A Conjecture Concerning Prime Numbers

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Abstract. The paper examines the conditions under which a second degree polynomial generates primes variable values between 0 and the constant term - 1. It is shown that for values of the constant term equal to or less than 41, such polynomials exist and we are proposing a conjecture that for the polynomials with the constant term greather than 41 the statement is not true.

Keywords: prime; polynomials

1. Introduction

The prime number theory dates back to ancient times (see the Rhind papyrus or Euclid's Elements).

A number $p\in\mathbf{N}$, $p\geq 2$ is called prime if its only positive divisors are 1 and p. The remarkable property of primes is that any nonzero natural number other than 1 can be written as a unique product (up to a permutation of factors) of prime numbers to various powers.

If there is not a formula, for the moment, generating prime numbers, there exist a lot of attempts (all unsuccessful) to determine it.

Unfortunately, many results about primes are at the stage of conjectures (theorems that seem to be valid, but remained unproven yet).

2. Main Results

Let $P(n)=an^2+bn+c$, $a,b,c\in\mathbf{Z}$, $c\geq 2$, $(a,b,c)=1$.

We propose the determination of $a,b,c\in\mathbf{Z}$ such that $P(n)$=prime $\forall n=\overline{0,c-1}$.

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Using the Wolfram Mathematica software, in order to determine the polynomials (for c≤41):

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Clear["Global`*"];
polynomial[a_,b_,c_,n_]=a*n^2+b*n+c;
lowerlimitc=2;
upperlimitc=41;
upperlimit=200;
For [c=lowerlimitc,c<=upperlimitc,c++,
        maximuma=0;
        maximumb=0;
        maximumc=0;
        maximumcounter=1;
        increase=2*upperlimit;
        If[PrimeQ[c],
                For[a=-upperlimit,a<=upperlimit,a++,
                For[b=-upperlimit,b<=upperlimit,b++,
                        counter=0;exit=0;
                        If[a≠0,For[n=0,n<c,n++,
                        If[PrimeQ[polynomial[a,b,c,n]]&&exit=0,counter=cou
                nter+1,exit=1]]];
                        If[exit=0&&counter≥
                maximumcounter,If[(Abs[a]+Abs[b])<
                increase,maximumcounter=counter;maximuma=a;maximumb
                =b;
                maximumc=c;increase=Abs[a]+Abs[b]]]]]];
If[maximumcounter≥2, Print["a=",maximuma," b=",maximumb,"
c=",maximumc];
For[n=0,n<c,n++,Print[polynomial[maximuma,maximumb,maximumc,x]]],If[
PrimeQ[c],Print["c=",c," accepts no polynomial"]]]]
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we get:

c	$P(n)=an^2+bn+c$	Module of Primes
2	$P(n)=n^2+2$	2,3
3	$P(n)=n^2-n+3$	3, 3, 5

5	$P(n)=n^2-n+5$	5, 5, 7, 11, 17
7	$P(n)=2n^2-2n+7$	7, 7, 11, 19, 31, 47, 67
11	$P(n)=n^2-n+11$	11, 11, 13, 17, 23, 31, 41, 53, 67, 83, 101
13	$P(n)=6n^2+13$	13, 19, 37, 67, 109, 163, 229, 307, 397, 499, 613, 739, 877
17	$P(n)=n^2-n+17$	17, 17, 19, 23, 29, 37, 47, 59, 73, 89, 107, 127, 149, 173, 199, 227, 257
19	$P(n)=2n^2-2n+19$	19, 19, 23, 31, 43, 59, 79, 103, 131, 163, 199, 239, 283, 331, 383, 439, 499, 563, 631
23	$P(n)=3n^2-3n+23$	23, 23, 29, 41, 59, 83, 113, 149, 191, 239, 293, 353, 419, 491, 569, 653, 743, 839, 941, 1049, 1163, 1283, 1409
29	$P(n)=2n^2+29$	29, 31, 37, 47, 61, 79, 101, 127, 157, 191, 229, 271, 317, 367, 421, 479, 541, 607, 677, 751, 829, 911, 997, 1087, 1181, 1279, 1381, 1487, 1597
31	$P(n)=-3n^2+45n+31$	31, 73, 109, 139, 163, 181, 193, 199, 199, 193, 181, 163, 139, 109, 73, 31, 17, 71, 131, 197, 269, 347, 431, 521, 617, 719, 827, 941, 1061, 1187, 1319
37	$P(n)=-4n^2+76n+37$	37, 109, 173, 229, 277, 317, 349, 373, 389, 397, 397, 389, 373, 349, 317, 277, 229, 173, 109, 37, 43, 131, 227, 331, 443, 563, 691, 827, 971, 1123, 1283, 1451, 1627, 1811, 2003, 2203, 2411
41	$P(n)=n^2-n+41$	41, 41, 43, 47, 53, 61, 71, 83, 97, 113, 131, 151, 173, 197, 223, 251, 281, 313, 347, 383, 421, 461, 503, 547, 593, 641, 691, 743, 797, 853, 911, 971, 1033, 1097, 1163, 1231, 1301, 1373, 1447, 1523, 1601

For c≥43 we have that from the condition that $P(n)=an^2+bn+c$ =prime ∀n= $\overline{0,c-1}$ implies that a+b=even. Indeed, if the prime is different from 2, we have that: an^2+bn =even for any n= $\overline{0,c-1}$ except possibly two values for which $P(n)$ =2.

But for two consecutive numbers n, n+1 such that $P(n)=an^2+bn+c$ =$prime_1$≠2, $P(n+1)=a(n+1)^2+b(n+1)+c$ =$prime_2$≠2 we have that: $an^2+bn=\text{even}$, $a(n+1)^2+b(n+1)=\text{even}$ therefore the difference: $2an+a+b=\text{even}$ that is $a+b=\text{even}$.

The structure of the polynomial becomes then:

$$P(n)=an^2+(2d-a)n+c\text{, } a,b,d\in\mathbf{Z}$$

Investigating for values of c greater than 41 to 1000, we have not found polynomials with this property, considering reasonable limits for a and b less than 200.

Finally we state the following:

Conjecture

There is not a polynomial $P(n) = an^2 + bn + c$, a,b,c∈**Z**, c≥43, $(a, b, c) = 1$ such that $P(n)$=prime ∀n=$\overline{0, c-1}$.

3. References

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