Mathematical and Quantative Methods

A Study of Integers Using Software Tools – III

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Abstract: The paper deals with a generalization of polite numbers that is of those numbers that are sums of consecutive integers to the numbers called almost polite numbers of order p,m which can be written in m ways as sum of two or more consecutive of same powers p of natural numbers.

Keywords: polite numbers; divisibility

1 Introduction

Let note for any $n\in\mathbf{N}^*$, $p\in\mathbf{N}^*$, $S_{n,p}=1^p+...+n^p$ and $S_{k,n,p}=k^p+...+n^p=S_{n,p}-S_{k-1,p}$, $k=\overline{2,n}$.

It is well known that:

$$S_{n,p}=\frac{(n+1)^{p+1}-1-\sum_{j=1}^{p}C_{p+1}^{j+1}S_{n,p-j}}{p+1}$$

and also:

$$S_{k,n,p}=\frac{(n+1)^{p+1}-k^{p+1}-\sum_{j=1}^{p}C_{p+1}^{j+1}S_{k,n,p-j}}{p+1}$$

It is easy to see that the first 10 sums are:

$$S_{n,1}=1+...+n=\frac{n(n+1)}{2}$$

$$S_{n,2}=1^2+...+n^2=\frac{n(n+1)(2n+1)}{6}$$

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$S_{n,3}=1^3+\ldots+n^3=\frac{n^2(n+1)^2}{4}$

$S_{n,4}=1^4+\ldots+n^4=\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$

$S_{n,5}=1^5+\ldots+n^5=\frac{n^2(n+1)^2(2n^2+2n-1)}{12}$

$S_{n,6}=1^6+\ldots+n^6=\frac{n(n+1)(2n+1)(3n^4+6n^3-3n+1)}{42}$

$S_{n,7}=1^7+\ldots+n^7=\frac{n^2(n+1)^2(3n^4+6n^3-n^2-4n+2)}{24}$

$S_{n,8}=1^8+\ldots+n^8=\frac{n(n+1)(2n+1)(5n^6+15n^5+5n^4-15n^3-n^2+9n-3)}{90}$

$S_{n,9}=1^9+\ldots+n^9=\frac{n^2(n+1)^2(n^2+n-1)(2n^4+4n^3-n^2-3n+3)}{20}$

$S_{n,10}=1^{10}+\ldots+n^{10}=$

$$\frac{n(n+1)(2n+1)(n^2+n-1)(3n^6+9n^5+2n^4-11n^3+3n^2+10n-5)}{66}$$

All over in this paper, the software presented was written in Wolfram Mathematica 9.0.

2 Almost Polite Numbers of Order p,m

A natural number N greather than 2 will be called almost polite number of order p if it can be written as sum of two or more consecutive of a same power p of natural numbers.

If N is odd it is natural that for N=2k+1 we have N=k+(k+1) therefore each odd natural number is polite of order 1.

A natural number N greather than 2 will be called almost polite number of order p,m if it can be written in m ways as a sum of two or more consecutive of same powers p of natural numbers.

Therefore N is a polite number of order p,m if:

$$N=S_{k_1,n_1,p_1}=S_{k_2,n_2,p_2}=\ldots=S_{k_m,n_m,p_m} \text{ with } p_1\neq p_2\neq\ldots\neq p_m$$

The software for determining the almost polite numbers of order p,m ($m\geq3$) limited to 30000 and powers less than or equal with 10 is:

```
Clear["Global`*"];
limit=30000;
pmax=10;
nrorimax=3;
nrk=Table[i,{i,nrorimax}];
nrn=Table[i,{i,nrorimax}];
nrp=Table[i,{i,nrorimax}];
S[0]=n;
(*The calculus of sums of powers from 1 to n*)
For[p=1,p≤pmax,p++,
 suma=0;
 For[j=1,j≤p,j++,suma=suma+Binomial[p+1,j+1]*S[p-j]];
 S[p]=Factor[((n+1)^(p+1)-1-suma)/(p+1)]]
(*The calculus of sums of powers from k to n*)
For[p=1,p≤pmax,p++,sumpower[n_,p]=S[p]];
For[p=1,p≤pmax,p++,sumpowerkn[n_,k_,p]=Factor[Simplify[sumpower[n,p]-
sumpower[k-1,p]]]]
(*The analysis*)
For[number=2,number≤limit,number=number+1,nrori=0;
 For[p=1,p≤pmax,p++,
  For[n=2,n≤number^(1/p),n++,
   For[k=1,k≤n-1,k++,

If[sumpowerkn[n,k,p]==number,nrori=nrori+1;nrk[[nrori]]=k;nrn[[nrori]]=n
;
nrp[[nrori]]=p];
    If[nrori≥2&& nrp[[nrori]]==nrp[[nrori-1]],nrori=nrori-1]]]];

If[nrori≥nrorimax,For[k=1,k≤nrori,k++,Print[number,"=\[Sum](power=",nr
p[[k]],") from ",nrk[[k]]," to ",nrn[[k]]]];Print[""]]]
```

We found (till 30000):

- $91=1+2+...+13=1^2+...+6^2=3^3+4^3$
- $559=9+10+...+34=7^2+...+12^2=6^3+7^3$
- $855=4+5+...+41=11^2+...+15^2=7^3+8^3$
- $6985=9+10+...+118=20^2+...+30^2=9^3+...+13^3$
- $19721=200+201+...+281=14^2+...+39^2=4^6+5^6$
- $24979=12489+12490=62^2+...+67^2=5^4+...+10^4$
- $29240=29+30+...+243=35^2+...+50^2=2^3+...+18^3$

3 References

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