Mathematical and Quantative Methods

Mathematics and Microeconomics

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Abstract: The article deals a number of issues regarding the use of mathematics in economics. The end of construction entails a different approach. Good organization of its with bright windows on each floor, gives confidence and calls the frightened yesterday, to come and admire both crystal mirrors (*outstanding results*) facing each other, which increases in a continuous recurring building details. On each floor, the visitor is coming from one of the windows and enjoys the scenery as you climb, always different, more comprehensive and fascinating.

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1. Introduction

"*The gate through which you can broach the Greek world, is not compulsive Homer. Greek geometry is a broad gate, in which the eye may include a landscape austere, but essential*" postulated half a century, one of the most complex personalities of this people: Dan Barbilian.

Mathematical abstractions arising often in practical problems, or even pure play, building a towering edifice in which binders are born inferences, occasionally appearing new milestones that ascends to heaven the human creation.

Like any building found during construction, mathematics frightens the viewer with its network, impregnable and sometimes difficult explicable. Network of beams and nodes (reasoning and theorems) depicts a set unsuspecting passersby scary transmitting the faint hearted unequivocal message: „Stay away and, especially, do not try to break any beam because the whole edifice will collapse !".

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The foolhardy which dares to venture through the maze of ideas will remain often ecstatic in front of nodal points that support the grandiose construction.

The end of construction entails a different approach. Good organization of its with bright windows on each floor, gives confidence and calls the frightened yesterday, to come and admire both crystal mirrors (*outstanding results*) facing each other, which increases in a continuous recurring building details. On each floor, the visitor is coming from one of the windows and enjoy the scenery as you climb, always different, more comprehensive and fascinating.

The share of the third dimension where the foolhardy stops, gives courage, detaching it from that point definitively dull bi-dimensional world of ordinary bystanders.

Might object at this point, based on justice: what does this approach to the economy?

Let's get imaginary on the road in the so concrete world economy. There are countless monuments of thought, human creation, which the visitor can not exclude from its itinerary. Among the various possible destinations are unpaved or potholed roads, other paved roads passing through intermediate cities or highways fast. Our "economic" traveler can take a rod and leave the way to an objective. In this situation, he can get in some time, if it is within a reasonable distance, or, in one day, would give sitting exhausted on the roadside.

But if he start driving a car on a road full of potholes, will certainly arrive faster, but will not be able to admire the beautiful scenery along the way, in the same situation being economist with large gaps in mathematics preparation.

On a paved road (*solid mathematical knowledge, but outdated*) he can enjoy the view over the valley bridges or tunnels connecting the knowledge unique economic views. Discomfort trip but will get tired, blurring sublime moments spent.

Traveler who chooses ordinary roads (*serious mathematical knowledge*) will arrive on time and in addition, may lie in different places, maybe not as great as the final, but that can reveal, often unexpected aspects.

Exceptional mathematical concepts and techniques will open to the researcher's fast highway that will enable achievement of objectives in a very short time. In this case, it is however possible that in the car chase, the economist can no longer admire the scenery, like the traveler through holes. How quickly will achieve its purpose but will compensate sometimes austere approach.

We can not conclude this parable not to mention a special category of "business travelers", i.e. those few that cross the land both by car and helicopter. If the first option will reveal new aspects of the landscape, the second will give the opening get, and interdisciplinary, allowing the birth of globalizing theories, systematizer that will open glances possible new economic destinations.

2. Trends pro and against the applications of mathematics in economics

The beginnings of serious application of mathematics in economics can be identified by mid-century XVI when, with the mathematical analysis of gambling begun by Gerolamo Cardano and continued by Blaise Pascal and Pierre de Fermat, was born the probability theory. Naturally, for us today, in 1662 was published the work „Natural and Political Observations Made upon the Bills of Mortality" by John Graunt, in which generating the mortality tables, he put actually the foundations of statistics. Essential contributions at the beginnings of statistics were made by Jacques Bernoulli in "Ars conjectandi" then continued research by Nicholas Bernoulli and Daniel Bernoulli in the early part of the XVIII century. In the same period, Gottfried Achenwall introduces the term "Staatswissenschaft" including the meaning to all the knowledge necessary for good governance, how will derive the word "statistics".

An essential landmark of the application of mathematics in economics is the work of Sir William Petty in the middle of the XVII century, which, influenced by the thinking of the founder of empiricism Francis Bacon, lay the groundwork of the so-called Political Arithmetic which stresses and use only quantitative methods and measurable phenomena, to the detriment of quality.

A first important application of mathematics was performed by Johann Heinrich von Thünen which, in "The Isolated State" published in 1826, lays the foundations of the theory of marginal productivity.

But the one who put the foundations of mathematics in economics application was, after Irving Fisher, the English economist and logician William Stanley Jevons who, in "The Theory of Political Economy", published in 1871, introduces the utility theory. Also, he had introduced in 1866 the concept of marginalism, continued idea in "Elements d' économie politique pure" by Léon Walras.

The nineteenth century brought to the fore the work of Antoine Augustin Cournot – French mathematician and philosopher who, in 1838, in "Researches on the Mathematical Principles of the Theory of Wealth" uses mathematics calculus to analyze duopolies, being one of the pillars of economic analysis. Also, Francis Ysidro Edgeworth in "Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences" introduces a number of elements of mathematics for economic analysis.

The modern era brings an effervescence in the treatment of economic phenomena using mathematics. Vilfredo Federico Damaso Pareto, author of efficiency, distribution or principle (80-20 rule) that today bears his name, Paul Samuelson, whose approach to determine a unifying mathematical structure of several economic sectors has shifted the scientific approach of a whole category scientists, John von Neumann and Wassily Leontief are personalities of economy who have very successfully used mathematical techniques.

In contrast marginalists, the Austrian School imposed by Ludwig von Mises and includes brand names such as Friedrich Hayek, Fritz Machlup, Carl Menger and Felix Kaufmann, categorically opposes to the use of mathematics in economics. The main argument in support of this attitude is that the complexity of human behavior generates countless difficulties of a mathematical approach.

A main objection of the Austrian School is relative to the nature of the goods. In the conception of representatives, concrete goods are indivisible. Arguments are that the act of consumption is usually conducted in complete units of product as opposed to infinitesimal quantities. Also, a pertinent observation is that the utility has not an intrinsic nature, being only specific to the asset, depending extrinsic from to the person who consumes it. The individuality of consumption makes it difficult to measure the utility aggregation at a community. Moreover, even assuming infinite divisibility of asset, during its use, the individual may change its preferences in favor of giving another. We could give an example, the act of choosing a television program. The consumer can assign different utilities before the actual process of program selection but after the election, during the course of it, he can change his preference. Judging the set of consumers, the aggregate utility is a dynamic phenomenon, difficult to control.

Also, the utility of a product may change from day to day or week to week. We here give the example of the usefulness of an umbrella on a rainy day and the same object in a cloudy day or a sunny day.

Another aspect that can be taken into account, in the Austrian School argument, is that of moving between different consumption goods without necessarily follow the mathematical logic of this act. Thus, consider an individual's choice between spending a holiday in a location that it will cost, say, 1000€ - when incurred or 1400€ - at the time of actual departure and staying at home to perform general cleaning house will cost 1200€. When an individual has an amount of 1000€, he has to choose between buying the ticket of leave or delay this act. It is possible that consumers may not save, finally, the amount of 1200€, at which point it will not be able to choose between any of the variants. Also, the character of substitutability of the two actions vary depending on the amount of time available. Thus, at an amount of 1000€, he can not choose but to leave, at 1200€ (*decision deferred in time*) has only alternative cleaning and at 1400€ may opt for either of the two variants.

At the level of aggregation, the utility becomes difficult to control and quantify. Thus, an individual working far away from home, has to choose between buying a car (*still living rent*) and buying a home. In this case, it is likely that the choice will be for the car (*assuming it has enough money to purchase one or the other but not both*). On the other hand, the same individual in the position to choose between buying two cars or two dwellings, will probably choose the second option, preferring renting the second place unnecessary holding of a second car. Even if he would rather purchase a pair car - housing, utility changes its specificity lower quantities.

3. Objective and Subjective Limitations of the Applications of Mathematics in Economics

Applying mathematical calculation in a series of economic situation requires great prudence, both conceptually (*what or how to use*) and the technical level or concluded.

We give here the example of a concrete situation. Consider the problem of purchasing two products A and B, with prices $p_A=11$ and $p_B=10$, the consumer's income being $V=184$€. If the utility function is of Cobb-Douglas type: $U(x,y)=x^{0.37}y^{0.63}$, the classical theory leads to the solution: $x_0=6.189$ pcs. of product A, $y_0=11.592$ pcs. of product B, the utility being: $U_{t0}=9.190$. Everything seems very fair here. What do we do if products A and B are indivisible? Basically there are two methods of choice. First, would be to rounding the values obtaining: $x_1=6$ and $y_1=12$ the total utility being $U_{t1}=9.285$. However, the necessary budget will be: $V_1=11\cdot 6+10\cdot 12=186>184=V$ so the solution is unacceptable. A second way would be to consider the integer values of the original solution that would meet within budget. Thus, at the values $x_2=6$ and $y_2=11$ the resulting utility is: $U_{t2}=8.790$. The algorithms presented in our the paper "Two methods of determination of an acquisition program in integer numbers" lead to an optimal solution (*in integers!*): $x_3=4$ and $y_3=14$ the budget condition $V_3=11\cdot 4+10\cdot 14=184$ being satisfied at the limit, and $U_{t3}=8.807$ greater than the previous.

Another example, which we consider to be treated, is those of production functions. Considering the general expression of such a function: $Q:\mathbf{R}_+\times\mathbf{R}_+\to\mathbf{R}_+$, $Q=Q(K,L)$ where Q is output, K - capital and L - work, the classical approach is to consider the isoquants, namely that curves on the surface $Q=Q(K,L)$ for which $Q=Q_0$ – constant. The results are interesting but, in our opinion, have a static character, reflecting only the degree of substitution between the two factors: labor and capital, but not the global nature of production. Curves obtained by varying the level of Q_0 allow a planar approach more qualitative than quantitative.

To better understand this phenomenon, consider the case of a hemisphere and a cone, height equal to the radius of the hemisphere, "placed" on a plane. The level curves for the different elevations will be circles in the plane of the base. Moving on the third axis with a constant pitch will generate a series of concentric circles close to each other in the case of cone and outlying in the case of the spheres, the limits being the large circles and the circle with null radius in the cone's case or the north pole of the hemisphere. Flat approach of these projections do not allow a highlighting of the properties of these areas, they being still fundamentally different.

For this reason, for a comprehensive and global approach of the phenomenon should be considered the production function overall like surface. The approach, from this point of view, involves a series of mathematical problems, difficult or sometimes hard to overcome. On one hand, highly specialized nature of the differential geometry of surfaces is inaccessible to the vast majority of researchers in economy, because the specific mathematical training at all the great universities of the world is insufficient. On the other hand, the three-dimensional approach involves a series of complex calculations including partial differential equations, traditionally more specific to applications in physics. Might object here the brutal overcoming of borders of application of mathematics in economics, but the work of Nicholas Georgescu-Roegen, "The Entropy Law and the Economic Process", published in 1971, masterfully applies entropy - a concept of thermodynamics, in economic theory.

Returning to the production functions, a study of their shows a number of properties thereof, such as:

- all points of production area are of parabolic type that is it is conical (*a surface generated by a straight line passing through a fixed point and a given curve*) or cylindrical (*a surface generated by a straight line rests on a given curve and parallel to a fixed direction*). The surface of a production function is ruled (*surface generated by a straight line moving again after a law*). The proof of the result is elementary. Considering a homogeneous function of degree 1, we have $Q(rK,rL)=rQ(K,L)$ where, with the notation $\chi=\frac{K}{L}$ we have: $Q(K,L)=Q(\chi L,L)=L\cdot Q(\chi,1)$. Noting $S(\chi,L)= Q(\chi L,L)=Q(K,L)$ and $q(\chi)=Q(\chi,1)$ we get: $u=S(\chi,L)=0+L\cdot q(\chi)$. Considering the position vector of the surface in the coordinate system (χ,L,u) we get: $(\chi,L,u)= (\chi,L,S(\chi,L))=(\chi,L,L\cdot q(\chi))= (\chi,0,0)+L(0,1,q(\chi))$ – the equation of a ruled surface.
- in any point of a surface production the total curvature is zero. The significance of this result is, according Minding 's theorem, that the production area is

locally isometric to a plane, i.e. there is a bijective application between points of the surface and those of a plane, keeping distance.

- the formal determination of the equations of the surface geodesics (*curves of minimum length*) determines the "short" way to transition from one production to another for the purposes of minimum capital and labor changes during this process.

The approach of the production functions with multiple factors, for example, by introducing the ground factor, complicates more from the point of mathematical view. The instrumentation of the differential geometry in the case of n-dimensional spaces is a very specialized, applied to the present, to the majority of relativistic researches.

Another problem particularly complicated, unresolved to the present, is the construction of a surface or an n –dimensional manifold, based on a series of discrete data generated by current practice, situation completely solved for two-dimensional case (regression functions, polynomial interpolation etc.). With the exception of multinomial regression, which generates a hyperplane in $\mathbf{R}^n$ and have a relatively simple solution, the phenomena in more than three dimensions involves difficulties, at least for now, insurmountable.

The analysis can be continued in many other situations. In our view, a major problem in mathematization of the economy is the lack of an axiomatic, on the, at one hand applicable to a large economic phenomena, and on the other coherent and not speculative. The argument of these ideas go, mainly from the same concept of utility, debated and disputed by all economic schools.

The axiomatization of preference or indifference relations necessary for the subsequent theory, especially the compatibility with a range of outcomes is punishable pretty strong criticism. Transitivity property relationship preference is often violated. Thus, it is possible that in the case of the temporal relationship, or preferably slightly offset applied to different situations, transitivity is not made. Thus, we prefer lemon to a teaspoon of sugar (*if it is a cup of tea*) and prefer a teaspoon of mustard instead of lemon (*if it is a portion of sausage*). It is unlikely that, in any event, to prefer one teaspoon sugar mustard!

Also, relatively to marginal issues, the First Gossen's law states that the marginal utility is a decreasing function with respect to increasing the quantity of a good consumed. We ought to do here a crucial observation. Usually, all the phenomena and analyzed indicators involve satisfy additional requirements arising from the observation of a large number of events (*along economic history*). On the other hand, there are many situations in which economic phenomenon does not satisfy the theoretical conditions. What then is to be done? The answer is basically simple: nothing! In the beginning of any economic phenomenon there are a number of factors that diverge evolves from normal economic regularities. After stabilization thereof in order to eliminate or at least diminish the economic laws disturbances, the process tends to approach the theoretical models such as the condition of application, as well as development.

Clarify now what was said by some examples. A trader opens a store in an area relatively isolated, with not too much competition. In the early days, record usually two types of sales: either buyers will come in large numbers (*though it has conducted an aggressive campaign advertising*) attracted not necessarily need to purchase but also out of curiosity or in the number very small because of the position which has not a very good location. As time passed, the store makes its segment and a well-defined volume of buyers that will stabilize and the daily sales. Just in this moment, that company may pose serious problem for law enforcement market economy.

Another class of examples is that of vices. In these cases, the first quantities consumed will increase the marginal utility obvious violation of the First law of Gossen. As the evolution of consumption, their harmful effects, reduces the utility gain finally reaching it to be negative.

Therefore, in many economic theories, we will consider the theoretical and practical situations occur at some point on when the economic laws are applicable.

A mathematized construction of the economy also requires a special attention in the act of modeling. A limitation of the aspects of the problem, for an easy mathematization will lead, often, rarely confirmed conclusions of economic practice. On the other hand, a consideration of a sufficient number of variables may result in an impossibility of solving analytical thereof. We can give the example of Krugman 's model space relative to the two regions which is not solvable analytically. Therefore, a compromise between scientific rigor and the ways of solving math must be done very carefully, removing if its mathematization sake.

4. Conclusions

The end of this paper should answer, inevitably, to the question: is mathematics for economics, a necessity or sufficiency? Following the arguments presented above, the answer comes somewhat natural: it is a necessity but not sufficiency take and place!

The need arises from the growing complexity of the current economic problems, the creation of new models of development, becoming more complex in the context of traditional resources exhaustion or in case of imbalances as one of major global economic crisis.

A non-mathematical approach can sometimes give satisfactory solutions, but high risk in their application at the macroeconomic level. On the other hand, solving math must take into account the qualitative and social aspects that can influence the final solution. Insufficient application without precautions mathematical methods in economics can be exemplified by the famous problem of diet solved by GB Dantzig. In order diet composition as cheap (*scientist must keep a diet*), it calculates the weight difference between the 500 own good food and its water content, aiming to maximize loads and results. The result was hilarious. Considering that the vinegar has a water content equal to 0, the model suggested consumption of 1893 liters / day (500 gallons / day) which obviously, for the sake of economics, but also his personal Dantzig did!

5. References

Barbu, I. (1968). *Pagini de proză/ Pages of prose.* EPL.

Fertig, L. (1971). *The Genius of Mises' Insights*, vol. Toward Liberty, pp. 123-131, Institute for Human Studies, Inc.

Ioan, C.A. (2008), *Matematică–II/Mathematics.* Galati: Sinteze

Ioan, C.A. (2009). The Production Functions from The Point of View of 3-Dimensional Geometry. *Acta Universitatis Danubius. Œconomica*, Vol. 8, No.1, pp. 7-15.

Ioan, G. & Ioan, C.A. (2010). Two methods of determination of an acquisition program in integer numbers. *Acta Universitatis Danubius. Œconomica*, Vol.9, No. 4, pp. 126-131.

Jevons, W.S. (1866). Brief Account of a General Mathematical Theory of Political Economy. *Journal of the Royal Statistical Society*, London, XXIX (June 1866), pp. 282-287.

Krugman, P. (1991). Increasing Returns and Economic Geography. *Journal of Political Economy*, 99, pp. 483-499.

Mises, L.v. (1998). *Human Action – A Treatise On Economics*. Ludwig von Mises Institute.

Schumpeter, J.A. (2006). *History of Economic Analysis*. Taylor&Francis e-Library.

Walker, D.A. (2006). *Walrasian Economics*. Cambridge University Press.