Tactical Assets Allocation: Evidence from the Nigerian Banking Industry

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Abstract: The core of portfolio selection theory centers on striking a balance between risk-return trade-off of a given investment layout so as to maximize benefits. Literature reveals that portfolio selection or asset allocation problems often involve the use of mathematical programming in propounding solution. This paper uses a blend of simultaneous equation and graphical approach to linear programming algorithm to help solve investors' problem in allocating assets among various alternatives when faced with problems associated with risk-return trade-off. We strongly suggest that practioners as well as policy makers use this approach to obtain optimal solution when faced with decision making given various investment alternatives.

Keywords: Tactical Asset Allocation; Linear programming; risk; return; investment

JEL Classification: D92

1. Introduction

The core of portfolio selection theory centers on striking a balance between risk-return relationship of a given investment layout. The basic theory of investment financing explained that the higher the expected return on an investment, the higher the level of risk associated with such investment, but higher risk does not necessarily connotes higher return. It is also a known fact that investors invest cash in portfolio of securities so as to earn a better return than would be earned if the money were retained as cash or as a bank deposit. Return from such activities may come by a way of regular income through dividend payments or interest or through the growth in capital value or a combination of both (Cohen and Zinberg (1967)). It is therefore clear that the core objective of portfolio selection deals with achieving the maximum return with minimum risk flow from a given set of investment (see Harvey (2001), Neveu (1985), Macedo (1995) and Grubel (1968) Enrique Ballestero (2012), Bogdan Rebiaz (2013) Hasuike et al (2009) Iskander M.G (2004)).

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Literature reveals that portfolio selection or asset allocation problem often use quadratic programming problem which entails minimization of associated risk of investment through the minimization of the total variance (a measure of risk), it is therefore expedient that adequate solution techniques be adopted to obtain optimal solution.

Magnus Dahlqust and Campbell R Harvey (2001) identified three distinguish level of portfolio selection/asset allocation hypothesis viz: Benchmark Asset Allocation; Strategic Asset Allocation; and Tactical Asset Allocation with each having its peculiarity.

This study advances literature by providing an optimal portfolio procedure of high-frequency tactical asset allocation using Nigeria data. It is devoted to the problem of selecting an efficient portfolio where the parameters in the calculation of efficiency are expresses in the form of linear programming.

The rest of this paper is structured as follows: Chapter two deals with review of existing literatures; Chapter three deals with data and methodology; Chapter four presents the results while Chapter five provides the summary and conclusion.

2. Literature Review

Mathematical approach to solving finance related problem dates back to Markowitz mean-variance theory of portfolio selection which offers a quantitative approach to quantifying the risk-return trade-off for general assets with correlated returns (see Oloyede (2002). Takashi, Hideroaki(2009), Bodgan Resiaz (2013). This was followed by the works of Loris and Savage (1955) which focuses their work of capital budgeting application. This was followed by works done by Sharpe and Linter's examinations of the equilibrium structure of asset prices and their Capital Asset Pricing Model (CAMP) which serve as the classical framework for mathematical modeling that helps in determining the risk of assets. Fama's efficient market hypothesis which classified capital market round the globe into three distinct classes viz: strong; semi-strong and weak efficient market hypothesis with each having its own distinguish features. As year come by, sophisticated quantitative techniques tools were introduced to the finance, prominent among them includes: Intertemporal and uncertainty analysis of valuation and optimal financial decision making, Dynamic portfolio theory which was an improvement on Markowitz mean variance model. Others include the intertemporal and international capital asset pricing models which modified and expanded the Sharpe and Linter's single risk measurement in Capital Asset Pricing Model to multidimensional measures of a security risk; Black and Scholes option pricing models (oloyede 2002)

Today, works relating to application of mathematical programming in assets price allocation or portfolio theory involves the use of linear programming, integral programming, Goal programming, fuzzy analysis and programming, decision theory, dynamic theory among other things. Their applications abound in literature, for instance, Date, Canepa and Abdel-Jawad (2011) used linear programming technique to propose a stochastic optimization based approach to determine the composition of portfolio issued over a series of government auctions for the fixed income debt, minimize the cost of servicing debt while controlling risk and maintaining market liquidity using UK conventional government debt portfolio data. Their work shows that the interactions between frequent re-calibration of the interest rate model and re-optimization of the issuance throughout the budgetary year facilitates changes in interest rate (see also Adamo, Amadori and Bernasci (2004), Consiglio, Staino (2010) Date and Wang (2009). Similarly, Mustafa (2011) used mixed integer linear programming as a quantitative tool to determine a minimum expected surplus criterion for hedging American contingent claims and established an optimal exercise and hedging policies.

In a related development, Mustafa et al (2009) in a study titled ‘expected gain-loss pricing and hedging of contingent claims in incomplete markets by linear programming’ used LP to analyze the problems associated with pricing and hedging contingent claims in the multi-period, discrete time and discrete state case based on the concept of a ‘gain- loss ratio opportunity’. Their results differs from the existing arbitrage pricing theorems as it provides a tighter price bounds on the contingent claims in an incomplete market which may converge to a unique price for a specific value of gain-loss preference parameter imposed by the market while the hedging policies may differ for different sides of the same trade. Other applications of linear programming to portfolio selection or asset allocation can be found in Shuo-Yan, Jennifer and Peterson (2009), Hop (2007), Juan WU and Xueqian GE (2012), Lofti et al (2010), Alireza et al (2009), Fiertz and Monico (2004), Hasuike et al (2009) Alireza Ghahtarani and Amir Abbas Najafi (2013).

2.2. Asset Allocation Procedures

As earlier stated, Magnus et al (2001) identified three distinguish level of asset allocation or portfolio selection procedures viz:

Benchmark Asset allocation which primarily replicates the investment weights of the benchmark index, for instance, if the financial manager of a firm is benchmarked on the Morgan world Stanley Capital International (MSCI) portfolio, the benchmark asset allocation assumes the same weights in this index (a typical indexing allocating procedure). Under this arrangement no information is used except the usual details of indexing which entails determining market weights, managing delisting, new listings, buy backs, secondary market offerings, dividends and warrants.

Strategic asset allocation in the second level of asset allocation which is typically long term in nature (usually five (5) years horizon). Here decisions are made based on the future expectation on the movement in the direction of asset prices. A good example includes a situation when the firm manager view that Nigerian Government bonds underperform over the next five years. Decisions here will centers on deviating from the benchmark which will introduce tracking error. The tracking error represents the standard deviation of the differences between benchmark return and portfolio return.

The Tactical Asset allocation is the third level of asset allocation. Under this approach, the investment manager will take short term bets usually one month to one quarter and deviate from the strategic weights. Tracking Error also occurs under this arrangement. The gap between the second and third asset allocation procedures / techniques induces tactical tracking error while the difference between the first and third weight is the total tracking error. However, it should be noted that the strategic and tactical tracking error standard deviation does not necessary equal total tracking error as a result of potential correlation between strategic and tactical weight over longer time horizons.

Our focus in this paper centers on the use of tactical asset allocation and mathematical programming to solve problems relating to portfolio selection using Nigerian data.

3. Data and Methodology

In this study, we used daily prices of two of the oldest bank stocks listed on the floor of the Nigerian Stock Exchange for the period September and October 2013. Specifically, the banks are Union Bank of Nigeria (UBN) and United Bank for Africa (UBA). First Bank of Nigeria being the oldest bank in the country was dropped because as at the time of this work, its stock is not traded on the floor of the exchange. As earlier stated, this study focuses on Tactical Asset Allocation technique which requires the use of current and short term data, this account for the use of data published in recent months. The data comprises of trading prices of these shares for forty three (43) days. Saturday, Sunday and public holidays were exempted. The Mean, Variance and Standard deviation of the stocks were calculated so as to generate outputs which were later used as variables for linear programming analysis.

3.1. Quadratic Programming Model

A quadratic programming model for portfolio selection

In developing a quadratic programming model for portfolio selection, following Etukudo (2011), we make the following assumptions:

n=number of stocks to be included in the portfolio

x_j = number of shares to be purchased in stocks j,j = 1, 2, ….,n

Y_j = returns per unit of money invested in stocks j at maturity

Assume the values of Yj are random variables, then

$$E(Y_j) = \bar{Y}_j; j=\overline{1,2}....,n \qquad (3.1)$$

$$V = \sigma_{ij} = E[(Y_i - \bar{Y}_i)(Y_j - \bar{Y}_j)] \qquad (3.2)$$

Where E (Y_j) is the mathematical expectation of Y_j and V is the variance – covariance matrix of the returns (See Gruyter (1987), Parsons (1977) and Etukudo et al (2009), Adedayo et al (2006)). Thus, the variance of the total returns or the portfolio variance is given by

$$f(x) = X'VX = \sum_{j=1}^{n} \sum_{j=1}^{n} {}_{ij}.x_i.x_j \qquad (3.3)$$

This measure the risk of the portfolio selected. The non-negativity constraints are

$$X_j \geq 0, \ j = 1, 2, \ldots, n \qquad (3.4)$$

Assuming the minimum expected return per unit of money invested in the portfolio is B, then

$$\sum_{j=1}^{n} Y_j x_j \geq K \qquad (3.5)$$

3.2. Minimization of the Total Risk Involved in the Portfolio

Hiller and Lieberman (2006) explained that by minimizing the total variance, $f(x)$ of the portfolio, the total risk involved in the portfolio is minimized. In order to obtain a minimum point of equation 3, $f(x)$ must be a convex function, (See also Etukudo (2011).

That is,

$$\frac{\partial^2 f(x_j)}{\partial x_1^2} \frac{\partial^2 f(x_j)}{\partial x_2^2} \ldots \frac{\partial^2 f(x_j)}{\partial x_j^2} - [\frac{\partial^2 f(x_j)}{\partial x_i \partial x_j}]^2 \geq 0 \qquad (6)$$

(3.6)

Where $$\frac{\partial^2 f(x_j)}{\partial x_1^2} \geq 0$$

$$\frac{\partial^2 f(x_j)}{\partial x_j^2} \geq 0 \qquad (3.7)$$

Where $i \neq j = 1, 2\ldots n$.

The strict inequalities of equations (6) and (7) explain that f(x) is strictly convex, thus has a global minimum at X^{*}. From equation 3 and inequalities 4 and 5, the portfolio selection model is given by;

Min $\quad f(x) = \sum_{i=i}^{n}\sum_{j=i}^{n} \sigma_{ij}.x_i.x_j$

Subject to: $\sum_{j=1}^{n} Y_j x_j \geq \overline{K}$; $X_j \geq 0,\ j = 1, 2, \ldots, n$

Remark: The expected values, Y_j and the variance – covariance matrix, σ_{ij} are based on date from historical records.

3.3. Numerical Example

A typical investor has a maximum of ₦1, 500:00 to invest by purchasing shares in UBA and Union Bank of Nigeria Plc. The columns 1 and 2 in the table below present the historical data of price per share on the floor of the exchange for the banks for 43 days for the months of September and October, 2013. Our focus is to obtain optimal allocation of the investible funds for purchase of shares in the portfolio in order to maximize the expected return (or minimize the total risk) in the portfolio mix. From the table it could be seen that mean prices per share for UBA and UBN are 7.45 and 11.39 respectively.

The difference between the mean price of a share and its price on the last day (i.e the 43rd day) is the expected return per share. Here the investor assumes that his expected returns would be at least ₦1500:00.

The share price deviations and the variance- covariance matrix for the share price are presented in columns 7 and 8 respectively.

Table 1. Historical prices of the assets with the Mean and Standard Deviations

Dates	Share price of UBA (X)	Share Price of UBN (Y)	X-*X*	Y-*Y*	$(X-X)^2$	$(Y-Y)^2$	(X-*X*)(Y-*Y*)
1	-	-	-	-	-	-	-
2	7.33	10.30	-0.12	-1.09	0.01	1.19	-0.1308
3	7.27	11.00	-0.18	-0.39	0.03	0.15	-0.702
4	7.35	10.80	-0.1	-0.59	0.01	0.35	0.059
5	7.50	10.80	-0.05	-0.59	0.003	0.35	0.0295
6	7.50	10.99	-0.05	-0.4	0	0.16	0.0295
7	7.45	10.66	0	-0.73	0	0.53	0.02
8							
9	7.45	10.60	0	-0.73	0	0.53	0
10	7.33	10.66	-1.09	-0.73	0.01	0.53	0.7957
11	7.30	10.66	.00	-0.73	0.02	0.53	0
12	7.10	10.66	-0.35	-0.73	0.12	0.53	0.0146
13	7.10	10.66	-0.35	-0.73	0.12	0.53	0.2555
14	-	-	-	-	-	-	-

15	-	-	-	-	-	-	-
16	7.05	10.39	-0.4	-1	0.16	1	0.4
17	7.10	10.39	-0.35	-1	0.12	1	0.35
18	7.20	10.39	-0.25	-1	0.06	1	0.25
19	7.07	10.39	-0.38	-1	0.144	1	0.38
20	7.30	10.22	-1.09	-1.17	0.01	1.37	1.2753
21	7.70	10.20	0.25	-1.19	0.03	1.42	0.2975
22	-	-	-	-	-	-	-
23	7.70	10.20	0.25	-1.19	0.06	1.42	0.2975
24	7.42	10.12	-0.03	-1.27	0	1.61	0.0381
25	7.40	10.15	-0.05	-1.24	0.003	1.54	0.062
26	7.40	10.25	-0.05	-1.14	0.003	1.30	0.057
27	7.35	10.16	-0.1	-1.23	0.01	1.51	0.123
28	-	-	-	-	-	-	-
29	-	-	-	-	-	-	-
30	7.40	10.16	0.05	-1.23	0.003	1.51	0.0615
1	-	-	-	-	-	-	-
2	7.55	10.25	0.1	-1.14	0.01	1.30	-0.114
3	7.60	10.20	0.15	-1.19	0.02	1.42	0.1785
4	7.50	10.21	0.05	-1.18	0.003	1.39	0.059
5	-	-	-	-	-	-	-
6	-	-	-	-	-	-	-
7	7.45	10.21	0	-1.18	0	1.39	0
8	7.30	10.42	-0.15	-0.97	0.02	0.94	0.1455
9	7.40	10.42	-0.05	-0.97	0.003	0.94	0.0485
10	7.40	10.31	-0.05	-1.08	0.003	1.17	0.054
11	7.60	10.30	0.15	-1.09	0.02	1.19	-0.1635
12	7.54	10.30	0.09	-1.09	0.008	1.19	-0.0981
13	7.50	10.45	0.05	-0.94	0.003	0.88	-0.047
14	-	-	-	-	-	-	-
15	-	-	-	-	-	-	-
16	7.50	10.45	0.05	-0.94	0.003	0.88	-0.047
17	-	-	-	-	-	-	-
18	7.65	10-52	0.2	-0.87	0.04	0.76	-0.174
19	-	-	-	-	-	-	-
20	-	-	-	-	-	-	-
21	7.85	10.50	0.4	-0.89	0.16	0.79	-0.356
22	7.70	10.50	0.25	-0.89	0.06	0.79	0.2225
23	7.75	10.52	0.3	-0.87	0.09	0.76	0.261
24	7.90	10.50	0.05	-0.89	0.003	0.79	0.0445
25	7.98	10.51	0.53	-0.88	0.28	0.79	-0.4664
26	-	-	-	-	-	-	-
27	-	-	-	-	-	-	-
28	7.60	10.50	0.15	-0.89	0.02	0.79	-0.1335
29	7.85	10.50	0.4	-0.89	0.16	0.79	-0.356
30	-	-					
Total	312.99	489.59			39.2	1.704	3.4372
Average	7.45	11.39			0.9561	0.0416	0.08383

Source: Authors Computation from data from the Nigerian Stock Exchange

From the table, the variance – covariance matrix is given by

$$\begin{matrix} \sigma 11 & \sigma 12 \\ \sigma 21 & \sigma 22 \end{matrix} = \begin{matrix} 0.9561 & 0.0838 \\ 0.0838 & 0.0416 \end{matrix}$$

Thus, we express the model for minimizing the total risk of the portfolio as this:

$$\text{Min } f(x) = x \quad y \begin{matrix} 0.9561 & 0.0838 \\ 0.0838 & 0.0416 \end{matrix} \begin{matrix} x \\ y \end{matrix}$$

$= 0.9561x^2 + 0.1676xy + 0.0416y^2$

Subject to:

$7.45x + 11.39y \leq 1500$

$7.98x + 10.51y \geq 1500$

$X, y, \geq 0$

Our solution algorithm entails the use of graphical approach of the linear programming model.

First, turn the inequalities signs to equality signs such that

$7.45x + 11.39y = 1500$ (3.8)

$7.98x + 10.51y = 1500$ (3.9)

In (3.8) let $X = 0$, then

$7.45(0) + 11.39y = 1500$

$= 11.39y = 1500$

$Y = \frac{1500}{11.39} = 131.69$

So we have coordinate (0, 131.69)

Also, in (3.8) solve for X when y= 0

$7.45x + 11.39(0) = 1500$

$7.45x = 1500$

$X = \frac{1500}{7.45} = 201.34$

Similarly we have coordinate (201.34, 0)

From (3.8) we have two coordinates viz: (201.34, 0) and (0, 131.69)

In equation (3.9), let us calculate the value of x and y as follows:

Given

$$7.98x + 10.51y = 1500 \quad (3.9)$$

If the same procedures are observed, we have coordinates (187.97, 0) and (0, 142.72).

Having gotten the required coordinates from each of the equations, we will proceed to plotting the graph (see Figure 1)

From the graph, it could be deduced that points ABCD represents the feasible region while point C is the point of equilibrium. We now evaluate each of the interceptions to determine the outlay that offers maximum returns with minimal total risk.

Points	X	Y	0.9561x	0.1676xy	0.0416x	Total
A	0	0	0	0	0	0
B	131.69	0	125.91	0	0	125.91
C	80	80	76.49	1072.64	3.328	1152.458
D	0	142	0	0	2.0732	2.0732

The optimal solution will be arrived at point C, which gives the highest return with minimal risk on investment of about ₦1152.46.

The tactical portfolio selection problem aim at minimization of portfolio variance was solved using graphical approach to linear programming, the results shows that X equals 80 and Y equal 80. The implication is that in order to minimize the risk associated with the portfolio to the minimal; the typical investor should allocate his shareholding by holding 80 shares of UBA and 80 shares of UBN respectively.

5. Summary and Conclusion

This paper uses linear programming model to find tactical solution to problems relating to portfolio risk minimization. Historical data from the Nigeria Stock Exchange on share prices of United Bank for Africa (UBA) and Union Bank of Nigeria (UBN) for the months of September and October 2013 were used. Our results shows how optimal allocations of investible funds could be made to each bank’s stock by minimizing the portfolio variance, thus by minimizing the total risk using graphical method of linear programming.

We recommend that investors use this approach to obtain optimal solution as an escape root out of business collapse.

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Appendix

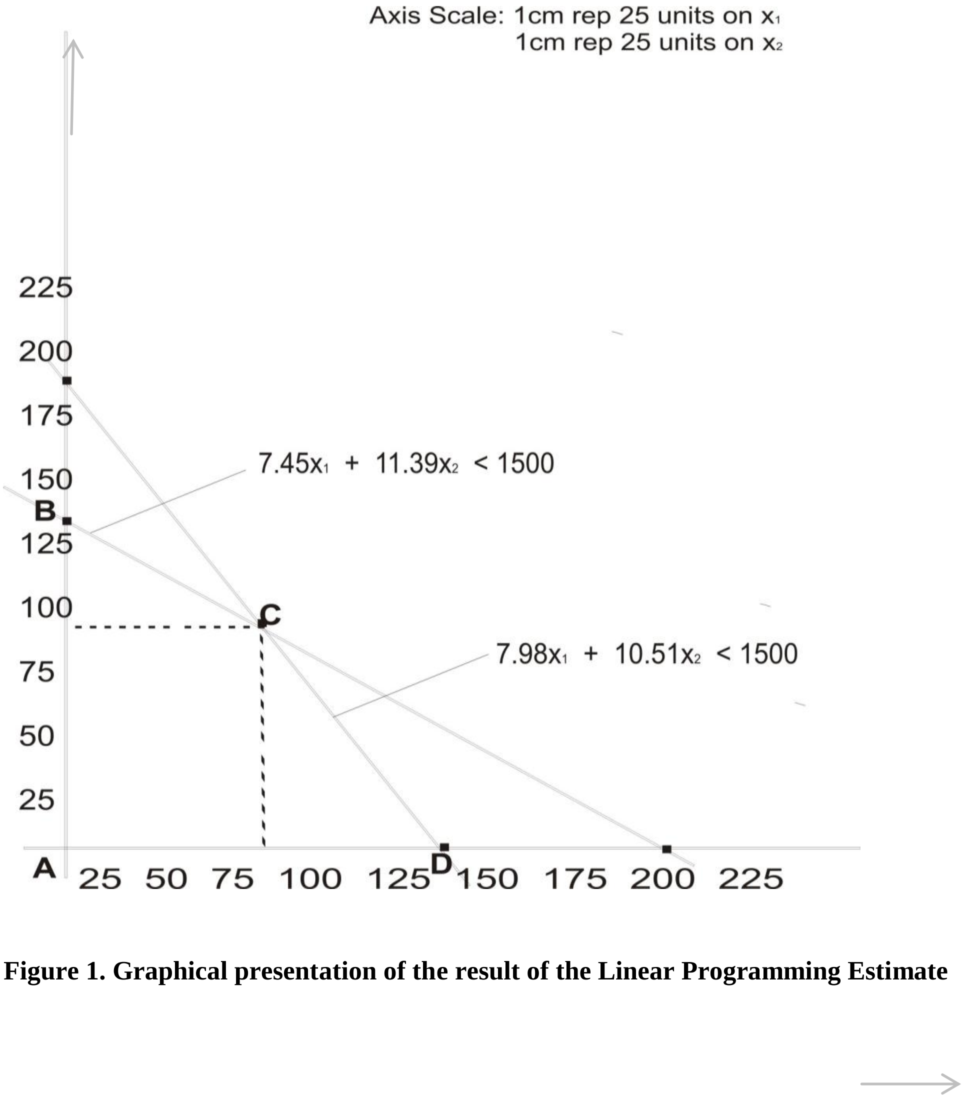

Figure 1. Graphical presentation of the result of the Linear Programming Estimate