Mathematical and Quantitative Methods

A Mathematical Model of an Open Economy with Applications in Romania

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Abstract: In this paper, we first study the static equilibrium of a a closed economy model in terms of dependence on national income and interest rate from the main factors namely the marginal propensity to consume, tax rate, investment rate and the rate of currency demand. In the second part, we study the dynamic equilibrium solutions in terms of stability. We thus obtain the variation functions of national income and interest rate variation and their limit values. Finally, we propose two scenarios of economic development of Romania.

Keywords: equilibrium; demand; income

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1. Introduction

In a previous paper, we proposed a model of economic equilibrium in an open economy. We will resume in the first part, the model equations to tie naturally, the case study of theoretical considerations.

The model presented below is a generalization of a closed economy model with government sector and money market (Stancu & Mihail, 2009) which the authors call M_3 - name that we still use when it references will be required.

Unlike the classical model, we consider net exports as the difference between exports and imports.

The essential differences compared to the M_3, which allow a more realistic analysis, are the following:

1. Government expenditures were proportional to the level of GDP (compared to the M_3 which are considered constant). In principle, we could consider a linear

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dependence of GDP, denoted by Y, but regression analysis could not justify the existence of a nonzero free term of the regression. It is clear from the mathematical calculus, that it arise naturally from the fact that, in the absence of potential GDP, government spending cannot exist.

2. The investments dependence is linear by GDP and interest rate without free term. Again, the difference to the M_3 is the renunciation of free term (because of the failure to check the null hypothesis), which, in economic terms, is that in the absence of output and monetary policy, investments are null.
3. Net exports were considered to be proportional to GDP, the absence of the constant term is due to inability to import or export in the absence of the output.
4. Government transfers were assumed to be proportional to GDP (compared to M_3 where are considered constant), again without free term because there cannot be an output without the necessary transfers.
5. The demand for money in the economy was regarded as linearly dependent on GDP and interest rate, with no free term. Again, the difference to the M_3 is the free term waiver, which in economic terms means that in the absence of output and monetary policy, money demand in the economy is null.

The first equation of the model is:

$$(1) D=C+G+I+NX$$

where:

- D – aggregate demand;
- C – actual final consumption of households;
- G – collective final consumption of general government;
- I – investments;
- NX – net exports

The second equation determines the relationship between consumption of households and disposable income:

$$(2) C=c_V V+C_0,\ C_0>0,\ c_Y \in (0,1)$$

where:

- c_V – marginal propensity to consumption, $c_V=\frac{dC}{dV} \in (0,1)$ and C_0 is autonomous consumption of households;

- V – disposable income

(3) $G=g_Y Y$, $g_Y \in (0,1)$

where:

- g_Y – marginal government consumption;
- Y – output

(4) $I=in_Y Y+i_r r$;

$in_Y \in (0,1)$, $i_r<0$

where:

- in_Y – investment rate, $in_Y \in (0,1)$;
- i_r – influence factor in the investments of the interest rate, $i_r<0$;
- r – interest rate

(5) $NX=\nu_Y Y$;

$\nu_Y \in (-1,1)$

where: ν_Y – marginal net exports

(6) $V=Y+TR-TI$;

$TR>0$

where:

- TR – government transfers;
- TI – taxes

(7) $TR=\theta_Y Y$;

$\theta_Y \in (0,1)$

where: θ_Y – marginal rate of government transfers

(8) $TI=ri_Y Y+T_0$, $ri_Y \in (0,1)$;

$T_0 \in \mathbf{R}$

where:

- ri_Y – tax rate, $ri_Y \in (0,1)$;
- T_0 – independent of income taxes (such as, for example, property taxes)

(9) $D=Y$ – the first equation of static equilibrium;

(10) $MD=md_Y Y+m_r r,\ md_Y>0,\ m_r<0$

where:

- MD – money demand in the economy;
- md_Y – rate of money demand in the economy;
- m_r – influence factor of the demand for money in relation to interest rate, $m_r<0$;

(11) $MD=M$ – the second static equilibrium equation

where: M – money supply.

(12) $\frac{dY}{dt}=\alpha(D-Y);$

$\alpha>0$ – the first dynamic equation;

(13) $\frac{dr}{dt}=\beta(MD-M)$

$\beta>0$ – the second dynamic equation

where:

- α - proportionality constant of the speed of variation of output relative to the gap between aggregate demand and GDP;
- β - proportionality constant of the speed of variation in interest rate in relation to the gap between demand and supply of money in the economy.

2. Static Equilibrium

Static equilibrium occurs when aggregate demand equals output (equation 9), and the supply and demand for money are also equal (equation 11).

From equations (1) – (8) follow:

(14) $D=C+G+I+NX=c_V V+C_0+g_Y Y+in_Y Y+i_r r+\nu_Y Y=c_V(Y+TR-TI)+C_0+g_Y Y+in_Y Y+i_r r+\nu_Y Y=$

$c_V(Y+\theta_Y Y-ri_Y Y-T_0)+C_0+g_Y Y+in_Y Y+i_r r+\nu_Y Y=$

$c_V Y+c_V\theta_Y Y-c_V ri_Y Y-c_V T_0+C_0+g_Y Y+in_Y Y+i_r r+\nu_Y Y=$

$Y(c_V+c_V\theta_Y-c_V ri_Y+g_Y+in_Y+\nu_Y)-c_V T_0+C_0+i_r r=$

$Y[c_V(1+\theta_Y-ri_Y)+g_Y+in_Y+\nu_Y]+C_0-c_V T_0+i_r r$

Noting:

(15) $E=C_0-c_V T_0$

(16) $\omega=1+\theta_Y-ri_Y$

(17) $\chi=1-c_V(1+\theta_Y-ri_Y)-g_Y-in_Y-\nu_Y=1-c_V\omega-g_Y-in_Y-\nu_Y$

results, first, from (2), (6), (7) and (8) and with (15) and (16):

$C=c_V V+C_0=c_V(Y+TR-TI)+C_0=c_V(Y+\theta_Y Y-ri_Y Y-T_0)+C_0=c_V[(1+\theta_Y-ri_Y)Y-T_0]+C_0=$

$c_V[\omega Y-T_0]+C_0=c_V\omega Y+C_0-c_V T_0=c_V\omega Y+E$ so:

(18) $C=c_V\omega Y+E$

How, in the absence of the output (Y=0) household consumption must be positive, it follows that $C=E\geq 0$.

Also, after the assumptions that $ri_Y\in(0,1)$, $\theta_Y\in(0,1)$ we obtain that: $\omega=1+\theta_Y-ri_Y\in(0,2)$ so it is still positive.

With the notations (15) - (17), equation (14) becomes:

(19) $D=Y(1-\chi)+i_r r+E$

The first static equilibrium equation D=Y is now> $Y(1-\chi)+i_r r+E=Y$ then:

$i_r r+E=Y-Y(1-\chi)=\chi Y$ from where:

(20) $Y=\frac{i_r}{\chi}r+\frac{E}{\chi}$

The natural condition of decreasing the output Y to an increasing of the interest rate (r) returns to $Y'(r)=\frac{i_r}{\chi}<0$ therefore, together with the hypothesis from (4) that is $i_r<0$, implies that $\chi>0$.

From the fact that $c_V, g_Y, in_Y, \theta_Y, ri_Y\in(0,1)$, $\nu_Y\in(-1,1)$ follows that $\chi>0$ or:

$\chi=1-c_V(1+\theta_Y-ri_Y)-g_Y-in_Y-\nu_Y>0$ if and only if:

$1-g_Y-in_Y-\nu_Y>c_V(1+\theta_Y-ri_Y)$ and how $1+\theta_Y-ri_Y\in(0,2)$ results, finally, that:

(21) $c_V<\frac{1-g_Y-in_Y-\nu_Y}{1+\theta_Y-ri_Y}$

Similarly, from equations (10) and (11): $MD=md_Y Y+m_r r=M$ is obtained: $md_Y Y=M-m_r r$ from where:

$$Y=-\frac{m_r}{md_Y}r+\frac{M}{md_Y} \quad (22)$$

The equilibrium condition in both markets (goods and services on the one hand and the money of the other) resulting from equations (20) and (22):

$$\begin{cases} Y=\frac{i_r}{\chi}r+\frac{E}{\chi} \\ Y=-\frac{m_r}{md_Y}r+\frac{M}{md_Y} \end{cases} \quad (23)$$

The solution of the system is:

$$\begin{cases} Y^*=\frac{Mi_r+Em_r}{i_r md_Y+m_r\chi} \\ r^*=\frac{M\chi-Emd_Y}{i_r md_Y+m_r\chi} \end{cases} \quad (24)$$

The equations (24) characterize the static equilibrium condition of the model.

From equations (2)-(8), (10), (24) result the values of main key indicators at the equilibrium:

$$G^*=g_Y Y^*=\frac{(Mi_r+Em_r)g_Y}{i_r md_Y+m_r\chi} \quad (25)$$

$$NX^*=\nu_Y Y^*=\frac{(Mi_r+Em_r)\nu_Y}{i_r md_Y+m_r\chi} \quad (26)$$

$$TR^*=\theta_Y Y^*=\frac{(Mi_r+Em_r)\theta_Y}{i_r md_Y+m_r\chi} \quad (27)$$

$$TI^*=ri_Y Y^*+T_0=\frac{(Mi_r+Em_r)ri_Y}{i_r md_Y+m_r\chi}+T_0 \quad (28)$$

$$I^*=in_Y Y^*+i_r r^*=\frac{(Mi_r+Em_r)in_Y}{i_r md_Y+m_r\chi}+\frac{(M\chi-Emd_Y)i_r}{i_r md_Y+m_r\chi}=\frac{(in_Y+\chi)i_r M+(m_r in_Y-md_Y i_r)E}{i_r md_Y+m_r\chi} \quad (29)$$

$$MD^*=md_Y Y^*+m_r r^*=\frac{(Mi_r+Em_r)md_Y}{i_r md_Y+m_r\chi}+\frac{(M\chi-Emd_Y)m_r}{i_r md_Y+m_r\chi}=M$$

$$V^* = Y^* + TR^* - TI^* = \frac{(Mi_r + Em_r)(1+\theta_Y - ri_Y)}{i_r md_Y + m_r \chi} - T_0 \quad (30)$$

$$C^* = c_V V^* + C_0 = \frac{(Mi_r + Em_r)(1+\theta_Y - ri_Y)c_V}{i_r md_Y + m_r \chi} - c_V T_0 + C_0 = \quad (32)$$

$$\frac{(Mi_r + Em_r)(1+\theta_Y - ri_Y)c_V}{i_r md_Y + m_r \chi} + E = \frac{i_r(1+\theta_Y - ri_Y)c_V M + [(1+\theta_Y - ri_Y)m_r c_V + i_r md_Y + m_r \chi]E}{i_r md_Y + m_r \chi} =$$

$$\frac{i_r(1+\theta_Y - ri_Y)c_V M + [i_r md_Y + m_r(1 - g_Y - in_Y - \nu_Y)]E}{i_r md_Y + m_r \chi}$$

Noting now, for simplicity:

$$\Lambda = \frac{1}{i_r md_Y + m_r \chi} < 0 \quad (33)$$

$$\Gamma = (Mi_r + m_r E)\Lambda^2 < 0 \quad (34)$$

follows, also:

$$M\chi - Emd_Y = \frac{\Gamma\chi - E\Lambda}{i_r \Lambda^2} = \frac{M\Lambda - md_Y \Gamma}{m_r \Lambda^2} \quad (35)$$

Substituting (33), (34) and (35) into formulas (24) follows:

$$\begin{cases} Y^* = \dfrac{\Gamma}{\Lambda} \\ r^* = \dfrac{\Gamma\chi - E\Lambda}{i_r \Lambda} = \dfrac{M\Lambda - md_Y \Gamma}{m_r \Lambda} \end{cases} \quad (36)$$

From the formulas (24) or (36) with notations (33)-(35) results, also, the partial derivatives of first and second orders of the equilibrium values of the output and interest rate respectively, required for the analysis of static equilibrium at a change of model parameters.

$$\frac{\partial Y^*}{\partial c_V} = m_r(\omega\Gamma - T_0\Lambda) \quad (37)$$

;

$$\frac{\partial Y^*}{\partial g_Y} = \frac{\partial Y^*}{\partial \nu_Y} = \frac{\partial Y^*}{\partial in_Y} = m_r \Gamma$$

;

$$\frac{\partial Y^*}{\partial \theta_Y} = -\frac{\partial Y^*}{\partial ri_Y} = m_r c_V \Gamma \quad ;$$

$$\frac{\partial Y^*}{\partial i_r} = M\Lambda - md_Y \Gamma \quad ;$$

$$\frac{\partial Y^*}{\partial md_Y} = -i_r \Gamma \quad ;$$

$$\frac{\partial Y^*}{\partial m_r} = -i_r \frac{M\Lambda - md_Y \Gamma}{m_r} \quad ;$$

$$\frac{\partial Y^*}{\partial M} = i_r \Lambda$$

$$\frac{\partial r^*}{\partial c_V} = -md_Y (\omega\Gamma - T_0\Lambda) \quad ; \quad (38)$$

$$\frac{\partial r^*}{\partial g_Y} = \frac{\partial r^*}{\partial \nu_Y} = \frac{\partial r^*}{\partial in_Y} = -md_Y \Gamma \quad ;$$

$$\frac{\partial r^*}{\partial \theta_Y} = -\frac{\partial r^*}{\partial ri_Y} = -md_Y c_V \Gamma \quad ;$$

$$\frac{\partial r^*}{\partial i_r} = -md_Y \frac{M\Lambda - md_Y \Gamma}{m_r} \quad ;$$

$$\frac{\partial r^*}{\partial md_Y} = -\chi\Gamma \quad ;$$

$$\frac{\partial r^*}{\partial m_r} = -\chi \frac{M\Lambda - md_Y \Gamma}{m_r} \quad ;$$

$$\frac{\partial r^*}{\partial M} = \chi\Lambda$$

$$\frac{\partial^2 Y^*}{\partial c_V^2} = 2(\omega\Gamma - T_0\Lambda)\omega m_r^2 \Lambda \quad ; \quad (39)$$

$$\frac{\partial^2 Y^*}{\partial g_Y^2} = \frac{\partial^2 Y^*}{\partial \nu_Y^2} = \frac{\partial^2 Y^*}{\partial in_Y^2} = 2m_r^2 \Gamma\Lambda \quad ;$$

$$\frac{\partial^2 Y^*}{\partial \theta_Y^2} = -\frac{\partial^2 Y^*}{\partial ri_Y^2} = 2c_V m_r^2 \Gamma\Lambda \quad ;$$

$$\frac{\partial^2 Y^*}{\partial i_r^2} = -2(M\Lambda - md_Y\Gamma)md_Y\Lambda \quad ;$$

$$\frac{\partial^2 Y^*}{\partial md_Y^2} = 2i_r^2\Gamma\Lambda \quad ;$$

$$\frac{\partial^2 Y^*}{\partial m_r^2} = 2\chi i_r \Lambda \frac{M\Lambda - md_Y\Gamma}{m_r} \quad ;$$

$$\frac{\partial^2 Y^*}{\partial M^2} = 0$$

$$\frac{\partial^2 r^*}{\partial c_V^2} = -2(\omega\Gamma - T_0\Lambda)\omega md_Y m_r \Lambda \quad ; \tag{40}$$

$$\frac{\partial^2 r^*}{\partial g_Y^2} = \frac{\partial^2 r^*}{\partial \nu_Y^2} = \frac{\partial^2 r^*}{\partial in_Y^2} = -2m_r md_Y \Gamma\Lambda \quad ;$$

$$\frac{\partial^2 r^*}{\partial \theta_Y^2} = -\frac{\partial^2 r^*}{\partial ri_Y^2} = -2c_V m_r md_Y \Gamma\Lambda \quad ;$$

$$\frac{\partial^2 r^*}{\partial i_r^2} = 2\frac{md_Y^2}{m_r}(M\Lambda - md_Y\Gamma)\Lambda \quad ;$$

$$\frac{\partial^2 r^*}{\partial md_Y^2} = 2i_r\chi\Gamma\Lambda \quad ;$$

$$\frac{\partial^2 r^*}{\partial m_r^2} = 2\chi^2\Lambda\frac{M\Lambda - md_Y\Gamma}{m_r} \quad ;$$

$$\frac{\partial^2 r^*}{\partial M^2}=0$$

For the analysis of the increasing or decreasing character of Y^* or r^* with respect to each parameter of the model (assuming that everyone else is constant), we can see that from the formulas (37)-(39) the only directly independent expressions from the model parameters are $\omega\Gamma-T_0\Lambda$ and $M\Lambda-md_Y\Gamma$. Therefore, it is necessary, first of all, to study their sign.

The condition that $\omega\Gamma-T_0\Lambda>0$ is equivalent to (from the formulas 33 and 34) with:

$$0<\omega\Gamma-T_0\Lambda=\omega(Mi_r+m_rE)\Lambda^2-T_0\Lambda=\Lambda^2\left(\omega(Mi_r+m_rE)-\frac{T_0}{\Lambda}\right)=$$

$$\Lambda^2\left[\omega(Mi_r+m_rE)-T_0(i_rmd_Y+m_r\chi)\right]=$$

$$\Lambda^2\left[\omega Mi_r+\omega m_r(C_0-c_VT_0)-T_0(i_rmd_Y+m_r\chi)\right]=$$

$$\Lambda^2\left[\omega Mi_r+\omega m_rC_0-T_0(i_rmd_Y+m_r\chi+c_V\omega m_r)\right]=$$

$$\omega m_r\Lambda^2\left(C_0-\frac{T_0(i_rmd_Y+m_r\chi+c_V\omega m_r)-\omega Mi_r}{\omega m_r}\right)$$

Noting:

$$(41)\quad \Phi_1=\frac{T_0(i_rmd_Y+m_r\chi+c_V\omega m_r)-\omega Mi_r}{\omega m_r}$$

follows that $\omega\Gamma-T_0\Lambda>0$ if and only if: $C_0<\Phi_1$.

We see now that $C_0=\Phi_1$ if and only if: $C_0=\frac{T_0(i_rmd_Y+m_r\chi+c_V\omega m_r)-\omega Mi_r}{\omega m_r}$ or, equivalently: $T_0=\frac{\omega m_rC_0+\omega Mi_r}{i_rmd_Y+m_r\chi+c_V\omega m_r}$.

Therefore:

- $\omega\Gamma-T_0\Lambda>0\Leftrightarrow C_0<\Phi_1\Leftrightarrow C_0<\frac{T_0(i_rmd_Y+m_r\chi+c_V\omega m_r)-\omega Mi_r}{\omega m_r}\Leftrightarrow$

$$T_0 > \frac{\omega m_r C_0 + \omega M i_r}{i_r m d_Y + m_r \chi + c_V \omega m_r}$$

- $\omega\Gamma - T_0\Lambda < 0 \Leftrightarrow C_0 > \Phi_1 \Leftrightarrow C_0 > \dfrac{T_0(i_r m d_Y + m_r \chi + c_V \omega m_r) - \omega M i_r}{\omega m_r} \Leftrightarrow$

$$T_0 < \frac{\omega m_r C_0 + \omega M i_r}{i_r m d_Y + m_r \chi + c_V \omega m_r}$$

- $\omega\Gamma - T_0\Lambda = 0 \Leftrightarrow C_0 = \Phi_1 \Leftrightarrow C_0 = \dfrac{T_0(i_r m d_Y + m_r \chi + c_V \omega m_r) - \omega M i_r}{\omega m_r} \Leftrightarrow$

$$T_0 = \frac{\omega m_r C_0 + \omega M i_r}{i_r m d_Y + m_r \chi + c_V \omega m_r}$$

Similarly, the condition that $M\Lambda - m d_Y \Gamma > 0$ is equivalent, successively, with:

$$0 < M\Lambda - m d_Y \Gamma = M\Lambda - m d_Y (M i_r + m_r E)\Lambda^2 = \Lambda^2\left[\frac{M}{\Lambda} - m d_Y (M i_r + m_r E)\right] =$$

$$\Lambda^2\left[M(i_r m d_Y + m_r \chi) - m d_Y (M i_r + m_r E)\right] =$$

$$\Lambda^2\left[M(i_r m d_Y + m_r \chi) - m d_Y (M i_r + m_r (C_0 - c_V T_0))\right] =$$

$$\Lambda^2\left[m_r M\chi - m d_Y m_r (C_0 - c_V T_0)\right] = m_r \Lambda^2 (M\chi - m d_Y C_0 + c_V m d_Y T_0) =$$

$$m_r m d_Y \Lambda^2\left(-C_0 + \frac{c_V m d_Y T_0 + M\chi}{m d_Y}\right).$$

Noting:

$$(42) \quad \Phi_2 = \frac{c_V m d_Y T_0 + M\chi}{m d_Y}$$

follows that $M\Lambda - m d_Y \Gamma > 0$ if and only if: $C_0 > \Phi_2$.

We note now that $C_0 = \Phi_2$ if and only if: $C_0 = \dfrac{c_V m d_Y T_0 + M\chi}{m d_Y}$ or, equivalently, $T_0 = \dfrac{m d_Y C_0 - M\chi}{c_V m d_Y}$.

Also:

- $M\Lambda - md_Y\Gamma > 0 \Leftrightarrow C_0 > \Phi_2 \Leftrightarrow C_0 > \frac{c_V md_Y T_0 + M\chi}{md_Y} \Leftrightarrow T_0 < \frac{md_Y C_0 - M\chi}{c_V md_Y}$
- $M\Lambda - md_Y\Gamma < 0 \Leftrightarrow C_0 < \Phi_2 \Leftrightarrow C_0 < \frac{c_V md_Y T_0 + M\chi}{md_Y} \Leftrightarrow T_0 > \frac{md_Y C_0 - M\chi}{c_V md_Y}$
- $M\Lambda - md_Y\Gamma = 0 \Leftrightarrow C_0 = \Phi_2 \Leftrightarrow C_0 = \frac{c_V md_Y T_0 + M\chi}{md_Y} \Leftrightarrow T_0 = \frac{md_Y C_0 - M\chi}{c_V md_Y}$

We have also the relations:

$$\Phi_1-\Phi_2 = \frac{T_0(i_r md_Y + m_r\chi + c_V\omega m_r) - \omega M i_r}{\omega m_r} - \frac{M\chi + c_V md_Y T_0}{md_Y} =$$

$$\frac{T_0(i_r md_Y^2 + m_r md_Y\chi + c_V\omega m_r md_Y) - \omega M md_Y i_r - M\omega m_r\chi - c_V\omega m_r md_Y T_0}{\omega m_r md_Y} =$$

$$\frac{T_0(i_r md_Y^2 + m_r md_Y\chi) - M\omega(md_Y i_r + m_r\chi)}{\omega m_r md_Y} =$$

$$\frac{T_0 md_Y(i_r md_Y + m_r\chi) - M\omega(md_Y i_r + m_r\chi)}{\omega m_r md_Y} =$$

$$\frac{(md_Y i_r + m_r\chi)(T_0 md_Y - M\omega)}{\omega m_r md_Y} = \frac{T_0 md_Y - M\omega}{\omega m_r md_Y\Lambda}.$$

Note now that $\Phi_1=\Phi_2$ if and only if $T_0 = \frac{M\omega}{md_Y}$. In this situation, the conditions relative to the position of C_0 relative to $\Phi_1=\Phi_2$ becomes:

- $C_0 < \Phi_1=\Phi_2 \Leftrightarrow T_0 = \frac{M\omega}{md_Y}$ and $C_0 < \frac{(c_V\omega+\chi)M}{md_Y}$
- $C_0 > \Phi_1=\Phi_2 \Leftrightarrow T_0 = \frac{M\omega}{md_Y}$ and $C_0 > \frac{(c_V\omega+\chi)M}{md_Y}$

- $C_0=\Phi_1=\Phi_2 \Leftrightarrow T_0=\frac{M\omega}{md_Y}$ and $C_0=\frac{(c_V\omega+\chi)M}{md_Y}$

Because $\Lambda<0$, $m_r<0$, $md_Y>0$, $\omega>0$ it follows that $\Phi_1-\Phi_2>0$ if and only if $T_0 md_Y - M\omega_{>0}$ then:

- $\Phi_1>\Phi_2 \Leftrightarrow T_0>\frac{\omega M}{md_Y}$
- $\Phi_2>\Phi_1 \Leftrightarrow T_0<\frac{\omega M}{md_Y}$
- $\Phi_2=\Phi_1 \Leftrightarrow T_0=\frac{\omega M}{md_Y}$

Before considering the various cases generated from the level of taxes independent of income (T_0) and the autonomous consumption of households (C_0) respectively, remark that from formulas (37)-(39) that:

- Y^* and r^* are strictly increasing and strictly convex with respect to the marginal government consumption g_Y, that is an increase in the share of government consumption in GDP will generate a stronger growth both GDP and interest rate. As a result of this situation, an increase in the budget drives to the increase in GDP but will affect also the growth of the interest rate, the last, with consequences in investments. Naturally the question arises whether the new level of investments will lead to a decrease or an increase in GDP. The differential of investments in relation to g_Y is:

$dI^*=in_Y dY^*+i_r dr^*=in_Y \frac{\partial Y^*}{\partial g_Y} dg_Y+i_r \frac{\partial r^*}{\partial g_Y} dg_Y=in_Y m_r \Gamma dg_Y-i_r md_Y \Gamma dg_Y=(in_Y m_r - i_r md_Y)\Gamma dg_Y$

The condition that $(in_Y m_r - i_r md_Y)\Gamma_{>0}$, due to the fact that $\Gamma<0$, is equivalent with $in_Y m_r - i_r md_Y < 0$ or: $in_Y > \frac{i_r md_Y}{m_r}$. Therefore, an investment rate higher than the threshold $\frac{i_r md_Y}{m_r}$ will result that the investment will increase, and for a

$\frac{i_r md_Y}{m_r}$ lower investment rate than the increase in the share of government consumption in GDP will lead to a decrease in investment. How, in formulas (1) and (9): $Y^*=C^*+G^*+I^*+NX^*$ will result, finally, an increasing respectively decreasing GDP.

- Y^* and r^* are strictly increasing and strictly convex with respect to ν_Y, that is an increase in the share of net exports to GDP will generate a stronger increase both of GDP and interest rate. The differential of investments in relation to ν_Y is:

$dI^*=in_Y dY^*+i_r dr^*=in_Y \frac{\partial Y^*}{\partial \nu_Y} d\nu_Y+i_r \frac{\partial r^*}{\partial \nu_Y} d\nu_Y=in_Y m_r \Gamma d\nu_Y - i_r md_Y \Gamma d\nu_Y=$ $(in_Y m_r - i_r md_Y)\Gamma d\nu_Y$. The condition that $(in_Y m_r - i_r md_Y)\Gamma > 0$, due to the fact that $\Gamma<0$, is equivalent with $in_Y m_r - i_r md_Y < 0$ or other: $in_Y > \frac{i_r md_Y}{m_r}$. Therefore, for an investment rate higher than the threshold $\frac{i_r md_Y}{m_r}$ will result that the investment will increase, and for a lower investment rate than $\frac{i_r md_Y}{m_r}$ the increase in the share of the marginal net exports in GDP will lead to a decrease in investment. Similarly to the above, would result in the end, an increase or, respectively, decrease in the GDP.

- Y^* and r^* are strictly increasing and strictly convex with respect to in_Y – the investment rate, that is an increase in the share of investment in GDP will generate a stronger increase both of GDP and interest rate. This later aspect is normal, because the acceleration of investments require additional funding sources leading to greater interest rate.
- Y^* and r^* are strictly increasing and strictly convex with respect to the marginal rate of government transfers θ_Y, that is an increase in the share of transfers in GDP will generate a stronger increase both of GDP and the interest rate. The differential of the investments in relation to θ_Y is:

$dI^*=in_Y dY^*+i_r dr^*=in_Y \frac{\partial Y^*}{\partial \theta_Y} d\theta_Y+i_r \frac{\partial r^*}{\partial \theta_Y} d\theta_Y=in_Y m_r c_V \Gamma d\theta_Y - i_r md_Y c_V \Gamma d\theta_Y=$ $(in_Y m_r - i_r md_Y)c_V \Gamma d\theta_Y$. The condition that $(in_Y m_r - i_r md_Y)\Gamma > 0$, due to the fact

that $\Gamma<0$, $c_V>0$, is equivalent with $in_Y m_r - i_r md_Y < 0$ or other: $in_Y > \frac{i_r md_Y}{m_r}$. Therefore, for an investment rate higher than the threshold $\frac{i_r md_Y}{m_r}$ will result that the investment will increase, and for a lower investment rate than $\frac{i_r md_Y}{m_r}$ the increase in the marginal rate of transfers in GDP will lead to a decrease in investment. Similarly to the above, would result in the end, an increase or, respectively, decrease in the GDP.

- Y^* and r^* are strictly decreasing and strictly concave with respect to ri_Y – tax rate, that is , an increase in the share of taxes in GDP will lead to a decrease becoming greater of GDP and interest rate. From formula (8) we have: $dTI^*=ri_Y dY^*=ri_Y \frac{\partial Y^*}{\partial ri_Y} dri_Y=-ri_Y m_r c_V \Gamma dri_Y<0$ therefore an increase in the tax rate will lead to a loss of tax revenue. Also, from the equations (2) and (6), we have $C^*=c_V V^*+C_0$ and $V^*=Y^*+TR^*-TI^*$ from where: $C^*=c_V(Y^*+TR^*-TI^*)+C_0=c_V Y^*+c_V TR^*-c_V TI^*+C_0$. Differentiating, assuming that transfers TR^* are constant, we obtain that $dC^*=c_V dY^*-c_V dTI^*=c_V \frac{\partial Y^*}{\partial ri_Y} dri_Y-c_V ri_Y \frac{\partial Y^*}{\partial ri_Y} dri_Y= -m_r(1-ri_Y)c_V^2 \Gamma dri_Y<0$ so the actual final consumption of households will decrease.
- Y^* is strictly decreasing and strictly convex with respect to the rate of money demand in the economy md_Y, that is an increase in demand for currency relative to GDP level will generate a decrease becoming more subdued of GDP. Also, r^* is strictly decreasing and strictly concave with respect to money demand rate md_Y, that is an increase in demand for currency relative to GDP level will generate a decrease becoming more pronounced of the interest rate. The differential of investments with respect to md_Y is:

$dI^*=in_Y dY^*+i_r dr^*=in_Y \frac{\partial Y^*}{\partial md_Y} dmd_Y+i_r \frac{\partial r^*}{\partial md_Y} dmd_Y=-in_Y i_r \Gamma dmd_Y-i_r \chi \Gamma dmd_Y= -(\chi+in_Y)i_r \Gamma dmd_Y<0$. Therefore, increases in the demand for money relative to GDP will (assuming constancy of other parameters) to a decrease in investment, GDP implicitly. Following this analysis, the dynamics of money demand will be lower to the GDP's growth.

- Y^* is is strictly increasing and linear, and r^* is strictly decreasing and linear with respect to the money supply M, i.e. an increase in the money supply will rise by virtue $dY^* = \frac{\partial Y^*}{\partial M} dM = i_r \Lambda dM > 0$, the growth of GDP, and since $dr^* = \frac{\partial r^*}{\partial M} dM = \chi \Lambda dM < 0$ a decrease of the interest rate.

For the remaining dependencies, we have now 13 cases with an appearance, at first sight, formal, but positioning in the following analysis, on concrete data, trends in both GDP and interest rate.

Case 1: $T_0 < \frac{\omega M}{md_Y}$, $C_0<\Phi_1<\Phi_2$. In this situation: $\omega\Gamma - T_0\Lambda > 0$ and $M\Lambda - md_Y\Gamma < 0$ from where:

- Y^* and r^* are strictly decreasing and strictly concave in relation to the marginal propensity to consumption (c_V)
- Y^* and r^* are strictly decreasing and strictly concave in relation to the factor of influence in the investment rate (i_r)
- Y^* is strictly increasing and strictly convex with respect to the factor that influence the demand for money in relation to interest rate (m_r)
- r^* is strictly decreasing and strictly concave in relation to the factor that influence the demand for money in relation to interest rate (m_r)

Case 2: $T_0 < \frac{\omega M}{md_Y}$, $C_0=\Phi_1<\Phi_2$. In this case: $C_0 = \frac{T_0(i_r md_Y + m_r\chi + c_V\omega m_r) - \omega M i_r}{\omega m_r}$ or, equivalent: $T_0 = \frac{\omega m_r C_0 + \omega M i_r}{i_r md_Y + m_r\chi + c_V\omega m_r}$, and $\omega\Gamma - T_0\Lambda = 0$ and $M\Lambda - md_Y\Gamma < 0$ from where:

- Y^* and r^* are constants with respect to the marginal propensity to consumption (c_V)
- Y^* and r^* are strictly decreasing and strictly concave in relation to the factor of influence in the investment rate (i_r)
- Y^* is strictly increasing and strictly convex with respect to the factor that influence the demand for money in relation to interest rate (m_r)

- r^* is strictly decreasing and strictly concave in relation to the factor that influence the demand for money in relation to interest rate (m_r)

Case 3: $T_0 < \frac{\omega M}{md_Y}$, $\Phi_1<C_0<\Phi_2$. In this case: $\omega\Gamma - T_0\Lambda < 0$ and $M\Lambda - md_Y\Gamma < 0$ from where:

- Y^* and r^* are strictly increasing and strictly convex with respect to the marginal propensity to consumption (c_V)
- Y^* and r^* are strictly decreasing and strictly concave in relation to the factor of influence in the investment rate (i_r)
- Y^* is strictly increasing and strictly convex with respect to the factor that influence the demand for money in relation to interest rate (m_r)
- r^* is strictly decreasing and strictly concave in relation to the factors that influence the demand for money in relation to interest rate (m_r)

Case 4: $T_0 < \frac{\omega M}{md_Y}$, $\Phi_1<\Phi_2=C_0$. In this case: $C_0 = \frac{c_V md_Y T_0 + M\chi}{md_Y}$, $T_0 = \frac{md_Y C_0 - M\chi}{c_V md_Y}$, $\omega\Gamma - T_0\Lambda < 0$ and $M\Lambda - md_Y\Gamma = 0$ from where:

- Y^* and r^* are strictly increasing and strictly convex with respect to the marginal propensity to consumption (c_V)
- Y^* and r^* are constants with respect to factors that influence interest rates in investments (i_r)
- Y^* is constant in relation to the factor that influence the demand for money in relation to interest rate (m_r)
- r^* is constant in relation to the factor that influence the demand for money in relation to interest rate (m_r)

Case 5: $T_0 < \frac{\omega M}{md_Y}$, $\Phi_1<\Phi_2<C_0$. In this case: $\omega\Gamma - T_0\Lambda < 0$ and $M\Lambda - md_Y\Gamma > 0$ from where:

- Y^* and r^* are strictly increasing and strictly convex with respect to the marginal propensity to consumption (c_V)
- Y^* and r^* are strictly increasing and strictly convex with respect to the factor of influence in the investment rate (i_r)

- Y^* is strictly decreasing and strictly concave in relation to the factor that influence the demand for money in relation to interest rate (m_r)
- r^* is strictly increasing and strictly convex with respect to the factor that influence the demand for money in relation to interest rate (m_r)

Case 6: $T_0 = \frac{\omega M}{md_Y}$, $C_0<\Phi_1=\Phi_2$. In this case: $C_0< \frac{(c_V\omega+\chi)M}{md_Y}$, $\omega\Gamma - T_0\Lambda>0$ and $M\Lambda - md_Y\Gamma<0$ from where:

- Y^* and r^* are strictly decreasing and strictly concave with respect to the marginal propensity to consumption (c_V)
- Y^* and r^* are strictly decreasing and strictly concave in relation to the factor of influence in the investment rate (i_r)
- Y^* is strictly increasing and strictly convex with respect to the factor that influence the demand for money in relation to interest rate (m_r)
- r^* is strictly decreasing and strictly concave in relation to the factor that influence the demand for money in relation to interest rate (m_r)

Case 7: $T_0 = \frac{\omega M}{md_Y}$, $C_0=\Phi_1=\Phi_2$. In this case: $C_0= \frac{(c_V\omega+\chi)M}{md_Y}$, $\omega\Gamma - T_0\Lambda=0$ and $M\Lambda - md_Y\Gamma=0$ from where:

- Y^* and r^* are constants with respect to the marginal propensity to consumption (c_V)
- Y^* and r^* are constants with respect to factors that influence interest rates in investments (i_r)
- Y^* and r^* are constants with respect to factors that influence the demand for money in relation to interest rate (m_r)

Case 8: $T_0 = \frac{\omega M}{md_Y}$, $\Phi_1=\Phi_2<C_0$. In this case: $C_0> \frac{(c_V\omega+\chi)M}{md_Y}$, $\omega\Gamma - T_0\Lambda<0$ and $M\Lambda - md_Y\Gamma<0$ from where:

- Y^* and r^* are strictly increasing and strictly convex with respect to the marginal propensity to consumption (c_V)
- Y^* and r^* are strictly decreasing and strictly concave in relation to the factor of influence in the investment rate (i_r)

- Y^* is strictly increasing and strictly convex with respect to the factor that influence the demand for money in relation to interest rate (m_r)
- r^* is strictly decreasing and strictly concave in relation to the factor that influence the demand for money in relation to interest rate (m_r)

Case 9: $T_0 > \frac{\omega M}{md_Y}$, $C_0<\Phi_2<\Phi_1$. In this case: $\omega\Gamma - T_0\Lambda > 0$ and $M\Lambda - md_Y\Gamma < 0$ from where:

- Y^* and r^* are strictly decreasing and strictly concave with respect to the marginal propensity to consumption (c_V)
- Y^* and r^* are strictly decreasing and strictly concave in relation to the factor of influence in the investment rate (i_r)
- Y^* is strictly increasing and strictly convex with respect to the factor that influence the demand for money in relation to interest rate (m_r)
- r^* is strictly decreasing and strictly concave in relation to the factor that influence the demand for money in relation to interest rate (m_r)

Case 10: $T_0 > \frac{\omega M}{md_Y}$, $C_0=\Phi_2<\Phi_1$. In this case: $C_0 = \frac{c_V md_Y T_0 + M\chi}{md_Y}$ or, equivalent: $T_0 = \frac{md_Y C_0 - M\chi}{c_V md_Y}$, $\omega\Gamma - T_0\Lambda > 0$ and $M\Lambda - md_Y\Gamma = 0$ from where:

- Y^* and r^* are strictly decreasing and strictly concave with respect to the marginal propensity to consumption (c_V)
- Y^* and r^* are constants with respect to the factor that influence interest rates of investments (i_r)
- Y^* and r^* are constants with respect to factors that influence the demand for money in relation to interest rate (m_r)

Case 11: $T_0 > \frac{\omega M}{md_Y}$, $\Phi_2<C_0<\Phi_1$. In this case: $\omega\Gamma - T_0\Lambda > 0$ and $M\Lambda - md_Y\Gamma > 0$ from where:

- Y^* and r^* are strictly decreasing and strictly concave with respect to the marginal propensity to consumption (c_V)
- Y^* and r^* are strictly increasing and strictly convex with respect to the factor of influence in the investment rate (i_r)

- Y^* is strictly decreasing and strictly concave in relation to the factor that influence the demand for money in relation to interest rate (m_r)
- r^* is strictly increasing and strictly convex with respect to the factor that influence the demand for money in relation to interest rate (m_r)

- **Case 12**: $T_0 > \frac{\omega M}{md_Y}$, $\Phi_2<\Phi_1=C_0$. In this case: $C_0= \frac{T_0(i_r md_Y + m_r\chi + c_V\omega m_r) - \omega M i_r}{\omega m_r}$ or, equivalent: $T_0 = \frac{\omega m_r C_0 + \omega M i_r}{i_r md_Y + m_r\chi + c_V\omega m_r}$, $\omega\Gamma - T_0\Lambda = 0$ and $M\Lambda - md_Y\Gamma > 0$ from where:
- Y^* and r^* are constants with respect to the marginal propensity to consumption (c_V)
- Y^* and r^* are strictly increasing and strictly convex with respect to the factor of influence in the investment rate (i_r)
- Y^* is strictly decreasing and strictly concave in relation to the factor that influence the demand for money in relation to interest rate (m_r)
- r^* is strictly increasing and strictly convex with respect to the factor that influence the demand for money in relation to interest rate (m_r)

Case 13: $T_0 > \frac{\omega M}{md_Y}$, $\Phi_2<\Phi_1<C_0$. In this case: $\omega\Gamma - T_0\Lambda < 0$ and $M\Lambda - md_Y\Gamma > 0$ from where:

- Y^* and r^* are strictly increasing and strictly convex with respect to the marginal propensity to consumption (c_V)
- Y^* and r^* are strictly increasing and strictly convex with respect to the factor of influence in the investment rate (i_r)
- Y^* is strictly decreasing and strictly concave in relation to the factor that influence the demand for money in relation to interest rate (m_r)
- r^* is strictly increasing and strictly convex with respect to factors that influence the demand for money in relation to interest rate (m_r)

3. The Determination of the Potential GDP. Okun's Law

The classical definition of potential GDP is one more or less formal in the sense that it is that level of GDP in the conditions of an optimal operation without imbalances in the economy. Following the model presented above, we define the equilibrium state as potential GDP.

Therefore, we define the potential GDP (Y^*) by the formula:

$$Y^* = \frac{Mi_r + Em_r}{i_r md_Y + m_r \chi} \tag{43}$$

Once determined the level of potential GDP, we naturally put the problem of calculating the natural rate of unemployment. The well-known expression of Okun's law is:

$$\frac{Y^* - Y}{Y^*} = c\left(u - u^*\right) \tag{44}$$

where:

- Y – current GDP;
- Y^* – potential GDP;
- u – the unemployment rate;
- u^* – the natural rate of unemployment;
- c – factor of proportionality

Due to the difficulties of Okun's law (in the sense that the determination of the constant c requires both knowledge of potential GDP - defined by formula (43) and the natural rate of unemployment - which is exactly the approach the front) is used in practice, a modified form of it (with the assumption that the economy is turning to potential GDP and the natural rate of unemployment knows no significant variations in short intervals):

$$\frac{\Delta Y}{Y} = a - c\Delta u \tag{45}$$

The determination of the constants a and c is made with relative ease, using linear regression, given that the statistics are known as the level of GDP (Y) and obvious the variations (ΔY) and the unemployment rate.

Substituting the value of c as determined by the formulas (43) and (44) we obtain the natural rate of unemployment:

$$u^* = u - \frac{Y^* - Y}{cY^*} = u - \frac{1}{c} + \frac{i_r m d_Y + m_r \chi}{c(Mi_r + Em_r)} Y \quad (46)$$

From equation (46) it is observed that the natural rate of unemployment increases in relation to the factor $\frac{i_r m d_Y + m_r \chi}{c(Mi_r + Em_r)} > 0$.

4. The Dynamic Equilibrium

Equations (12) and (13) are dynamic equilibrium laws. It is observed that for the values of Y and r in the static equilibrium, follows: $\frac{dY}{dt} = \frac{dr}{dt} = 0$ so the dynamic process becomes stationary.

Consider then the system of differential equations of first order from the formulas (12) and (13):

$$\begin{cases} \frac{dY}{dt} = \alpha(D - Y) \\ \frac{dr}{dt} = \beta(MD - M) \end{cases}, \alpha, \beta > 0 \quad (47)$$

From (10) and (19) we can rewrite (47) as:

$$\begin{cases} \frac{dY}{dt} = -\alpha\chi Y + \alpha i_r r + \alpha E \\ \frac{dr}{dt} = \beta m d_Y Y + \beta m_r r - \beta M \end{cases} \quad (48)$$

From (48) follows: $\lim_{t\to\infty} Y(t) = \tilde{Y}$, $\lim_{t\to\infty} r(t) = \tilde{r}$, $\tilde{Y}, \tilde{r} \in \mathbf{R}_+$ if and only if:

1. $\Delta = (\alpha\chi + \beta m_r)^2 + 4\alpha\beta i_r m d_Y = 0$:

$$(49)\quad \begin{cases} Y = \left(-\dfrac{\alpha\chi+\beta m_r}{2} Y_0 + \alpha i_r r_0 + \alpha \dfrac{2 i_r \beta M + E(\alpha\chi+\beta m_r)}{\alpha\chi-\beta m_r} \right) te^{\frac{-\alpha\chi+\beta m_r}{2}t} + \\ \left(Y_0 + 4\alpha\beta \dfrac{m_r E + i_r M}{(\alpha\chi-\beta m_r)^2} \right) e^{\frac{-\alpha\chi+\beta m_r}{2}t} + \dfrac{i_r M + m_r E}{\chi m_r + i_r m d_Y} \\ r = \left(r_0 - 4\alpha\beta \dfrac{-\chi M + m d_Y E}{(\alpha\chi-\beta m_r)^2} \right) e^{\frac{-\alpha\chi+\beta m_r}{2}t} + \\ \left(-\dfrac{\alpha\chi+\beta m_r}{2} Y_0 + \alpha i_r r_0 + \alpha \dfrac{2 i_r \beta M + E(\alpha\chi+\beta m_r)}{\alpha\chi-\beta m_r} \right) \dfrac{\alpha\chi+\beta m_r}{2\alpha i_r} te^{\frac{-\alpha\chi+\beta m_r}{2}t} + \dfrac{\chi M - m d_Y E}{\chi m_r + i_r m d_Y} \end{cases}$$

and:
$$\begin{cases} \tilde{Y} = \dfrac{m_r E + i_r M}{\chi m_r + i_r m d_Y} \\ \tilde{r} = \dfrac{\chi M - m d_Y E}{\chi m_r + i_r m d_Y} \end{cases}$$

2. $\Delta=(\alpha\chi+\beta m_r)^2+4\alpha\beta i_r m d_Y>0$ and $\lambda_1\neq\lambda_2$ are real roots of the equation: $\lambda^2+(\alpha\chi-\beta m_r)\lambda-\alpha\beta(\chi m_r+i_r m d_Y)=0$:

$$(50)\quad \begin{cases} Y = k_1 e^{\lambda_1 t} + k_2 e^{\lambda_2 t} + \dfrac{m_r E + i_r M}{\chi m_r + i_r m d_Y} \\ r = \dfrac{\lambda_1 + \alpha\chi}{\alpha i_r} k_1 e^{\lambda_1 t} + \dfrac{\lambda_2 + \alpha\chi}{\alpha i_r} k_2 e^{\lambda_2 t} - \dfrac{m d_Y E - \chi M}{\chi m_r + i_r m d_Y} \end{cases}$$

where:

$$k_1 = \frac{(\lambda_2+\alpha\chi)Y_0 - (\lambda_2+\alpha\chi)\dfrac{m_r E + i_r M}{\chi m_r + i_r m d_Y} - \alpha i_r r_0 - \alpha i_r \dfrac{m d_Y E - \chi M}{\chi m_r + i_r m d_Y}}{\lambda_2 - \lambda_1}$$

$$k_2 = \frac{\alpha i_r r_0 + \alpha i_r \dfrac{m d_Y E - \chi M}{\chi m_r + i_r m d_Y} - (\lambda_1+\alpha\chi)Y_0 + (\lambda_1+\alpha\chi)\dfrac{m_r E + i_r M}{\chi m_r + i_r m d_Y}}{\lambda_2 - \lambda_1}$$

and:
$$\begin{cases} \tilde{Y} = \dfrac{m_r E + i_r M}{\chi m_r + i_r m d_Y} \\ \tilde{r} = \dfrac{\chi M - m d_Y E}{\chi m_r + i_r m d_Y} \end{cases}$$

3. $\Delta=(\alpha\chi+\beta m_r)^2+4\alpha\beta i_r md_Y<0$ and $\lambda_1=\mu+i\nu$, $\lambda_2=\mu-i\nu$, $\nu\neq 0$ are imaginary roots of the equation: $\lambda^2+(\alpha\chi-\beta m_r)\lambda-\alpha\beta(\chi m_r+i_r md_Y)=0$:

$$(51)\quad \begin{cases} Y=\left(Y_0-\dfrac{m_rE+i_rM}{\chi m_r+i_r md_Y}\right)e^{\mu t}\cos\nu t+ \\ \dfrac{1}{\nu}\left(\alpha i_r r_0-\dfrac{\beta m_r+\alpha\chi}{2}Y_0+\dfrac{(\beta m_r+\alpha\chi)(i_rM+m_rE)+2\alpha i_r(-\chi M+md_YE)}{2(\chi m_r+i_r md_Y)}\right)e^{\mu t}\sin\nu t+ \\ \dfrac{i_rM+m_rE}{\chi m_r+i_r md_Y} \\ r=\left(r_0+\dfrac{-\chi M+md_YE}{\chi m_r+i_r md_Y}\right)e^{\mu t}\cos\nu t+ \\ \dfrac{1}{\nu}\left(\beta md_Y Y_0+\dfrac{\beta m_r+\alpha\chi}{2}r_0+\dfrac{(\beta m_r+\alpha\chi)(-\chi M+md_YE)-2\beta md_Y(m_rE+i_rM)}{2(\chi m_r+i_r md_Y)}\right)e^{\mu t}\sin\nu t+ \\ \dfrac{\chi M-md_YE}{\chi m_r+i_r md_Y} \end{cases}$$

and:

$$\begin{cases} \tilde{Y}=\dfrac{m_rE+i_rM}{\chi m_r+i_r md_Y} \\ \tilde{r}=\dfrac{\chi M-md_YE}{\chi m_r+i_r md_Y} \end{cases}$$

It is observed in the three cases above that the limit $\tilde{Y}$ of the output is just Y^* and those of interest rate $\tilde{r}$ is r^*.

5. The Analysis of the Romanian Economy

In this section we will apply the theoretical model outlined above for the Romanian economy. The data taken into account shall relate to the period 2001-2012 for the simple reason that to the year 2000, the economy went through a string "forever" restructuring and remodeling.

In order to correlate the real data collected from Romanian official sources (Romanian Statistical Yearbook, Monthly Statistical Bulletins of NSI and NBR) or international (World Bank), we first determine the cumulative deflator and inflation factor relative to a reference period, such as 2000.

Considering the GDP's deflator corresponding to the year "n": $GDP_{deflator,n}=\frac{GDP\ nominal_n}{GDP\ real_n}$, we will compute the cumulative deflator relative to 2000, by the formula:

$$GDP_{cumulative\ deflator,n}=\frac{GDP_{cumulativedeflator,n-1}}{GDP_{deflator,n}}=\frac{1}{\prod_{k=1}^{n}GDP_{deflator,k}}$$

where $GDP_{deflator,2000}=1$.

The obtained date is shown in Table 1.

Table 1. The Determination of Cumulative Deflator of GDP

Year (n)	Deflator GDP ($GDP_{deflator,n}$)	Cumulative deflator ($GDP_{deflator\ cumulative,n}$)
2000	-	1
2001	1.374	0.727802038
2002	1.234	0.589790954
2003	1.24	0.475637867
2004	1.15	0.413598145
2005	1.123	0.368297547
2006	1.108	0.332398508
2007	1.13	0.294157971
2008	1.116	0.263582412
2009	1.065	0.247495222
2010	1.036	0.238895002
2011	1.071	0.223057892
2012	1.052	0.212032217

Source: World Bank

Considering, also, the consumer price index: CPI for the year "n": CPI_n, $\pi_n=CPI_{n-1}-1$ – inflation, we will compute the cumulative consumer price index $CPI_{cumulative}$, relative to the reference period 2000 by the formula:

$$CPI_{cumulative,n}=\frac{CPI_{cumulative,n-1}}{CPI_n}=\frac{1}{\prod_{k=1}^{n}CPI_k}$$

where $CPI_{2000}=1$ we obtain:

Table 2. The Determination of Cumulative Inflation

Year (n)	The Consumer Price Index ($CPI_n=1+\pi_n$)	Inflation factor (π_n)	The cumulative Consumer Price Index ($CPI_{cumulative,n}$)
2000	-	-	1
2001	1.345	0.345	0.743494424
2002	1.225	0.225	0.606934224
2003	1.153	0.153	0.526395684
2004	1.119	0.119	0.470416161
2005	1.09	0.09	0.431574459
2006	1.065	0.065	0.405234234
2007	1.0484	0.0484	0.386526358
2008	1.0785	0.0785	0.358392544
2009	1.0559	0.0559	0.33941902
2010	1.0609	0.0609	0.31993498
2011	1.0579	0.0579	0.302424596
2012	1.0333	0.0333	0.292678405

Source: INSSE

5.1The Determination of the Linear Regression $C=c_VV+C_0$

During 2001-2012, the final individual consumption of households (C) and the disposable income (V) had the following values:

Table 3. The Actual Final Consumption of Households and the Disposable Income in the Period 2001-2012

Year	Actual final consumption of households (mil. current) C	Actual final consumption of households (mil. lei-2000) C	National disposable income (mil. current) V	National disposable income (mil. lei-2000) V
2001	92177.3	67086.83	102486.7	74590.0
2002	116895.7	68944.03	132454.7	78120.6
2003	149395.8	71058.30	167428.1	79635.1

2004	191499.0	79203.63	204571.9	84610.6
2005	226928.7	83577.28	243518.1	89687.1
2006	268441.3	89229.49	269977.5	89740.1
2007	313223.3	92137.13	326148.3	95939.1
2008	381108.1	100453.40	430582.8	113494.1
2009	362749.9	89778.87	417915.8	103432.2
2010	382446.2	91364.49	439887.3	105086.9
2011	401336.8	89521.34	456694.8	101869.4
2012	418716.6	88781.41	478353.2	101426.3

Source: INSSE

The corresponding regression analysis of data in Table 3 (in million-2000) provides the following results:

- The empirical correlation coefficient is ρ=0.93971118. The critical value of the correlation coefficient rc for 12 values of exogenous variable is 0.576 for a significance level of more than 0.95, so how $|\rho|$>rc a linear dependence between variables may exist.
- The R Square=0.8831 means that 88.31% of the total variation of consumption variable is explained by the variation of the national disposable income, the remaining 11.69% being due to other factors.
- The Fisher-Snedecor statistics F allows the analysis of the null hypothesis H_0 which states that all regression coefficients are equal to 0. Computing $F_{\alpha,k,N-(k+1)}$ where α=0.05, k=1 (the number of degrees of freedom corresponding regression (explanatory factor), N-(k+1)=10 (the number of degrees of freedom corresponding to residual factor (unregistered factors), if $F \leq F_{\alpha,k,N-(k+1)}$ then the null hypothesis H_0 with probability 1-α will be rejected, that is at least one of the coefficients can be nonzero. If $F > F_{\alpha,\ k,\ N-(k+1)}$ then the null hypothesis H_0 states that all coefficients are null, the regression being not valid. In this case, F=75.511819 and $F_{0.05;\ 1;\ 10}$=0.004134. Therefore, the null hypothesis H_0 is rejected with probability 0.95.

- Significance F value represents the probability that the regression equation cannot explain the evolution of the endogenous variable (links coincidental phenomenon). If Significance $F<\alpha$ then the null hypothesis H_0 is rejected with probability $1-\alpha$, so it is possible that at least one coefficient be different from 0. In the present model we have Significance $F=5.66615\cdot10^{-6}<0.05$ so the null hypothesis H_0 is rejected with probability 0.95.
- Relative to the values P-value, if one value is less than α then the variable significantly influences the process. In this case: P-value(C_0)=0.288568 and P-value(c_V)=$5.666151\cdot10^{-6}$ so both autonomous consumption of households and national disposable income affects household final consumption.
- The intervals [Lower 71%, Upper 71%] are the confidence intervals in which belong the coefficients. If 0 belongs to the range then do not reject the null hypothesis relative to the coefficient, so the variable is further removed from the model. In the case of our regression, $C_0\in[30.45076;19357.90806]$ and $c_V\in[0.69769,0.90357]$ so, besides the rejected of null hypothesis, it can be stated that the values of C_0 and c_V with a higher probability of 0.71 belong in the respective intervals.

Therefore, after the regression analysis, we obtain that the average of the household final consumption dependence of national disposable income is:

(52) $C=0.80063\cdot V+9694.17941$

where:

(53) $c_V=0.80063$

(54) $C_0=9694.17941$

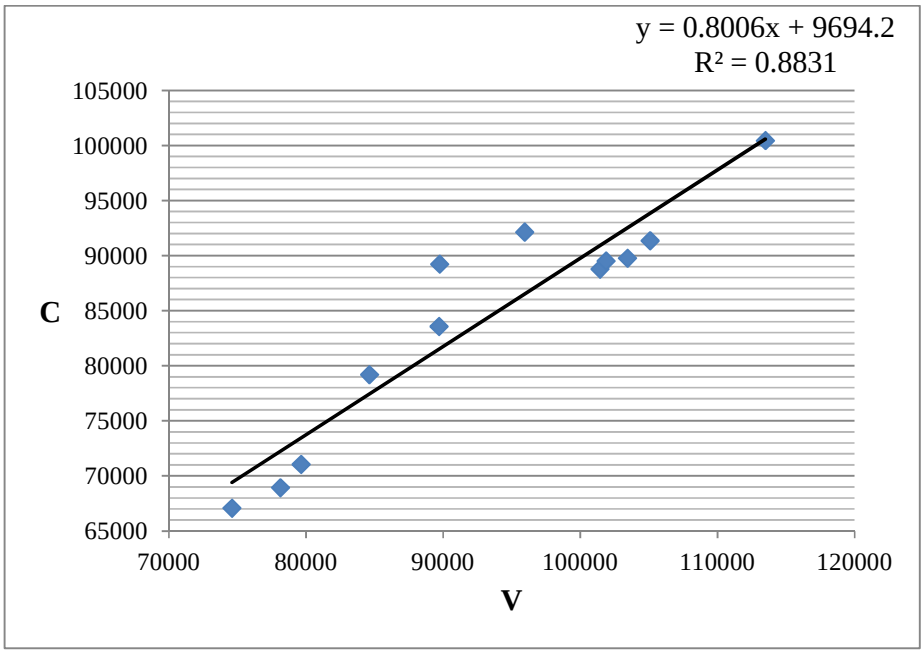


Figure 1. The dependence of household final consumption by the national disposable income during 2001-2012

From equation (52) we obtain that at an increase of 1000 million lei-2000 of the disposable income, the final consumption of households will increase, on average, with 800.63 million-2000 (in terms of autonomous consumption household constant).

5.2. The Determination of the Linear Regression $G=g_Y Y$

During the period 2001-2012, the collective final consumption of general government (G) records the following values:

Table 4. The Collective Final Consumption of General Government during 2001-2012

Year	The collective final consumption of general government (mil. current) G	The collective final consumption of general government (mil. lei-2000) G
2001	8554.4	6225.9098
2002	10223.1	6029.4919
2003	19422.9	9238.2667
2004	19555.6	8088.1599
2005	24109.4	8879.4329
2006	26426.3	8784.0627
2007	31713.7	9328.8377
2008	39809.4	10493.0577
2009	43873.4	10858.4569
2010	37355.0	8923.9228
2011	35148.2	7840.08340
2012	39869.2	8453.5549

Source: INSSE

Also in the same period, the Gross Domestic Product (denoted in the model with Y) has the following values:

Table 5. Gross Domestic Product during 2001-2012

Year	GDP (mil. current) Y	GDP (mil. lei-2000) Y
2001	117945.8	85841.1936
2002	152017.0	89658.2515
2003	197427.6	93904.0425
2004	247368.0	102310.9459
2005	288954.6	106421.2703
2006	344650.6	114561.3451
2007	416006.8	122371.7164
2008	514700.0	135665.8673
2009	501139.4	124029.6072
2010	523693.3	125107.7120
2011	556708.4	124178.2021
2012	587466.2	124561.7606

Source: INSSE

The corresponding regression analysis of data from tables 4 and 5 (in million-2000) provides the following results:

- The empirical correlation coefficient is $\rho=0.993465047>0.576$ for a significance level of more than 0.95, so that the linear dependence between variables may exist.
- The R Square=0.4836 means that 48.36% of the total variance of the variable collective final consumption of government is explained by the variation in GDP, the remaining 51.64% being due to other factors.
- The Fisher-Snedecor statistics $F=833.3871101$ and $F_{0.05;1;11}=0.004116$, so how $F> F_{0.05;1;11}$ then the null hypothesis H_0 will be rejected with probability 0.95, so the coefficient g_Y can be nonzero.
- Significance $F=5.79592\cdot10^{-11}<0.05$ therefore it is possible that the value g_Y be different from 0.
- P-value$(g_Y)=1.01269\cdot10^{-11}$ so the GDP affects the collective final consumption of government.
- Studying the interval [Lower95%,Upper95%] we have that $g_Y\in$ [0.070413161,0.082036189] with a probability greater than 0.95.

After the regression analysis, we obtain that, on average, the collective final consumption of government dependence from GDP is:

(55) $G=0.076224675\cdot Y$

where:

(56) $g_Y=0.076224675$

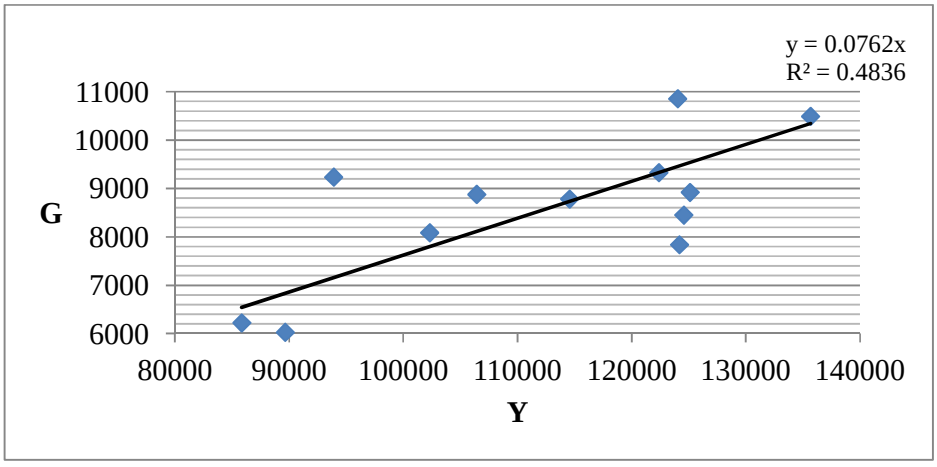


Figure 2. The dependence of the collective final consumption of government from GDP (mil. lei-2000)

From equation (55) we obtain that at an increase of 1000 million lei-2000 of GDP, the general government final consumption will increase by an average of 76.2 million-2000.

5.3 The Determination of the Linear Regression $I=in_Y Y+i_r r$

Given the existence of significant inflation, we first calculate the real interest rate (r_n) through the formula $r_n=\frac{rd_n-\pi_n}{1+\pi_n}$ where rd_n is the nominal interest rate.

Table 6. The Nominal and Real Interest Rates

Year (n)	The nominal interest rate (rd_n)	The real interest rate (r_n)
2001	0.3880	0.03197
2002	0.2847	0.04873
2003	0.1884	0.03070
2004	0.2027	0.07480
2005	0.0959	0.00541
2006	0.0844	0.01822
2007	0.0746	0.02499
2008	0.0946	0.01493
2009	0.0933	0.03542
2010	0.0667	0.00547
2011	0.0625	0.00435
2012	0.0531	0.01916

Source: INSSE

During 2001-2012, investments (I) have the following values:

Table 7. Investments during 2001-2012

Year	Investments (mil. current) I	Investments (mil. lei-2000) I
2001	26186.20	19058.37
2002	33446.10	19726.21
2003	43370.20	20628.51
2004	58551.40	24216.75
2005	67286.60	24781.49
2006	91188.30	30310.85
2007	128858.70	37904.81
2008	160896.90	42409.59
2009	127137.40	31465.90
2010	133898.60	31987.71
2011	149909.40	33438.47
2012	158727.80	33655.41

Source: INSSE

The corresponding regression analysis of data from tables 5, 6 and 7 (in million-2000) provides the following results:

- The empirical correlation coefficient is $\rho=0.994753707>0.576$ for a significance level of more than 0.95, so that the linear dependence between variables may exist.
- The Fisher-Snedecor statistics $F=472.7802825$ and $F_{0.05;2;10}=0.051557$, so how $F> F_{0.05;2;10}$ then the null hypothesis H_0 will be rejected with probability 0.95, so at least one coefficient can be nonzero.
- Significance $F=7.67318\cdot10^{-10}<0.05$ therefore it is possible that the values in_Y and i_r be different from 0.
- P-value(in_Y)$=1.05276\cdot10^{-9}$ and P-value(i_r)$=0.134714194$ means that the level of GDP and the real interest rate influences the level of investment with a degree of confidence over 86 %.
- Studying the interval [Lower86%, Upper86%] we have that $in_Y\in[0.257663378,0.299164024]$ and $i_r\in[-145195.7337,-1092.50004]$ with a probability greater than 0.86.

After the regression analysis, we obtain that, on average, the investment dependence from GDP and the real interest rate is:

(57) $I=0.278413701 \cdot Y-73144.11685 \cdot r$

where:

(58) $in_Y=0.278413701$

(59) $i_r=-73144.11685$

From equation (57) we obtain that an increase of 1000 million lei-2000 of GDP, given in the conditions of a constant real interest rate, investments will grow, on average, by 278.4 million lei- 2000. Also, in terms of GDP constant, an increase in the real interest rate by 0.01 will generate a decrease in investments of 731.4 million lei-2000.

5.4. The Determination of the Linear Regression $NX=\nu_Y Y$

During 2001-2012, Net Exports (NX) have record the following values:

Table 8. Net Exports of Romania during 2001-2012

Year	Net Exports (mil. current) NX	Net Exports (mil. lei-2000) NX
2001	-8972.10	-6529.9
2002	-8547.90	-5041.5
2003	-14761.30	-7021.0
2004	-22238.00	-9197.6
2005	-29370.10	-10816.9
2006	-41405.30	-13763.1
2007	-57788.90	-16999.1
2008	-67114.40	-17690.2
2009	-30273.50	-7492.5
2010	-30006.50	-7168.4
2011	-29686.00	-6621.7
2012	-29847.40	-6328.6

Source: INSSE

The corresponding regression analysis of data from tables 5 and 8 (in million-2000) provides the following results:

- The empirical correlation coefficient is $\rho=0.934871694>0.576$ for a significance level of more than 0.95, so that the linear dependence between variables may exist.

- The R Square=0.2034 means that only 20.34% of the total variance of the variable Net Exports is explained by the variation in GDP, the remaining 79.66% being due to other factors.
- The Fisher-Snedecor statistics F=76.29125409 and $F_{0.05;1;11}$=0.004116, so how F> $F_{0.05;1;11}$ then the null hypothesis H_0 will be rejected with probability 0.95, so the coefficient ν_Y can be nonzero.
- Significance F=$5.41218 \cdot 10^{-6}$<0.05 therefore it is possible that the value ν_Y be different from 0.
- P-value(ν_Y)=$2.80703 \cdot 10^{-6}$ so GDP affects Net Exports.
- Studying the interval [Lower95%, Upper95%] we have that $\nu_Y \in$[-0.107436875,-0.06418918] with a probability greater than 0.95.

After the regression analysis, we obtain that, on average, the Net Exports dependence from GDP is:

(60) NX=-0.085813028·Y

where:

(61) ν_Y=-0.085813028

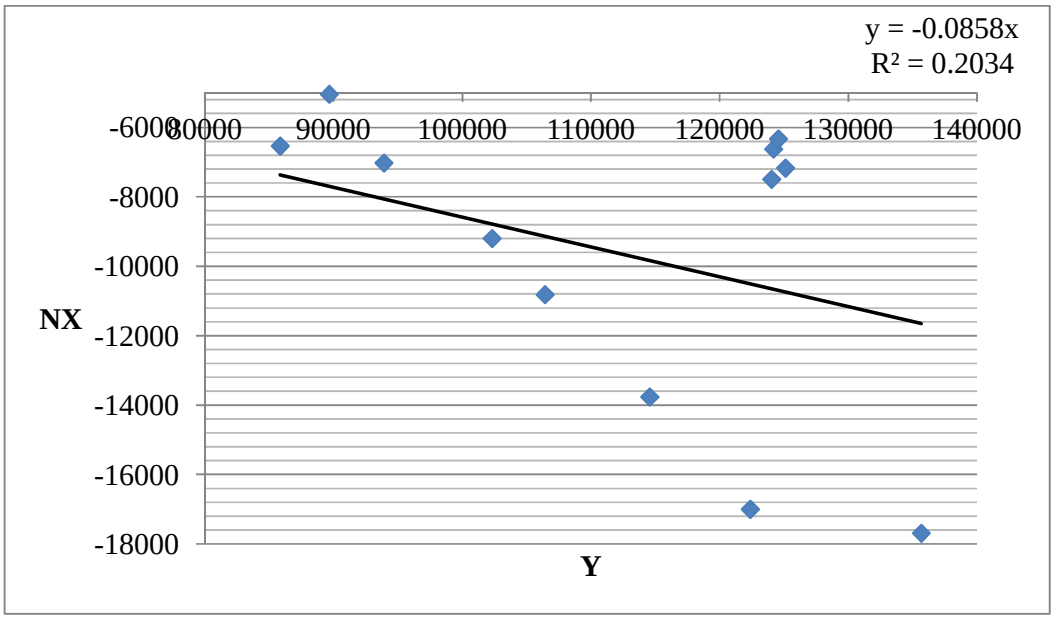


Figure 3. Net Exports dependence from GDP (mil. lei-2000)

From equation (60) we obtain that at an increase of 1000 million lei-2000 of GDP, Net Exports will decrease by an average of 85.8 million-2000.

5.5. The Determination of the Linear Regression $TR=\theta_Y Y$

During 2001-2012, government transfers (TR) have recorded the following values:

Table 9. Government Transfers of Romania during 2001-2012

Year	Government transfers (mil. current) TR	Government transfers (mil. lei-2000) TR
2001	-891.9	-649.1
2002	-1602.8	-945.3
2003	-4571.3	-2174.3
2004	-10366.2	-4287.4
2005	-8490.8	-3127.1
2006	-11536.1	-3834.6
2007	-14925.4	-4390.4
2008	8362.4	2204.2
2009	1660.9	411.1
2010	7041.7	1682.2
2011	4673.4	1042.4
2012	4931.6	1045.7

Source: INSSE

Because after a period of negative transfers (2001-2007) follows a reversal of direction caused by the entry of Romania into the European Union and labor migration to more economically developed countries, we perform regression analysis only on the period 2008-2012, government transfers marginal rate thus being determined much closer to the current trend.

The regression analysis for the period 2008-2012 (tables 5 and 9) provides the following results:

- The empirical correlation coefficient is $\rho=0.913128008>0.878$ (corresponding to a total of 5 values of exogenous variable) for a significance level of more than 0.95, so that the linear dependence between variables may exist.
- The R Square value=0.1137 means that only 11.37% of the total variance of the government transfers is explained by the variation in GDP, the remaining 88.63% being due to other factors.

- The Fisher-Snedecor statistics F=20.06778828 and $F_{0.05;1;4}$=0.004453, so how F> $F_{0.05;1;4}$ then the null hypothesis H_0 will be rejected with probability 0.95, so the coefficient θ_Y can be nonzero.
- Significance F=0.020740723<0.05 therefore it is possible that the value θ_Y be different from 0.
- P-value(θ_Y)=0.010992314 so GDP affects government transfers.
- Studying the interval [Lower95%, Upper95%] we have that $\theta_Y \in$ [0.003879799,0.016528517] with a probability greater than 0.95.

Therefore after the regression analysis, we obtain that, on average, the dependence of government transfers on GDP is:

(62) $TR=0.010204158 \cdot Y$

where:

(63) $\theta_Y=0.010204158$

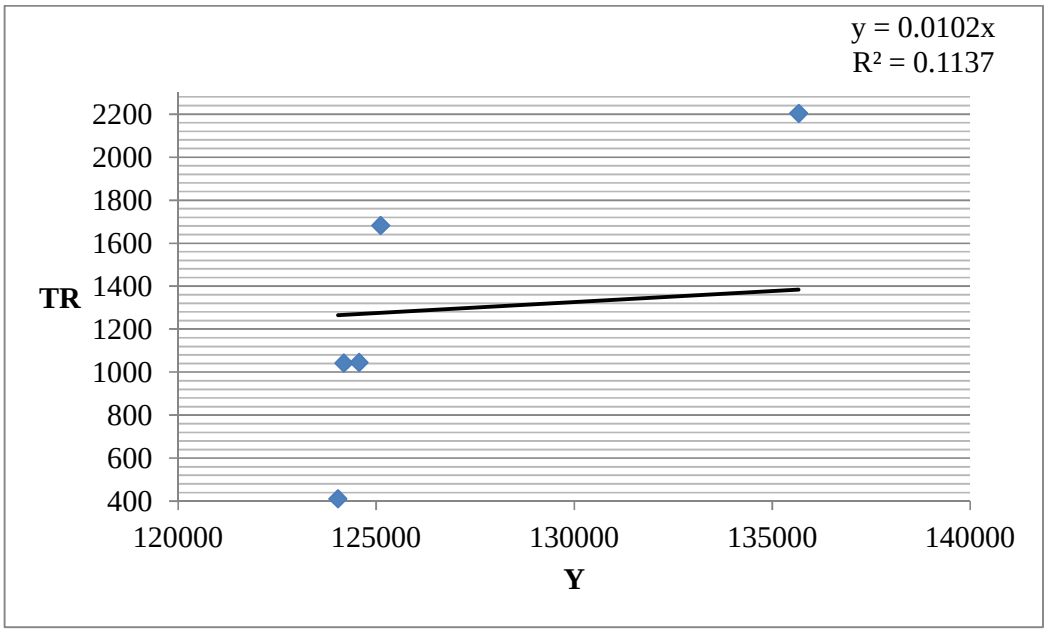


Figure 4. Government transfers dependence from GDP (mil. lei-2000)

From equation (62) we obtain that at an increase of 1000 million lei-2000 of GDP, the government transfers will increase, on average, by 10.2 million lei-2000.

5.6. The Determination of the Linear Regression $TI=ri_Y Y+T_0$

During 2001-2012, the level of taxes (TI) has the following values:

Table 10. Taxes during 2001-2012

Year	Taxes (mil. current) TI	Taxes (mil. lei-2000) TI
2001	14567.2	10602.1
2002	17959.5	10592.4
2003	25428.2	12094.6
2004	32429.9	13412.9
2005	36945.7	13607.0
2006	63137.0	20986.6
2007	74933.1	22042.2
2008	92479.6	24376.0
2009	84884.5	21008.5
2010	90847.7	21703.1
2011	104687.0	23351.3
2012	114044.6	24181.1

Source: INSSE

The corresponding regression analysis of data from tables 5 and 10 (in million-2000) provides the following results:

- The empirical correlation coefficient is $\rho=0.963749>0.576$ for a significance level of more than 0.95, so that the linear dependence between variables may exist.
- The R Square value=0.9288 means that 92.88% of the total variance of the taxes is explained by the variation in GDP, the remaining 7.12% being due to other factors.
- The Fisher-Snedecor statistics F=130.473850 and $F_{0.05;1;10}=0.004134$, so how $F> F_{0.05;1;10}$ then the null hypothesis H_0 will be rejected with probability 0.95, so at least one of the coefficients can be nonzero.
- Significance $F=4.6389661\cdot10^{-7}<0.05$ therefore it is possible that at least one coefficient to be different from 0 with a probability greater than 0.95.

- P-value(θ_Y)=0.010992314 so GDP affects government transfers.
- Studying the interval [Lower95%, Upper95%] we have that $\theta_Y \in$ [0.003879799,0.016528517] with a probability greater than 0.95.
- P-value(T_0)=0.00018686 and P-value(ri_Y)=$4.63896615 \cdot 10^{-7}$, so both independent of income taxes and GDP influence (with a higher probability than 0.95) the collection of taxes.
- Intervals [Lower95%, Upper95%] are: $T_0 \in$ [-25992.56186,-11461.97351], $ri_Y \in$ [0.39228,0.27617].

Following regression analysis, we obtain that, on average, the GDP dependence of taxes is:

(64) TI=0.32825·Y-18727.26768

where:

(65) ri_Y=0.32825

(66) T_0=-18727.26768

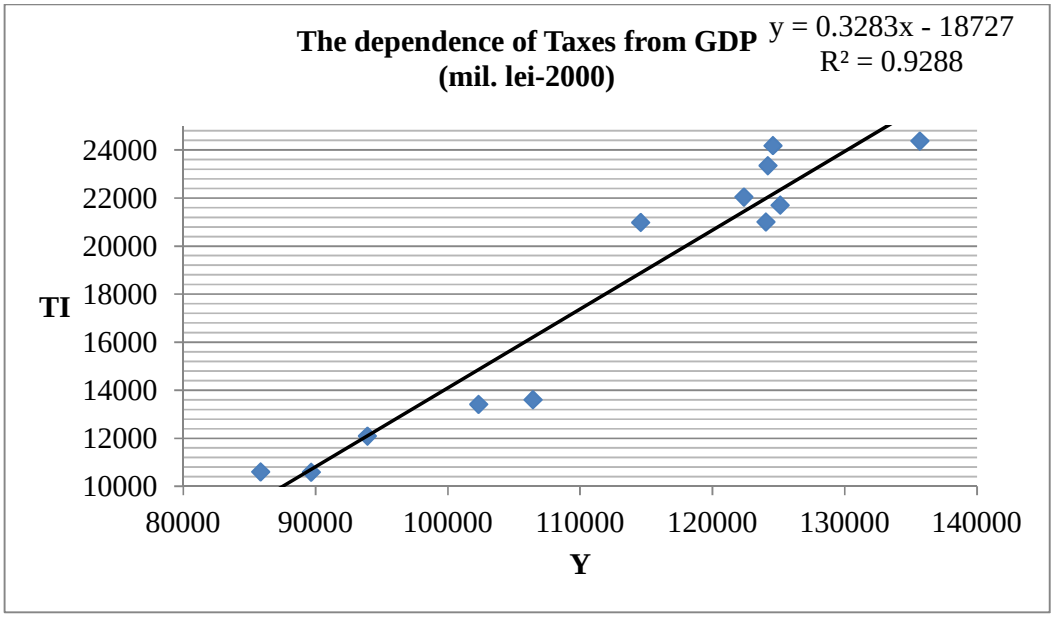


Figure 5.

From equation (64) we obtain that at an increase of 1000 million lei-2000 of GDP, taxes will increase, on average, with 328.25 million-2000 (in the hypothesis of independent of income taxes constancy).

5.7. The Determination of the Linear Regression $MD=md_Y Y+m_r r$

During 2001-2012, the demand for currency in the Romanian economy (MD) had the following values:

Table 11. The Money Demand in the Period 2001-2012

Year	The money demand (mil. current) MD	The money demand (mil. lei-2000) MD
2001	4643.90	3379.8
2002	6547.09	3861.4
2003	9209.40	4380.3
2004	12700.50	5252.9
2005	27633.77	10177.4
2006	39275.04	13055.0
2007	62200.55	18296.8
2008	87864.34	23159.5
2009	81441.49	20156.4
2010	78946.89	18860.0
2011	81308.22	18136.4
2012	87601.43	18574.3

Source: INSSE

The corresponding regression analysis of data from Tables 5, 6 and 11 (in million-2000) provides the following results:

- The empirical correlation coefficient is $\rho=0.967385979>0.576$ for a significance level of more than 0.95, so that the linear dependence between variables may exist.
- The Fisher-Snedecor statistics $F=72.92486928$ and $F_{0.05;2;10}=0.051557$, so how $F> F_{0.05;2;10}$ then the null hypothesis H_0 will be rejected with probability 0.95, so at least one coefficient can be nonzero.
- Significance $F=2$, $75102\cdot10^{-6}<0.05$ therefore it is possible that the values md_Y and m_r be different from 0.

- P-value(md_Y)=$1.60695 \cdot 10^{-6}$ and P-value(m_r)=0.013059076 means that the level of GDP and the real interest rate influences the level of money demand with a degree of confidence over 95%.
- Studying the interval [Lower95%, Upper95%] we have that $md_Y \in$ [0.123318438,0.194135677], $m_r \in$ [-289187.5838,-43287.96074] with a probability greater than 0.95.

After the regression analysis, we obtain that, on average, the money demand from GDP and the real interest rate is:

(67) $MD=0.158727057 \cdot Y-166237.7723 \cdot r$

where:

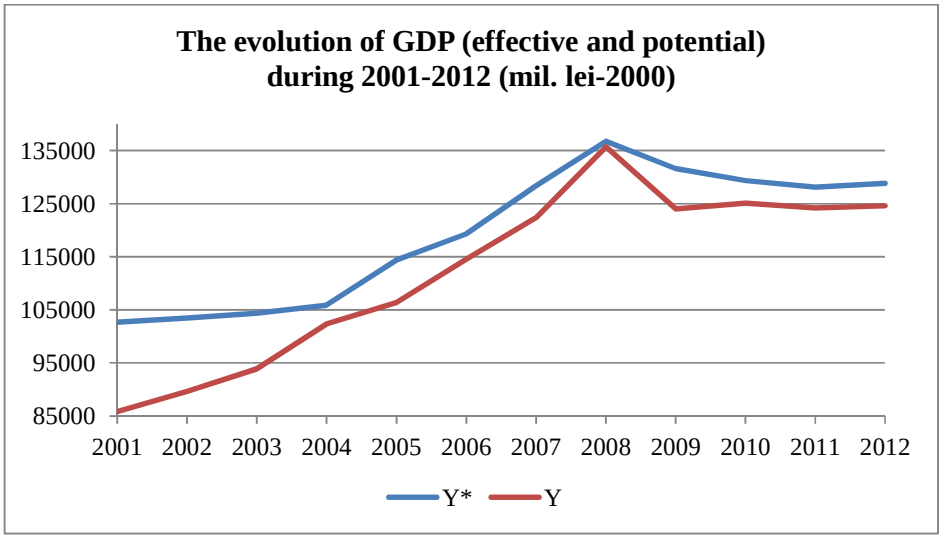
(68) $md_Y=0.158727057$

(69) $m_r=-166237.7723$

From equation (67) we obtain that at an increase of 1000 million lei-2000 of GDP, given a constant real interest rate, the demand for money will increase, on average, with 158.7 million-2000. Also, in terms of GDP constant, an increase in the real interest rate by 0.01 will generate a decrease in demand for currency by 1662.4 million lei-2000.

5.8. The Determination of Static Equilibrium during 2001-2012

In previous sections, we saw that model parameters were determined in the linear regressions with one or two variables based on the dynamics of the main economic indicators in the period 2001-2012. Due to high levels of correlation coefficients, we can consider constant parameter values so determined. However in the analyzed period, money has where gone considerable fluctuations from year to year (with extremes -12.97% - 2009 and -93.75% - 2005) with an annual average of 19.59%. As a result of this situation, we will determine the static equilibrium values of the output, the real interest rate and other economic indicators, comparing them with the actual values recorded, all calculations being performed for comparability in the currency of 2000.



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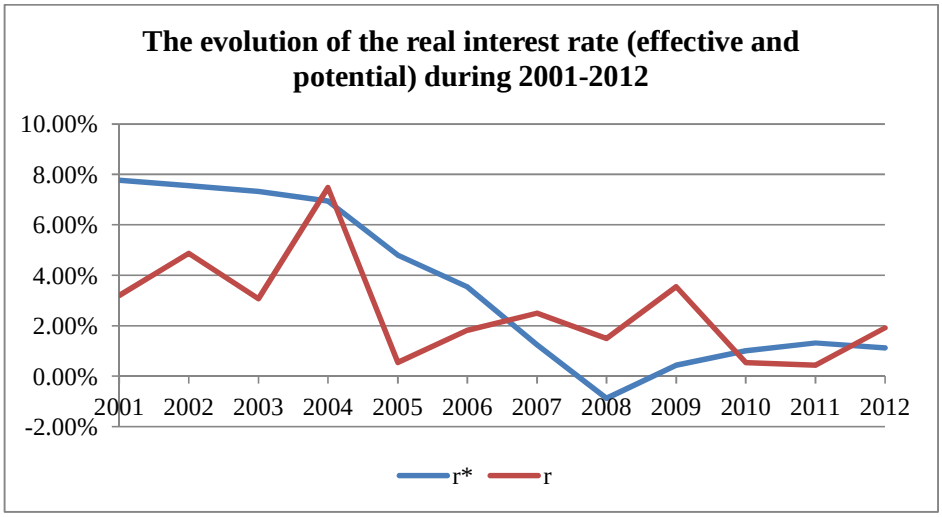

Figure 7.

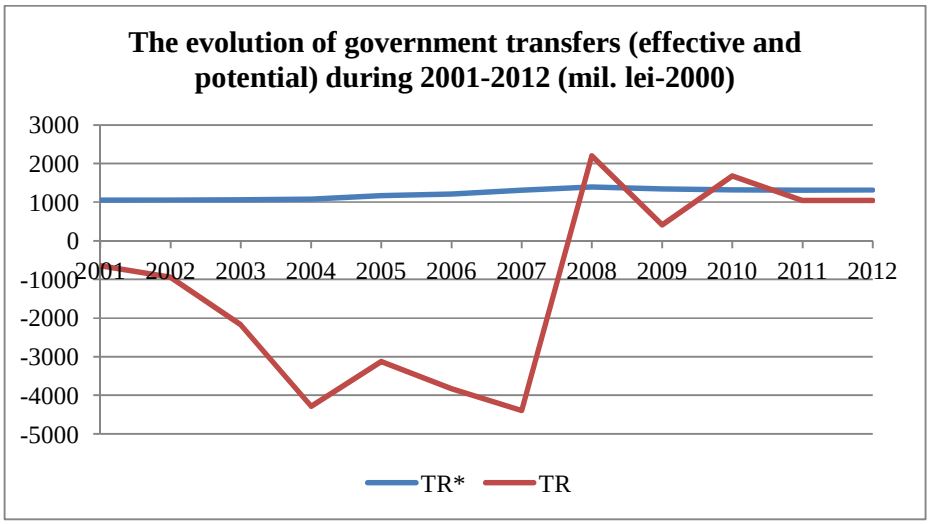

Figure 8.

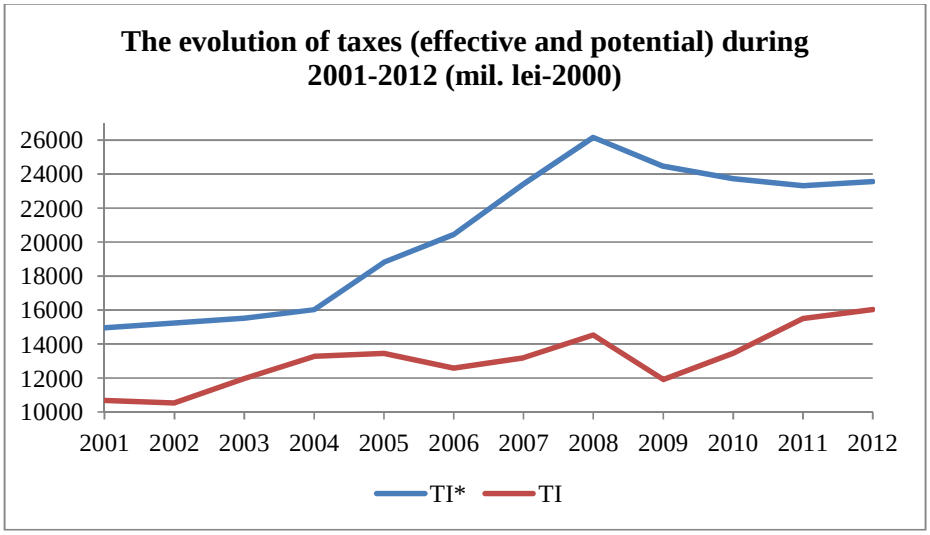

Figure 9.

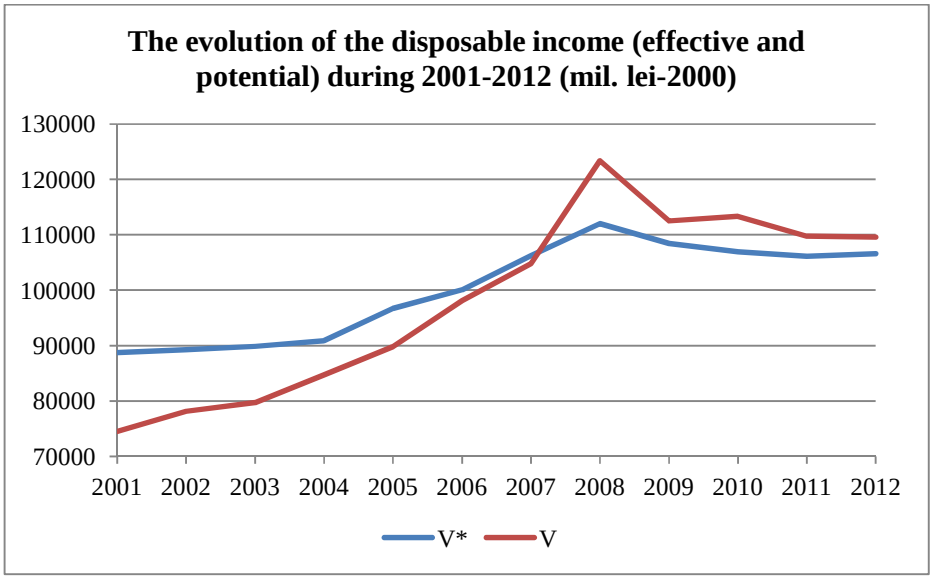

Figure 10.

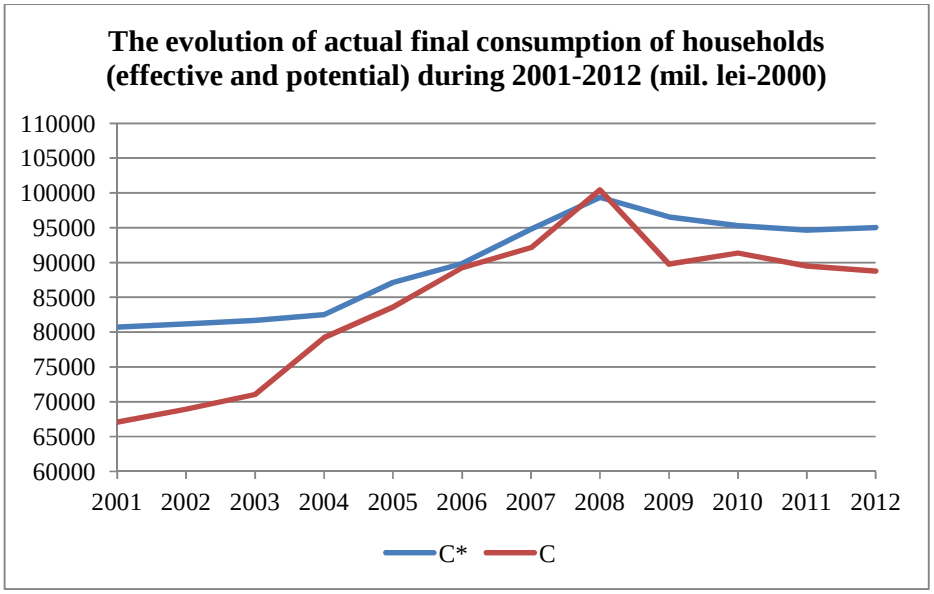

Figure 11.

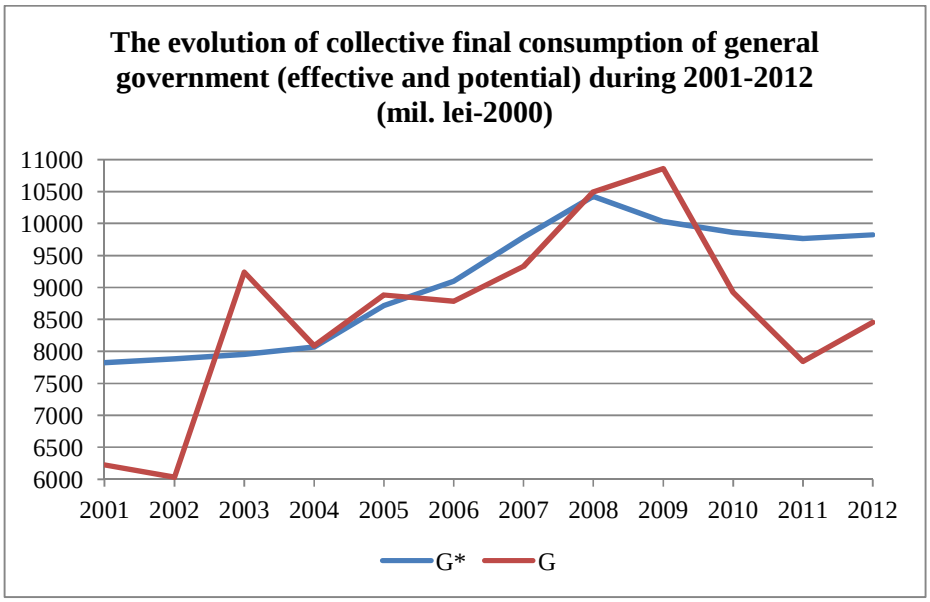

Figure 12.

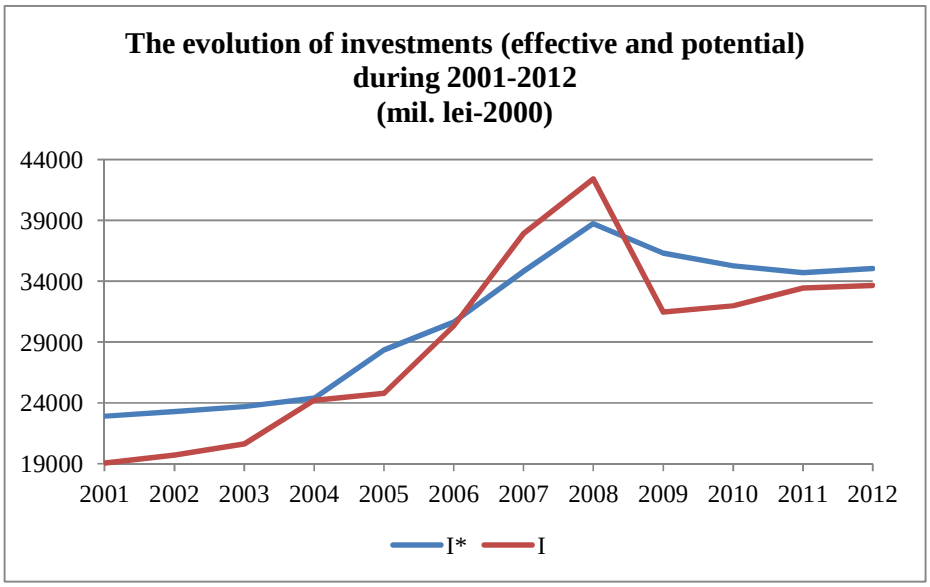

Figure 13.

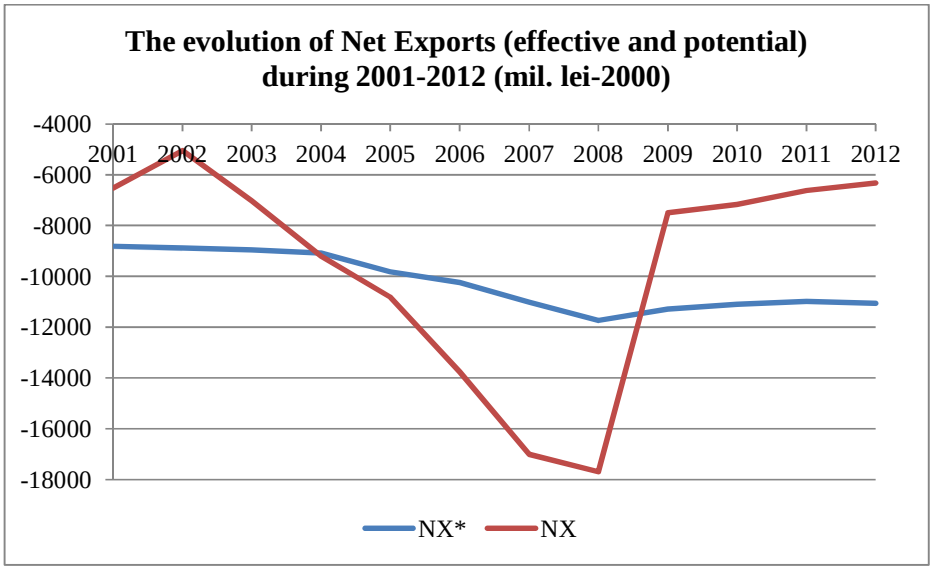

Figure 14.

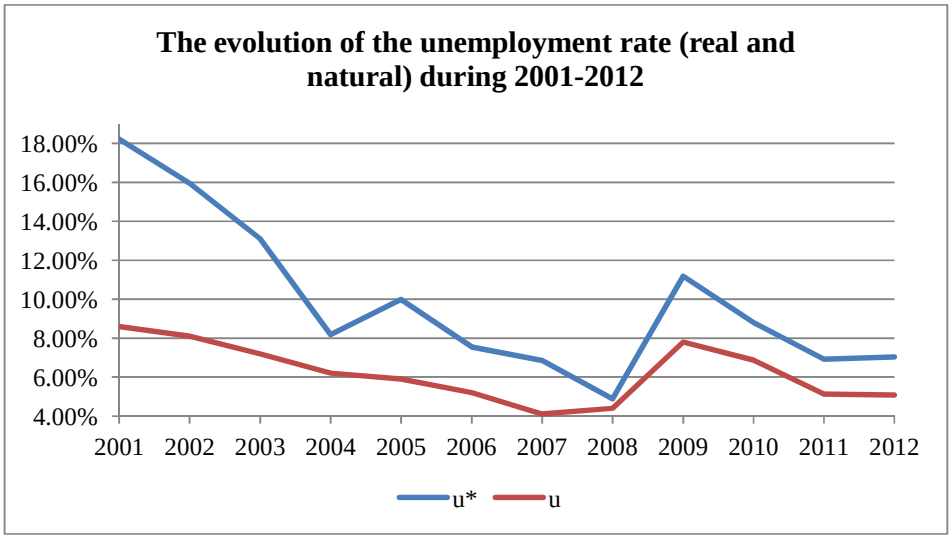

Figure 15.

We will divide the length of this period in three equal intervals, corresponding to enhance economic stages, expansion and the crisis. Coincidentally or not, these periods coincide with election cycles that have distinct developmental strategies, unfortunately less in line with the requirements of economic theory.

1. Period 2001-2004

Table 12. Main Economic Indicators of Romania in 2001-2004

Year/ Indicator	2001			2002		
	effective	potential	effective/ potential	effective	potential	effective/ potential
Y	85841.1936	102636.4617	83.64%	89658.25153	103467.3315	86.65%
r	3.20%	7.77%	41.16%	4.87%	7.56%	64.49%
TR	-649.1266376	1047.318658	-61.98%	-945.3169419	1055.796985	-89.54%
TI	10687.88537	14963.38707	71.43%	10536.6154	15236.12202	69.16%
V	74504.18159	88720.39325	83.98%	78176.31919	89287.0065	87.56%
C	67086.82678	80726.13279	83.10%	68944.02648	81179.77873	84.93%
G	6225.909753	7823.430952	79.58%	6029.491907	7886.763738	76.45%
I	19058.36972	22894.44344	83.24%	19726.20724	23279.63404	84.74%
NX	-6529.912664	-8807.545515	74.14%	-5041.4741	-8878.844975	56.78%
u	8.60%	18.23%	47.19%	8.10%	15.95%	50.78%
M	3379.8			3861.4		
Year/ Indicator	**2003**			**2004**		
	effective	potential	effective/ potential	effective	potential	effective/ potential
Y	93904.04246	104362.6415	89.98%	102310.9459	105868.0835	96.64%
r	3.07%	7.33%	41.89%	7.48%	6.95%	107.65%
TR	-2174.283379	1064.932869	-204.17%	-4287.441089	1080.294637	-396.88%
TI	11978.70185	15530.00956	77.13%	13279.437	16024.17438	82.87%
V	79751.05723	89897.56477	88.71%	84744.0678	90924.20378	93.20%
C	71058.29958	81668.60824	87.01%	79203.63113	82490.56327	96.02%
G	9238.266718	7955.008447	116.13%	8088.159881	8069.760278	100.23%
I	20628.5094	23694.69901	87.06%	24216.75042	24392.62074	99.28%
NX	-7021.033239	-8955.674231	78.40%	-9197.595544	-9084.860771	101.24%
u	7.20%	13.09%	54.98%	6.20%	8.18%	75.83%
M	4380.3			5252.9		

The first remark, after the analysis of the Table 12, is that in the period of economic consolidation, the effective GDP was approach continuously to the potential, from 83.64% in 2001 to 96.64% in 2004.

In the period 2001-2003, the real interest rate was much below potential, with differences of 3-4%, confirming the procyclical policies reported in the previous chapter when the macroeconomic analysis was based on official statistics. The negative gap between real interest rate and that potential, led to the beginning of the Romanian economy overheating that boosted the next period (2005-2008). Linked to this negative phenomenon, the final consumption of households increased much exaggerated, reaching a potential relationship to 83.1% in 2001 and to 96.02% in 2004.

Tax revenue (taxes) is not close to the potential threshold, equally evolved GDP. Thus, if in 2001 the ratio $GDP_{effective}/GDP_{potential}$ was 83.64%, the ratio $TI_{effective}/TI_{potential}$ was only 71.43%, while the end of the period in which $GDP_{effective}/GDP_{potential}$=96.64% the ratio $TI_{effective}/TI_{potential}$ was 82.87%, the gap between the two ratios increasing. These differences can be explained either by inefficient collection system at national level or on account of tax evasion growing.

Relative to government spending stands, at first sight, a paradoxical fact. If in the first two years they were placed at odds of 79.58% and 76.45% of the potential, in the last two years they have exceeded the maximum level stood at 116.13% and 100.23% of the economic balance dictates. The explanation is simple but, as we shall see in the next period, the phenomenon is characteristic of electoral timetables.

The ratio of effective investments to potential reveals a fairly close correlation relative to GDP growth (the effectiveness remaining questionable).

The effective unemployment rate in 2001-2003 was much below the natural (as defined above, relative to the potential level of GDP) stood at about half the forecast model. In 2004, the difference between the two rates has decreased noticeably (6.20% - effective rate to 8.18% - the natural rate). On the other hand, during this period, the analysis based on the National Institute of Statistics and the Ministry of Labor reveals a discrepancy between the relative dynamics of the unemployed and the employed in the economy.

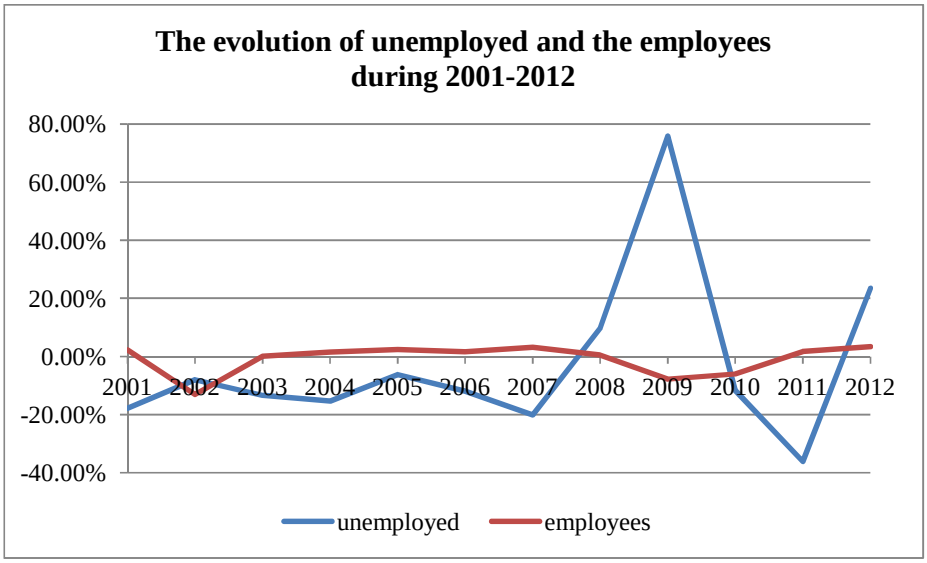

Figure 16.

Source: INSSE, Labor Ministry

In the analyzed period, the average of the relative dynamics of the number of unemployed was -13.65% (representing thus a decrease in the number of unemployed), while the average growth in the number of employees was only -2.34% (the number of jobs reducing therefore with an average 2.34% every year). The question is the absorption of the unemployed into the labor market, in the period where review has been a very big gap.

On the one hand, the rigidity and inflexibility recorded at all levels of the labor market and the high level of taxes led to a reduced employability in this period. On the other hand, a regression analysis between dynamic collection of taxes and the evolution of unemployment shows a very interesting situation. Regression equation:

$$\frac{\Delta TI}{TI} = -0.2470 \frac{\Delta PS}{PS} + 0.03916$$

where: TI – collected taxes, and PS – number of unemployed persons show an inverse dependence between tax collection and increased unemployment. At first glance, it seems a normal phenomenon, because the income from unemployment benefits being greatly reduced compared to the period of employment, the rates will decrease. From the regression equation, we note that at an increase in the number of unemployed, collecting taxes decreased by 24.7%. On the other hand, the level of taxation, in 2001-2004, showed an average of 12.54%. The difference

between these two values can have two causes: either the amplification of the phenomenon of emigration (and therefore a segment of the population goes out of the system) or amplification of "black" work, justifying also the employability gap reported above.

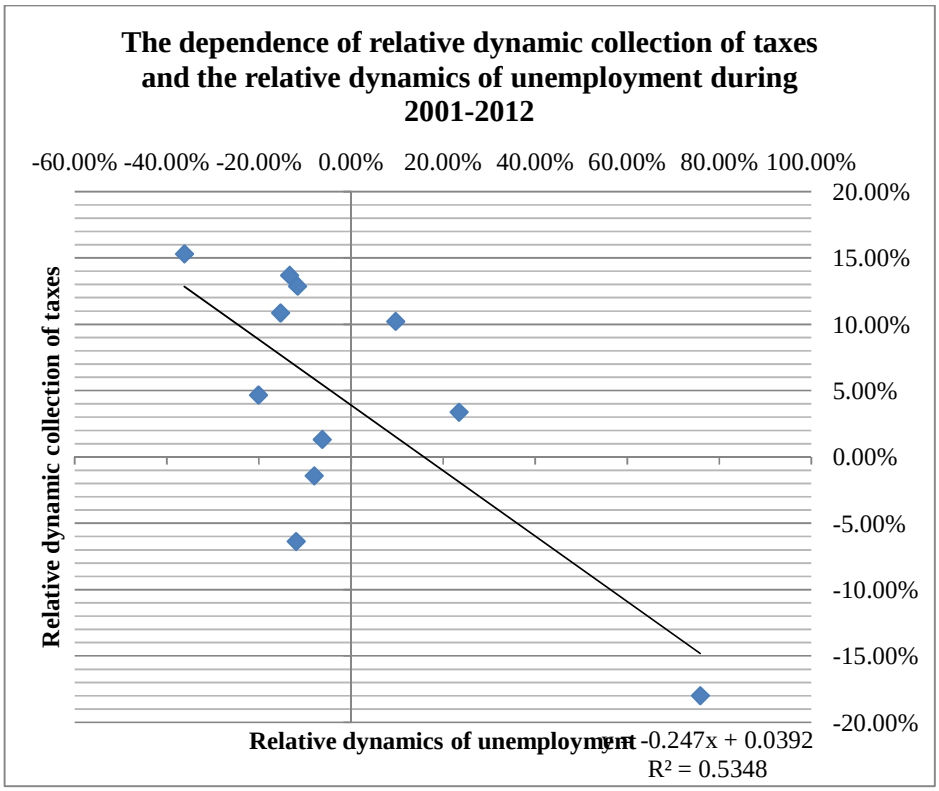

Figure 17.

2. Period 2005-2008

Table 13. Main Economic Indicators of Romania in 2005-2008

Year/ Indicator	2005			2006		
	effective	potential	effective/ potential	effective	potential	effective/ potential
Y	106421.2703	114364.448	93.05%	114561.3451	119329.0557	96.00%
r	0.54%	4.80%	11.28%	1.82%	3.54%	51.45%
TR	-3127.14081	1166.99288	-267.97%	-3834.58243	1217.65252	-314.92%
TI	13453.98304	18813.12559	71.51%	12597.96992	20442.76947	61.63%
V	89840.14641	96718.31534	92.89%	98128.79278	100103.9387	98.03%
C	83577.28346	87129.48615	95.92%	89229.48754	89840.10808	99.32%
G	8879.432869	8717.392904	101.86%	8784.062685	9095.818509	96.57%
I	24781.4897	28331.52852	87.47%	30310.85484	30633.11664	98.95%
NX	-10816.93577	-9813.959534	110.22%	-13763.05993	-10239.98755	134.41%
u	5.90%	9.99%	59.09%	5.20%	7.55%	68.87%
M	10177.4			13055.0		
Year/ Indicator	**2007**			**2008**		
	effective	potential	effective/ potential	effective	potential	effective/ potential
Y	122371.7164	128372.8227	95.33%	135665.8673	136762.4991	99.20%
r	2.50%	1.25%	199.78%	1.49%	-0.87%	-170.96%
TR	-4390.42539	1309.93655	-335.16%	2204.18156	1395.54613	157.94%
TI	13185.39574	23411.40682	56.32%	14532.24725	26165.33741	55.54%
V	104795.8953	106271.3525	98.61%	123337.8016	111992.7079	110.13%
C	92137.13054	94777.9068	97.21%	100453.3921	99358.57913	101.10%
G	9328.837659	9785.176714	95.34%	10493.05766	10424.67707	100.66%
I	37904.8138	34825.79979	108.84%	42409.59294	38715.24705	109.54%
NX	-16999.0656	-11016.06058	154.31%	-17690.17541	-11736.00411	150.73%
u	4.10%	6.85%	59.86%	4.40%	4.87%	90.32%
M	18296.8			23159.5		

The analysis of Table 13 shows that in the period of economic expansion began in 2004, the actual GDP was close to the potential from 93.05% in 2005 to 99.20% in 2008.

Contrary to economic theory and practice, providing that in periods of economic expansion the actual GDP must exceed the potential and the actual unemployment rate being below the natural rate in Romania was a paradoxical situation. If actual unemployment rate remained below the natural (5.90% versus 9.99% - 2005, 5.20% to 7.55% - 2006, 4.10% to 6.85% - 2007, 4.40% to 4.87% in 2008), the actual GDP has remained below the potential.

On the other hand, relative to the evolution of the number of employees in relation to ownership, one can speak of two distinct periods. In the first of these (2005-2006) the number of employees in the public sector fell considerably from year to year (8.23% - 2005 3.68% - 2006), while the private sector has increased staff 7.89% in 2005 and 5.32% in 2006. In the second period (2007-2008) the budget unit increased by 1.67%, while in 2008 and to decrease 0.68%. Meanwhile, growth in private units' staff maintained the trend falling to 6.01% in 2007 and 5% in 2008. On the whole employed population is observed, however, a continuing decline in the public share of 32.79% in 2005 to 28.81% in 2008.

In absolute values, it is observed that the number of employees made redundant in the public sector over this period far below of the new employees in the private sector. Thus, in 2005, 134 thousand people came out of the public system corresponded to 224000 new private sector jobs, in 2006: 55000 exits from the public to the private face to 163000 entries, in 2007 there was excess job employment in both sectors, as in 2008 to register 10000 departures from the public and 171000 private arrivals.

Because the actual unemployment rate remained below the natural rate, and the actual GDP was below the potential, we consider that inputs were either poorly allocated or insufficiently. In support of this testimony is the dynamic evolution of the most important sector of the economy, namely industry, where the number of employees decreased continuously in the period, which means that staffing were made in sectors with low added value.

Relative to the real interest rate can be observed again two periods. The first of these (2005-2006) it was far below the potential rate (0.54% versus 4.80% in 2005 and 1.82% to 3.54% in 2006) which led to the easing lending, as reflected in the evolution of household consumption from a ratio of 95,92% of potential in 2005, reached almost maximum in 2006 - 99.32% (being uncorrelated with the actual GDP share in the potential of only 96%). The explanation is very simple, following the evolution of net exports. From a trade deficit of 10817 million lei-2000 recorded in 2005, in 2006 it increased to 13763 million lei-2000, thus encouraging lending was made not to stimulate domestic production, but in the purchase of imported products.

The second period (2007-2008) can be described as a blending of opposites. On the one hand, the restrictive policy of the National Bank which used benchmark

interest rate increase (from the nominal 7.46% or actual 2.50% in 2007 compared to the potential of 1.25%, in 2008 recorded 9.46%, 1.49% and -0.87% respectively) could not counteract the relaxation of fiscal policy, budgetary and revenue, resulting in wage increases above productivity gains. Turning to household consumption, it has reached an alarming rate in 2008 of 101.10% of the potential.

Government spending after a timid decline in 2006, they beginning to have an upward trend, culminating in 2008 with a percentage of 100.66% compared to the potential, due, as mentioned above, to electoral calendars.

With the accession of Romania to the European Union, the transfers experienced a spectacular development from negative values recorded until 2007, being located at a positive level so far.

Although, as of 1 January 2005, the flat tax was introduced and despite the fact that absolute revenues from taxes increased from year to year, the ratio to the potential decreased continuously from 71.51% in 2005 to 55.54% in 2008. Easing the tax burden led to a series of consequences for businesses and individuals. Investment growth both in absolute terms and as a percentage of potential was apparently positive, however being not reflected in the actual GDP growth, which means that it has not followed the principle of economic efficiency. Relative to population, lower income tax rates coupled with wage increase led to an artificial increase in disposable income (110.13% of potential), which resulted, as we have seen above, in an oversized consumer based mainly on imports. All this have a result in reducing the country's competitiveness indicators externally.

3. Period 2009-2012

Table 14. Main Economic Indicators of Romania in 2009-2012

Year/ Indicator	2009			2010		
	effective	potential	effective / potential	effective	potential	effective / potential
Y	124029.6072	131581.1957	94.26%	125107.712	129344.5693	96.72%
r	3.54%	0.44%	807.57%	0.55%	1.00%	54.41%
TR	411.0648146	1342.675293	30.62%	1682.226937	1319.852405	127.46%
TI	11917.61269	24464.56262	48.71%	13450.91143	23730.38488	56.68%
V	112523.0593	108459.3083	103.75%	113339.0275	106934.0369	105.99%
C	89778.86712	96529.64363	93.01%	91364.48578	95308.46991	95.86%
G	10858.45688	10029.7339	108.26%	8923.922806	9859.247784	90.51%
I	31465.89907	36313.19892	86.65%	31987.70634	35276.30074	90.68%

NX	-7492.546611	-11291.38077	66.36%	-7168.402883	-11099.4491	64.58%
u	7.80%	11.18%	69.79%	6.87%	8.80%	78.10%
M	20156.4			18860.0		
Year/ Indicator	**2011**			**2012**		
	effective	potential	effective / potential	effective	potential	effective / potential
Y	124178.2021	128096.1724	96.94%	124561.7606	128851.6625	96.67%
r	0.43%	1.32%	32.92%	1.92%	1.13%	169.61%
TR	1042.438752	1307.113565	79.75%	1045.658079	1314.822706	79.53%
TI	15508.70219	23320.59571	66.50%	16032.98568	23568.58709	68.03%
V	109711.9386	106082.6903	103.42%	109574.433	106597.8982	102.79%
C	89521.34053	94626.85872	94.60%	88781.40882	95039.34815	93.42%
G	7840.083394	9764.089133	80.30%	8453.55485	9821.676124	86.07%
I	33438.47473	34697.54492	96.37%	33655.40727	35047.78955	96.03%
NX	-6621.696577	-10992.32038	60.24%	-6328.610381	-11057.15127	57.24%
u	5.12%	6.92%	74.00%	5.08%	7.04%	72.18%
M	18136.4			18574.3		

The analysis of this period will be conducted on three times: 2009, 2010-2011 and 2012 due to their peculiarities from a mix of economic policy and political factor.

In retrospect, 2009 may be considered a denial of economic realities by policymakers, although the crisis was visible since its beginning. This approach, coupled with electoral character of the period, led to increased depression phase.

The macroeconomic indicators deceleration was due to the insustainability of the main components of GDP: consumption and investment, i.e. an excessive aggregate demand.

Thus, in terms of growth over the previous year, the household consumption decreased by 10.63%, the share face potential consumption (derived in the mathematical model) being 93.01%. Relative to investments, there is also a reduction in their levels of 25.8% compared to 2008, the share to potential be only 86.65%. The decrease in trade deficit by 57.65% (from 17 690 million lei-2000 in 2008 to 7493 million lei-2000 in 2009) was based on the reduction, especially in imports (to an extent greater than falling exports). On the other hand, the consumption reduce in Romania, as we saw above, very much dependent on

imports and not based only to a small extent on domestic saving, could not be offset by lower trade deficit.

The flagrant violation of any economic theory (economic policy adopted in 2009 cannot be assigned, with one exception, even in the liberal models supported, but even in the Keynesian) is reflected both in terms of monetary, fiscal, and in the budget. Thus, the real interest rate increased from 1.49% to 3.54%, with the decrease of foreign capital flows which affect the investment sector that dropped from 42410 million lei-2000 in the year 2008 to 31466 million lei-2000 in 2009. Because of the economic crisis of the real economy experienced, a large number of firms have closed or suspended operations (particularly in the construction sector), the unemployment rate increased from 4.40% to 7.80%, and consumption decreased by 10.63%, the main funding sources of the state (tax revenue) decreasing significantly by 17.99% compared to 2008.

Despite this cruel reality, government spending increased by 3.48%, mainly due to increased overall budget unit with 0.62% (4.23% in public administration, education and health). In parallel, production personnel in industry and agriculture - the main sectors that could improve net exports - fell by 19.14%.

Interestingly, is the fact that, opposite to the inertia public sector, private sector started adjusting since 2008, feeling the first signs of the crisis long before declaring it official.

The period 2010-2011 can be called the period of recognition and awareness of the crisis, with more or less economic stability measures.

Monetary policy is one that, in this period, was decisive for the evolution of the main macroeconomic indicators. Thus real interest fell, first, from 3.54% in 2009 to 0.55% in 2010 and becoming in 2011: 0.43%. This was reflected in the investment immediately began a slight upward trend (increase of 1.66% in 2010 and 4.54% in 2011).

With increasing taxation (16.95% - 2009, 17.35% - 2010, 18.80% - 2011) the taxes collected increased significantly from 12.87% (2010 face to 2009), reaching to 15.30% (2011 face to 2010). High taxation put his mark on disposable income and thus on consumption that experienced after a timid increase of 1.77% in 2010, a decrease of 2.02% in 2011, returning to its level of 2009. These austerity measures, considered by authorities at the time, needed affect Romanian main engine of economic growth (albeit unsustainable) – consumption.

A positive aspect of this period is the reducing of the trade deficit (decrease of 4.33% in 2010 compared to 2009 and by 7.63% in 2011 compared to 2010) supported by strong exports (whose development was the increase 28.84% in 2010 and to 21.26% in 2011) compared to imports whose dynamics, however, was lower (21.88% in 2010 and 18.05% in 2011).

These percentages creates a misleading appearance due to the fact that the analysis in depth of the phenomenon reveals that the evolution of exports was due, on the one hand, to the exchange rate (RON/EUR, fig.18) which has had a trend upward and, on the other hand, to the economic difficulties of the European Union - Romania's main trading partner, which allowed better penetration on the foreign market local goods, comparatively cheap.

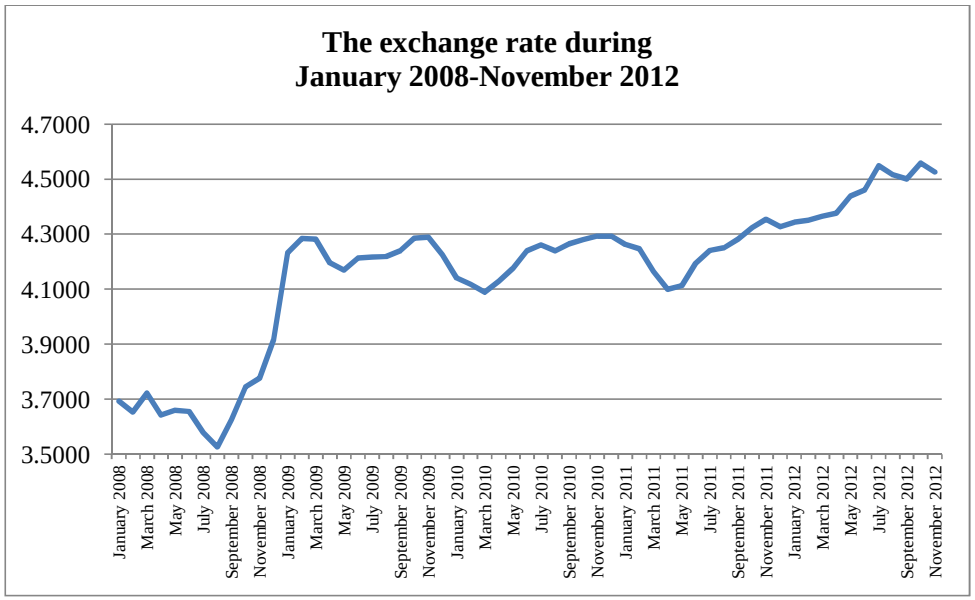

Figure 18.

Source: INSSE

A second measure, which would have been welcome, was to reduce government spending to 17.82% in 2010 and 12.15% in 2011. But studying the dynamics of public sector employees in a decrease of only 5%, the difference is actually reflected in the decline in public-sector wages, which inevitably led to a decline in consumption.

The year 2012 was distinguished by a mix of policies, primarily due to changes in political strategy. After four months of liberal policy, Romania has focused (after tipping the political balance of forces) on a strategy essentially social-democratic. Political instability, reflected by twists and power relations, as well as feed-back from the European Union have led to developments weak, although positive, the economic performance.

The GDP growth was modest (after three years of economic crisis, during which a number of states have overcome difficulties) of 0.3% respecting 2011, household

consumption fell by 0.83%, investments with a modest rate (compared to the previous year) growth of 0.65%, all due to higher real interest rate from 0.43% to 1.92%.

The trade balance continues the trend of deficit reduction to 4.43% respecting to 2011.

In parallel, the collection of taxes has increased by 3.38%, but the increasing of the government spending; without an economic justification, with 7.82% (consisting primarily of salary increase) was due to the electoral calendar.

During this period (2010-2012) the unemployment rate experienced a continuous decline from 6.87% to 5.08%.

From the foregoing, it emerges that is essential to the conclusion that it cannot be a visible progress without high investments. On the other hand, the problem is that the sources of funding for the various projects. With the accession to the European Union have been allocated sufficient funds to start serious investment projects. Unfortunately, due to excessive bureaucracy and a coordinating effective device for writing projects, the absorption rate was the lowest of all European countries, with an average of 16.51% during 2007-31 March 2013, far below the 33.36% European. Also, in recent years, the rate of absorption continuously decreased from 15.08% in 2011 to 11.47% in 2012.

5.9. The Analysis of Dynamic Equilibrium

The desire of each economy is to reach potential GDP when all inputs are used to the maximum. On the other hand, the balance cannot be achieved instantly with differences between aggregate demand and output, and between demand and supply of money. Following these considerations, the dynamic equations (12) and (13) study the time variation of just two main indicators of economy namely GDP and real interest rate.

In section 4, we determined the temporal variations, depending on the sign of the expression: $\Delta=(\alpha\chi+\beta m_r)^2+4\alpha\beta i_r md_Y$ obtaining the equations (49)-(51).

To determine the parameters α and β we considered first, numerical approximations:

$\frac{dY}{dt}\Big|_{t=n} = \Delta Y = Y_n - Y_{n-1}$ where Y_n is the level of GDP at the time n, respectively

$\frac{dr}{dt}\Big|_{t=n} = \Delta r = r_n - r_{n-1}$ where r_n – real interest rate at time n. They were then

calculated, considering the ratios $\alpha_n = \frac{\frac{dY}{dt}}{D-Y}, \beta_n = \frac{\frac{dr}{dt}}{MD-M}$ the averages of α_n and β_n in the analyzed period, being considered like values for α and β respectively. There were thus obtained the following values:

(70) α=10.32709

(71) β=5.31171·10^{-6}

From the formulas (53), (54), (56), (58), (59), (61), (63), (65), (66), (68), (69) we saw that:

- c_V=0.80063
- C_0=9694.17941
- g_Y=0.076224675
- in_Y=0.278413701
- i_r=-73144.11685
- ν_Y=-0.085813028
- θ_Y=0.010204158
- ri_Y=0.32825
- T_0=-18727.26768
- md_Y=0.158727057
- m_r=-166237.7723

From the formulas (15), (17), we obtain:

(72) $\chi = 1 - c_V(1 + \theta_Y - ri_Y) - g_Y - in_Y - \nu_Y$ =0.18519

(73) $E = C_0 - c_V T_0$ =24687.7379

(74) $\Delta = (\alpha\chi + \beta m_r)^2 + 4\alpha\beta i_r md_Y$=-1.48773

As a result of the value of Δ we have the situation of dynamic equations (51).

The roots λ_1 and λ_2 present in the equations are:

(75) λ_1=-1,39772+0,60986·i

(76) λ_2=-1,39772-0,60986·i

from where:

(77) μ=-1,39772

(78) ν=0,60986

Substituting in the formulas (51), we get for the money supply, in the year 2012, M=18574.3261 (lei-2000):

(79) $Y=-4289.901993\cdot e^{-1.39772\cdot t}\cdot\cos(0.60986\cdot t)-435.9515998\cdot e^{-1.39772\cdot t}\cdot\sin(0.60986\cdot t)+ 128851.6625$

(80) $r=0.00372897\cdot e^{-1.39772\cdot t}\cdot\cos(0.60986\cdot t)+0.00098035\cdot e^{-1.39772\cdot t}\cdot\sin(0.60986\cdot t)+0.011296585$

The graphical representation of the function Y and r with respect to t is the following:

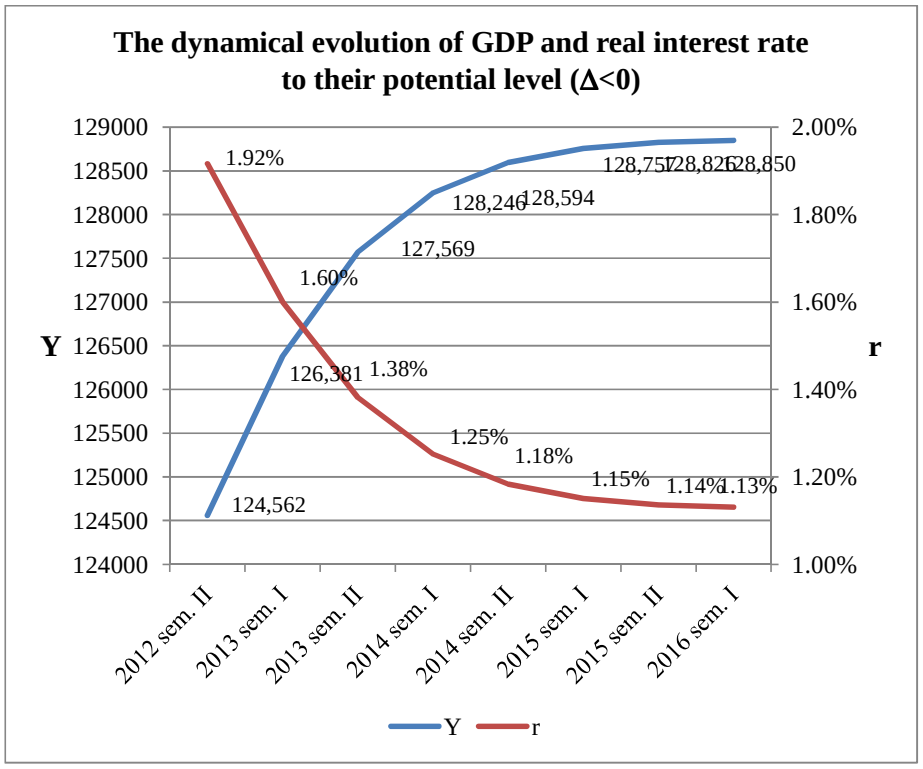

Figure 19.

The significance of formulas (79) and (80) and the evolution represented in fig.19 is that, given the constancy of model parameters, and the money supply (the year 2012), both GDP and the real interest will tend asymptotically to static equilibrium

levels determined by formula (24) as follows: Y^*=128851,6625 (lei-2000) and r^*=1.13%. It is noted that, after only two years, GDP reached 99.8% of the potential, while the real interest rate: 95.76% of the optimum. Therefore, in terms of constancy of all parameters, in order to achieve the potential of the economy, will be pursued that semis economic indicators above (Y and r) along with everyone else present in model to meet the theoretical values.

In semis the dynamic situation of the Romanian economy should be as follows:

Table 15. Main economic indicators of Romania (after the Dynamical Analysis) in 2013

Year/ Indicator	2013 sem. I			2013 sem. II		
	effective	potential	effective/ potential	effective	potential	effective/ potential
Y	126381.3638	128851.6625	98.08%	127569.0389	128851.6625	99.00%
r	1.60%	1.13%	141.69%	1.38%	1.13%	122.33%
TR	1289.615388	1314.822706	98.08%	1301.734612	1314.822706	99.00%
TI	22757.70585	23568.58709	96.56%	23147.56292	23568.58709	98.21%
V	104913.2734	106597.8982	98.42%	105723.2106	106597.8982	99.18%
C	93690.59183	95039.34815	98.58%	94339.04953	95039.34815	99.26%
G	9633.378406	9821.676124	98.08%	9723.908551	9821.676124	99.00%
I	34015.56549	35047.78955	97.05%	34506.14996	35047.78955	98.45%
NX	-10845.16746	-11057.15127	98.08%	-10947.08545	-11057.15127	99.00%

Table 16. Main Economic Indicators of Romania (after the Dynamical Analysis) in 2014

Year/ Indicator	2014 sem. I			2014 sem. II		
	effective	potential	effective/ potential	effective	potential	effective/ potential
Y	128245.7375	128851.6625	99.53%	128593.9959	128851.6625	99.80%
r	1.25%	1.13%	110.88%	1.18%	1.13%	104.80%
TR	1308.639752	1314.822706	99.53%	1312.193435	1314.822706	99.80%
TI	23369.69081	23568.58709	99.16%	23484.00743	23568.58709	99.64%
V	106184.6865	106597.8982	99.61%	106422.1819	106597.8982	99.84%
C	94708.51967	95039.34815	99.65%	94898.66497	95039.34815	99.85%
G	9775.489687	9821.676124	99.53%	9802.035571	9821.676124	99.80%
I	34789.20458	35047.78955	99.26%	34936.41041	35047.78955	99.68%
NX	-11005.15501	-11057.15127	99.53%	-11035.04012	-11057.15127	99.80%

Table 17. Main Economic Indicators of Romania (after the Dynamical Analysis) in 2015

Year/ Indicator	2015 sem. I			2015 sem. II		
	effective	potential	effective/ potential	effective	potential	effective/ potential
Y	128757.0432	128851.6625	99.93%	128825.6073	128851.6625	99.98%
r	1.15%	1.13%	101.86%	1.14%	1.13%	100.58%
TR	1313.857196	1314.822706	99.93%	1314.556834	1314.822706	99.98%
TI	23537.52809	23568.58709	99.87%	23560.03439	23568.58709	99.96%
V	106533.3724	106597.8982	99.94%	106580.1297	106597.8982	99.98%
C	94987.68703	95039.34815	99.95%	95025.12226	95039.34815	99.99%
G	9814.463798	9821.676124	99.93%	9819.690071	9821.676124	99.98%
I	35006.06566	35047.78955	99.88%	35035.74568	35047.78955	99.97%
NX	-11049.0317	-11057.15127	99.93%	-11054.91539	-11057.15127	99.98%

5.10. A First Scenario of Economic Growth

The working hypothesis in the previous section relative to the dynamic equilibrium, assumed constant money supply. Hypothesis may seem forced, but we must not neglect the fact that if the money supply (M) would be variable, the system of differential equations (12) - (13) would not have had constant coefficients, its integration being particularly difficult. In what follows, however, we take into account the variability of money supply, but only studying static balance (otherwise, the dynamic limit).

In this first scenario, we propose, first, to determine how the money supply trend.

The analysis of data from the period 2006-2012 (considered because in 2005 there was an increase in the supply of currency aberrant 93.75%) reveals a rather uniform increase in the money supply, with the exception of 2007 (an increase of 40.15%). Substituting this last variation interpolated values we obtain an average increase of 8.78% in the money supply. Data analysis last four years (2009-2012) reveals that economic indicators were performed on an average of 96.15% of the potential. As a result, we determine the potential level of main indicators, and we will adjust this percentage. Also, taking into account the forecasted GDP deflator: 1.048 – 2013, 1.037 – 2014, 1.025 – 2015, the cumulative deflator is obtained since 2000: 0.202320817 – 2013, 0.195102042 – 2014, 0.190343455 – 2015. Also the forecast for CPI is 1.035 – 2013, 1.03 – 2014, 1.025 – 2015 from where the current inflation factor compared to 2000 results: 0.282781068 – 2013, 0.274544726 – 2014, 0.267848513 – 2015. Taking into account of these, we will express the results of analysis in addition to coin and current coin 2000.

Table 18. The Forecast of the Main Economic Indicators of Romania - Scenario I in 2013

Year/ Indicator	2013			
	potential (mil. lei-2000)	effective=potential· 0,9615 (mil. lei-2000)	potential (mil. lei-2013)	effective=potential· 0,9615 (mil. lei-2013)
M	20205		**99867**	
Y	126596	650775	126596	**650775**
r	0.42%		**3.93%**	
TR	1292	6641	1292	**6641**
TI	23549	121056	23549	**121056**
V	104339	536359	104339	**536359**
C	92857	477339	92857	**477339**
G	9650	49605	9650	**49605**
I	34953	179676	34953	**179676**
NX	-10864	-55845	-10864	**-55845**

Table 19. The Forecast of the Main Economic Indicators of Romania - Scenario I in 2014

Year/ Indicator	2014			
	potential (mil. lei-2000)	effective=potential· 0,9615 (mil. lei-2000)	potential (mil. lei-2014)	effective=potential· 0,9615 (mil. lei-2014)
M	21979		**108635**	
Y	134726	129539	134726	**129539**
r	-0,36%		**2,63%**	
TR	1375	1322	1375	**1322**
TI	25497	24515	25497	**24515**
V	110604	106346	110604	**106346**
C	98247	94464	98247	**94464**
G	10269	9874	10269	**9874**
I	37771	36317	37771	**36317**
NX	-11561	-11116	-11561	**-11116**

Table 20. The Forecast of the Main Economic Indicators of Romania - Scenario I in 2015

Year/ Indicator	2015			
	potential (mil. lei-2000)	effective=potential· 0,9615 (mil. lei-2000)	potential (mil. lei-2015)	effective=potential· 0,9615 (mil. lei-2015)
M	23909		**118173**	
Y	138056	132740	138056	**132740**
r	-1,20%		**1,27%**	
TR	1409	1355	1409	**1355**
TI	26590	25566	26590	**25566**
V	112874	108529	112874	**108529**
C	100065	96212	100065	**96212**
G	10523	10118	10523	**10118**
I	39315	37801	39315	**37801**
NX	-11847	-11391	-11847	**-11391**

A comparison with the "Projection of main macroeconomic indicators for the period 2013-2016" conducted by the National Commission for Prognosis, reveals a consistent correspondence, the present model being slightly more pessimistic on long term.

Thus, in Tables 18-20, the rate of GDP growth is forecast to be 1.63% - 2013 2.32% - 2014, 2.47% - 2015 while the National Commission forecast: 1, 6% - 2013, 2.2% - 2014 and 2.8% - 2015.

The individual consumption of households is projected to model the growth of 4, 59% - 2013, 1.73% - 2014 and 1.85% - 2015, while the report mentioned above states: 2.3% - 2013, 1.7% - 2014, 2.0% - 2015.

5.11. The Second Scenario of Economic Growth

In section 2, we studied changes in potential GDP and potential real interest rate relative to changes in the model constants (derived from linear regressions), resulting in a total of 13 cases of behavior depending on parameter values.

From the parameter values are obtained immediately following derived quantities are necessary for the analysis of the monotony of the above functions. We have therefore (for M=18574.32609 - money supply in 2012):

$$(81)\quad \omega = 1 + \theta_Y - ri_Y = 1 + 0.010204 - 0.328252 = 0.681952$$

(82) $\Phi_1=\frac{T_0\left(i_r md_Y+m_r\chi+c_V\omega m_r\right)-\omega M i_r}{\omega m_r}=-30169.51536$

(83) $\Phi_2=\frac{c_V md_Y T_0+M\chi}{md_Y}=6676.94849$

(84) $\frac{\omega M}{md_Y}=79802.37497$

How $T_0=-18727.26768<79802.37497=\frac{\omega M}{md_Y}$

$C_0=9694.17941>\Phi_2>\Phi_1$ follows the case of monotony 5, that is:

- Y^* and r^* are strictly increasing and strictly convex with respect to the marginal propensity to consumption (c_V), marginal net exports ν_Y, the investment rate in_Y, the marginal rate of government transfers θ_Y, the government consumption g_Y and the marginal factor influence in the investment rate (i_r);
- Y^* is strictly decreasing and strictly concave in relation to the factor that influence the demand for money in relation to interest rate (m_r) and with respect to the tax rate ri_Y;
- r^* is strictly increasing and strictly convex with respect to the factor that influence the demand for money in relation to interest rate (m_r);
- Y^* is strictly decreasing and strictly convex with respect to the rate of money demand in the economy md_Y;
- r^* is strictly decreasing and strictly concave with respect to the tax rate ri_Y and the rate of money demand in the economy md_Y.

Computing the first order partial derivatives of Y and r, we obtain the following graphs (corresponding to 1% of variation of c_V, g_Y, ν_Y, ri_Y, in_Y, θ_Y and with 10000 of i_r and m_r – the multiplicative factor in this case being taken arbitrary just for exemplifying monotony).

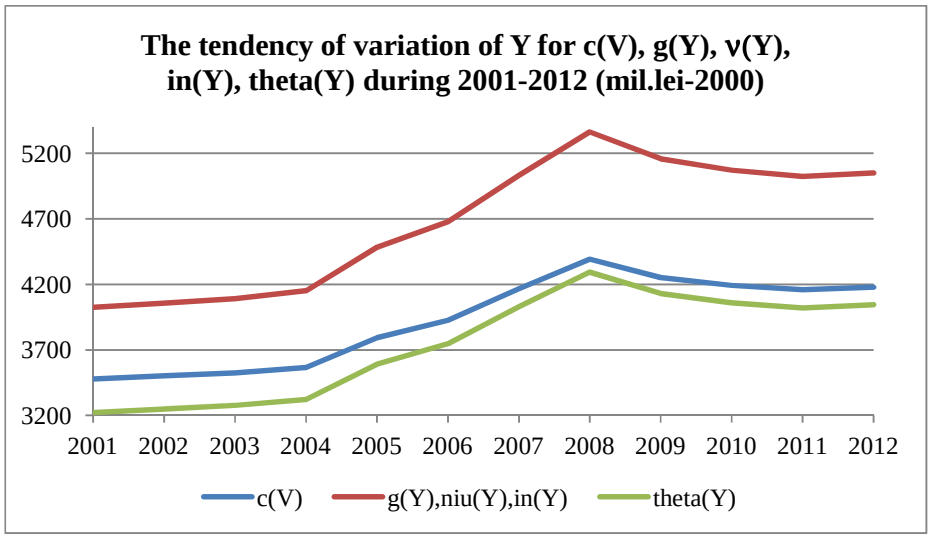

Figure 20.

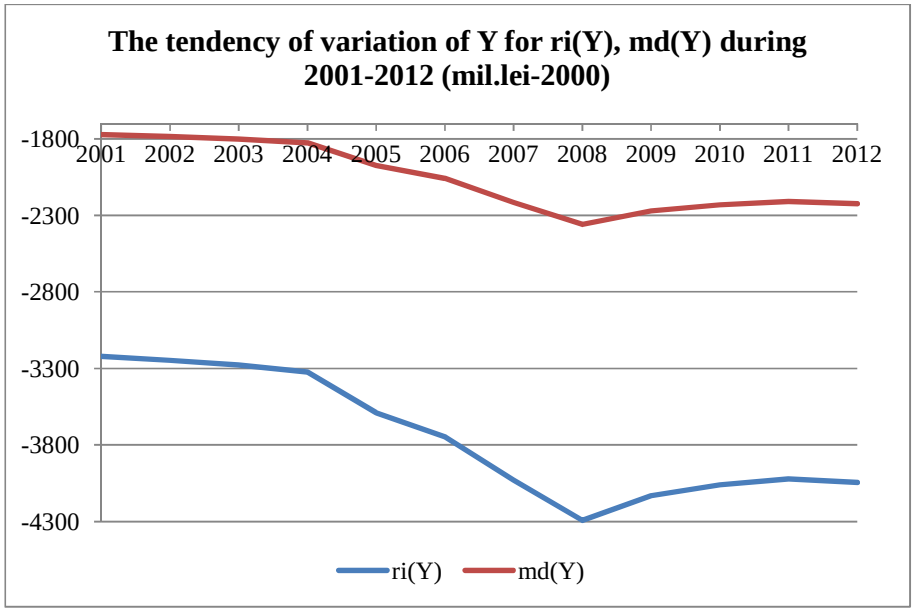

Figure 21.

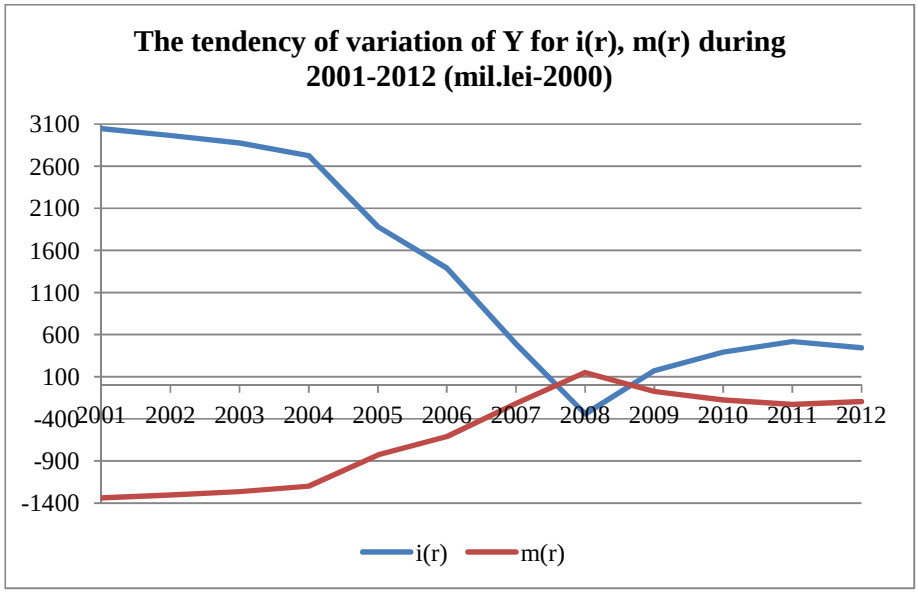

Figure 22.

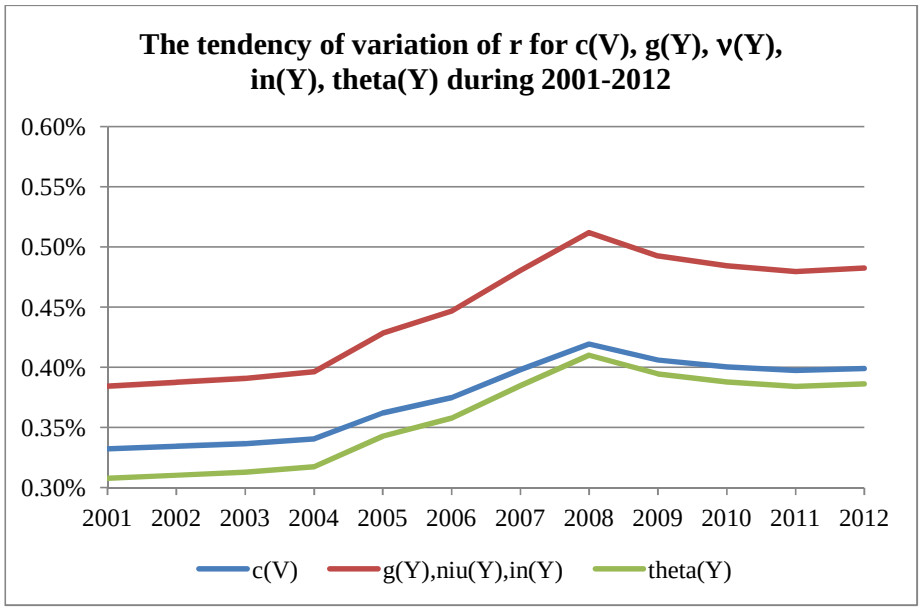

Figure 23.

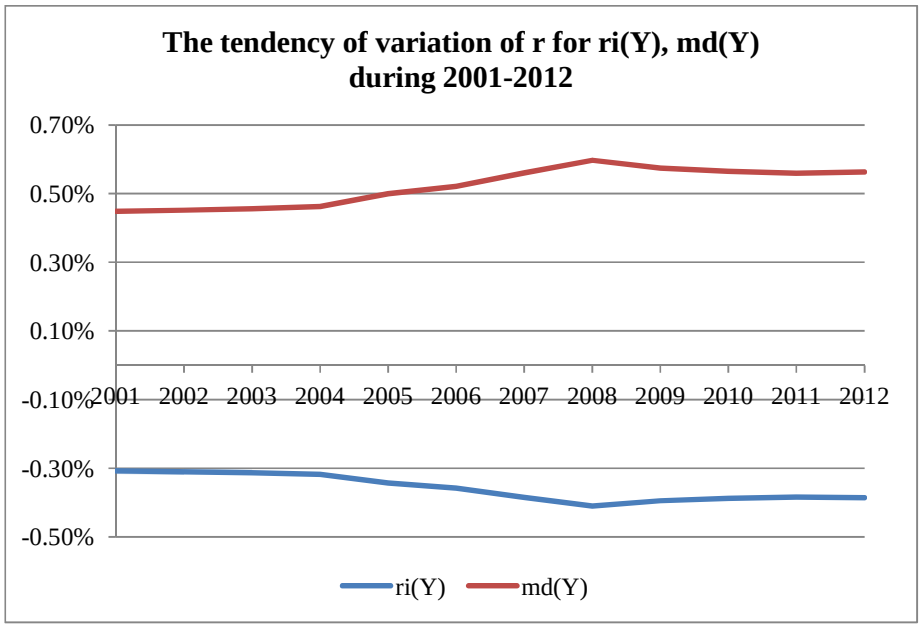

Figure 24.

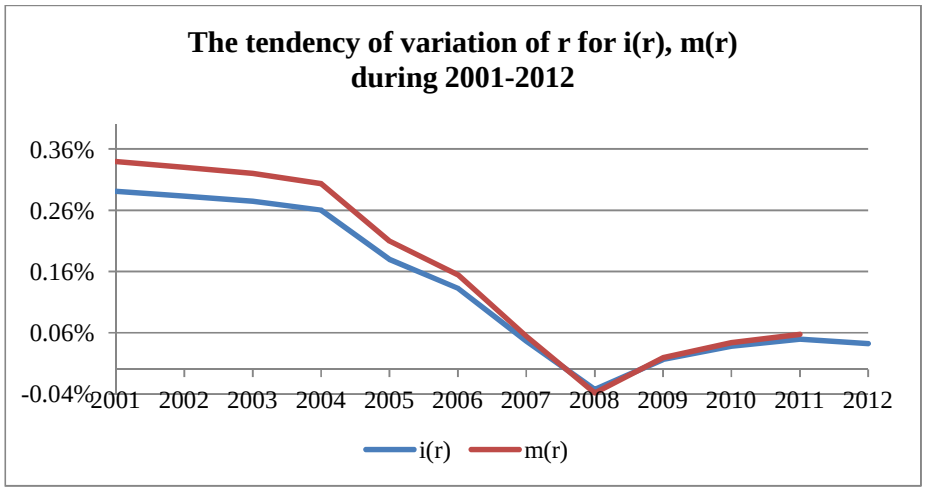

Figure 25.

From the above graphs, it is noted that, for example, an increase in the marginal propensity to consumption from 0.80 to 0.81 will result, in the conditions of constancy of all other parameters of the model, to an increase in GDP of 4180 million lei-2000. An increase in GDP by 5053 mil. lei-2000 will occur when

marginal increase in government consumption, net exports and a marginal rate of investment, where the marginal government transfers with an increase of only 4045 mil.lei-2000. Meanwhile, the increase of these parameters produces an increase in the real interest rate of 0.40% for the marginal propensity to consumption growth, of 0.48% increase in the case of marginal government consumption, net exports and the marginal rate of investment and 0.39% increase when government transfers marginal rate grow by 1%.

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