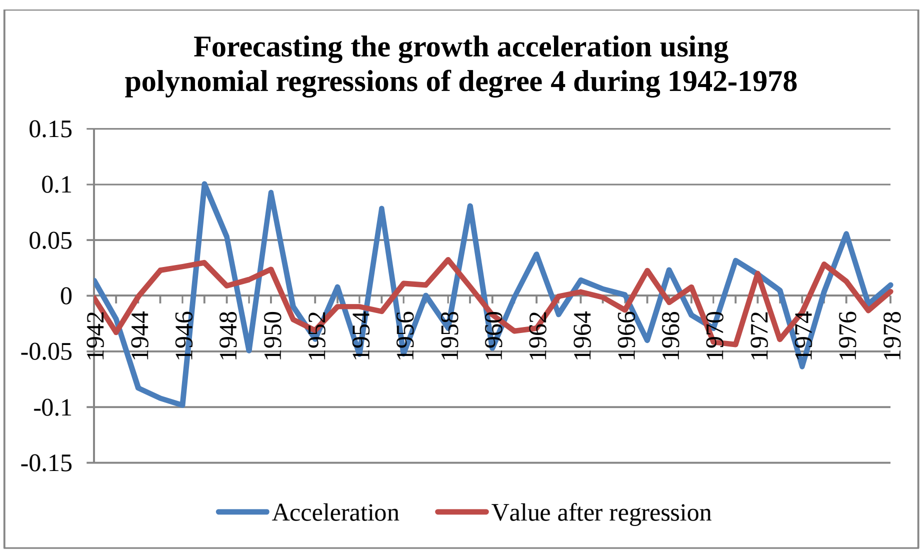

Figure 3. Forecasting the growth acceleration using polynomial regressions of degree 4 during 1942-1978

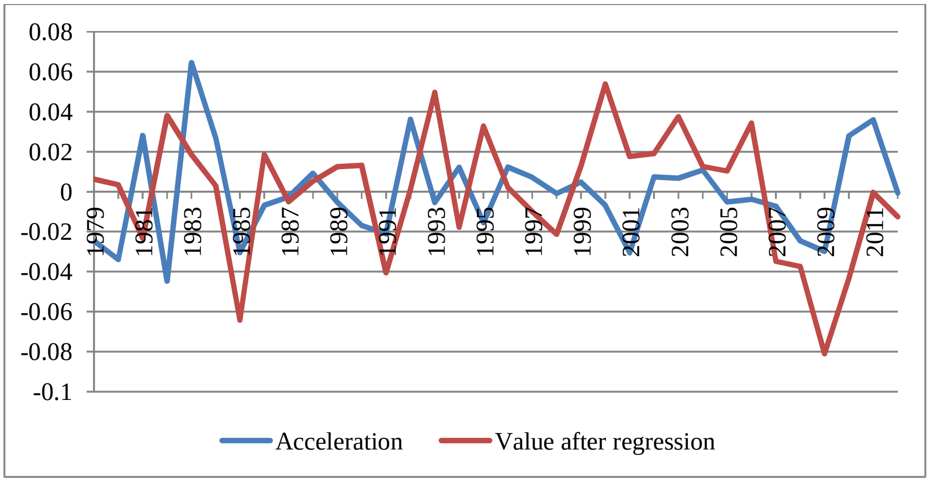

Figure 4. Forecasting the growth acceleration using polynomial regressions of degree 4 during 1979-2012

Naturally, we can put the question, of the apparent ineffectiveness of the method in the presence of such errors. On the other hand, the distribution of errors, have the following values:

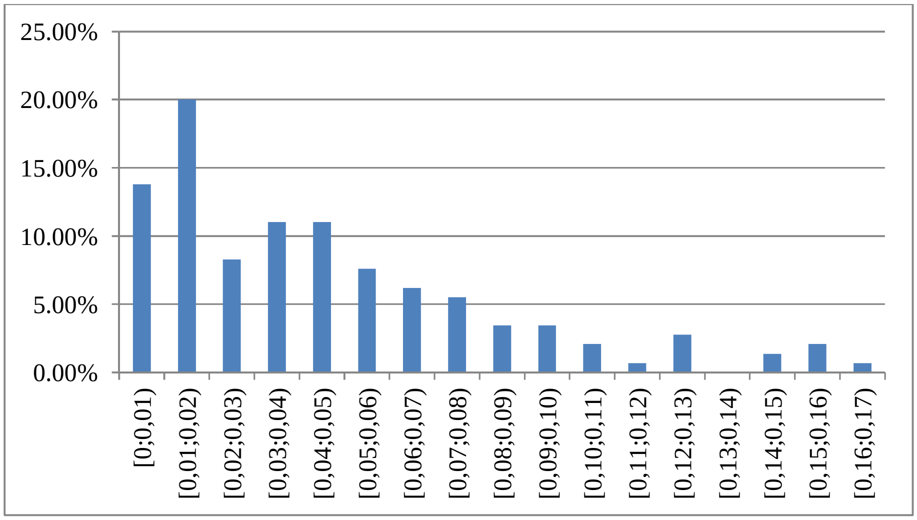

Figure 5. Distribution of Error Intervals in Total Forecasting

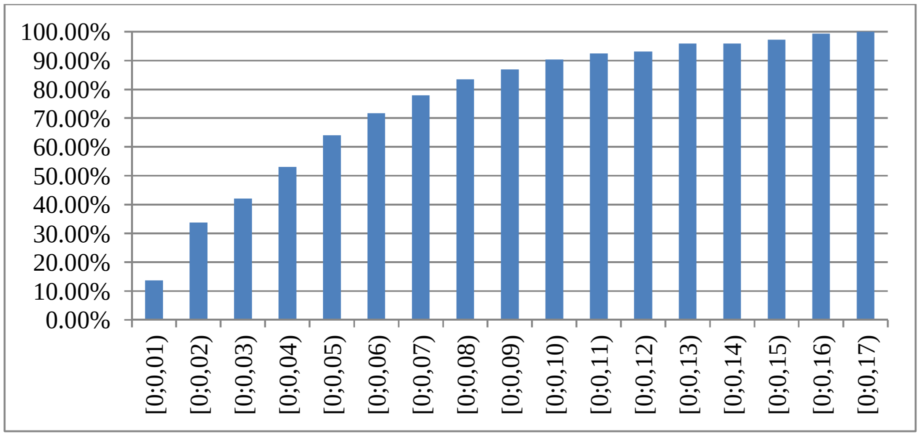

Figure 6. The Cumulative Distribution of Error Intervals in Total Forecasting

It is observed in Figures 5 and 6, that more than 50% (i.e. 53.1%) of the forecasts have absolute errors less than 4%, 33.79% having absolute error less than 2%. It also should be noted that the absolute error can be at low levels, but the direction of the prediction to be opposite (in the sense that an increase/ decrease in the real can be predicted to decrease/ increase). From Table 3 it follows, however, that 57.24% of the predictions are correct, which is somewhat reasonable. Based on this method, we intend to determine the expected value of the USA's GDP in 2013. Considering the data for the period 1937-2012 (76 consecutive records) it is obtained an estimate of v_k for 2013 of 0.002884. How $v_k=\Delta r_k$ we have:

$r_{2013}=r_{2012}+v_{2013}$=0.0284+0, 002884=0, 031284=3, 13%

Data from economywatch.com, evaluate an increase of 2.72% which confirms the correctness of our analysis.

3. Conclusions

The above analysis suggests a comparison between different types of polynomial regression, in the direction of the best determination, both in terms of least square and maximum absolute error between the actual and forecasted data.

4. References

Harrison M. & Waldron P. (2011). *Mathematics for Economics and Finance*. Routledge.

Ioan, C. A. (2011). *Econometrics*. Galati: Zigotto.

Simon, C. P. & Blume, L. E. (2010). *Mathematics for Economists*. W.W.Norton & Company.