Mathematical and Quantative Methods

The Prediction of Exchange Rates with the Use of Auto-Regressive Integrated Moving-Average Models

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Abstract: Currency market is recently the largest world market during the existence of which there have been many theories regarding the prediction of the development of exchange rates based on macroeconomic, microeconomic, statistic and other models. The aim of this paper is to identify the adequate model for the prediction of non-stationary time series of exchange rates and then use this model to predict the trend of the development of European currencies against Euro. The uniqueness of this paper is in the fact that there are many expert studies dealing with the prediction of the currency pairs rates of the American dollar with other currency but there is only a limited number of scientific studies concerned with the long-term prediction of European currencies with the help of the integrated ARMA models even though the development of exchange rates has a crucial impact on all levels of economy and its prediction is an important indicator for individual countries, banks, companies and businessmen as well as for investors. The results of this study confirm that to predict the conditional variance and then to estimate the future values of exchange rates, it is adequate to use the ARIMA (1,1,1) model without constant, or ARIMA [(1,7),1,(1,7)] model, where in the long-term, the square root of the conditional variance inclines towards stable value.

Keywords: ADF; stationarity; ARIMA; EUR; prediction

JEL Classification: C32; C53; F31

1 Introduction: Literature Review

In today's global economy, the crucial importance for any future investments is the accuracy in predicting the foreign exchange rates or at least the correct prediction of the trend. There already are a great number of methods for predicting the exchange rates. It was shown by Robert Meese (MEESE R., 1983) that models based on the random walk hypothesis in predicting exchange rates are better than those based on macroeconomic indicators. However, this does not apply for the

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long-term prediction, which was proved by examining the prediction of USD exchange rate against four other currencies during seventeen years (MARK N., 1995).

Predictions of exchange rates made with the use of ARIMA models were started in nineties by economists Bellgard and Goldschmidt (BELLGARD C., 1999). However, they concluded that these models are not very suitable for predicting the exchange rates. Dunis and Huang (DUNIS C., 2002) who were using ARMA (4,4) were of the opposite opinion; their results were, however, insignificant.

Another example of a study using Box Jenkins models is for instance the paper "Exchange-rates forecasting: exponential smoothing techniques and ARIMA models", in which the authors investigated the behavior of daily exchange rates of the Romanian Leu against Euro, United States Dollar, British Pound, Japanese Yen, Chinese Renminbi and the Russian Ruble (FĂT M., 2011).

Weisang and Awazu (WEISANG G., 2008) presented three ARIMA models which used macroeconomic indicators to model the USD/EUR exchange rate. They discovered that over the time period from January 1994 to October 2007, the monthly USD/EUR exchange rate was best modeled by a linear relationship between its preceding three values and the current value. These authors also concluded that ARIMA (1,1,1) is the most suitable model for the prediction of the time series of USD/EUR exchange rate.

Another often used method for predicting the trend of exchange rates is the ANN model (Artificial Neural Network). Kamruzzaman J. a Ruhul A. Sarker (KAMRUZZAMAN J., 2003) developed and investigated three ANN based forecasting models using Standard Backpropagation (SBP), Scaled Conjugate Gradient (SCG) and Backpropagation with Baysian Regularization (BPR) for Australian Foreign Exchange to predict six different currencies against Australian dollar.

One of the recent studies (ROUT M., 2013) uses the hybrid model combining an adaptive autoregressive moving average (ARMA) architecture and differential evolution (DE) based on training of its feed-forward and feed-back parameters. The results of the developed model are compared with other four competitive methods such as ARMA-particle swarm optimization (PSO), ARMA-ca t swarm optimization (CSO), ARMA-bacterial foraging optimization (BFO) and ARMA-forward backward least mean square (FBLMS). The derivative based ARMA-FBLMS forecasting model exhibits the least suitable prediction performance of the exchange rates. Compared to that, ARMA-DE exchange rate prediction model possesses superior short and long range prediction potentiality compared to others. Many studies are dealing with the prediction of USD/EUR, USD/YEN or

USD/RON. The originality of this paper lays in the prediction of EUR against other European currencies for the long-term time horizon (2014-2020).

2. Methodology

In this paper, there are models of time series of monthly exchange rates of the national currencies not including the common European currency, for the time period of 12/1998 to 12/2013. These currencies are of the Czech republic (CZK), Poland (PLN), Great Britain (GBP), Romania (RON), Sweden (SEK) and Hungary (HUF). The data were obtained from the ECB database and they contain values of the selling price of each currency, specifically the average value for each month at the foreign exchange market FOREX. In total, there are 180 observations in the time series. Countries that do not use Euro but have fixed exchange rate were not included in this prediction. This applies for Bulgaria (1EUR=1,95583BGN), Denmark (1EUR=7,46038DKK) and Lithuania (1EUR=3,4528LTL).

To obtain the adequate ARIMA (p, d, q) model, the series stationarity was tested by applying the ADF-Augmented Dickey-Fuller (DICKEY&FULLER, 1979) and PP-Phillips-Perron unit root tests (PHILLIPS P., 1988). ADF was performed for the scenario with a constant, without a constant and with a trend. The most suitable appears to be the model with a constant, the results of which are shown below in the table no. 1 for different currencies. The results of these tests regarding non-stationarity of the indices are the same, namely the series EUR/RON, EUR/SEK, EUR/GBP, EUR/HUF are non-stationary (the null hypothesis of the unit root existence cannot be rejected, i.e. it is not a stationary time series).

Table 1. Augmented Dickey-Fuller (ADF) test with a constant for the currencies CZK/SEK/GBP/PLN/HUF/RON

Results/currency	CZK	SEK	GBP	PLN	HUF	RON
Estimated value γ	-0,001	-0,03	-0,01	-0,06	-0,01	0,003
Test statistics: t	-0,36	-1,81	-0,75	-2,82	-0,22	0,96
Asymptotic p-value	0,56	0,38	0,39	0,06	0,61	0,91

Source: author (SW Gretl)

If we do not reject the null hypothesis and the given series is non-stationary, it is necessary to proceed to its transformation, as the Box Jenkins (AR, MA, ARMA or ARIMA) models are based on the time series stationarity, in the form of

$$Y_n = a_1Y_{n-1} + a_2Y_{n-2} + \ldots + a_p\, Y_{n-p} - b_{1n-1} - b_{2n-2} - \ldots b_{qn-q} +_n \quad (1)$$
$$(1-a_1L-a_2L^2 - \ldots a_pL^p)Y_n = (1-b_1L- \ldots - b_qL^q)_n \quad (2)$$
$$\phi(L)Y_n = \theta(L)\varepsilon_n \quad (3)$$

Where *p* is the order of the autoregressive part, while q is the order of the moving average part, and ε_n represents the white noise.

Validation of ARMA (p,q) models is based on minimizing the AIC (Akaik's information criterion) and BIC (Schwarz's information criterion) criteria, as well as on the verification of the correlation of the error terms of the model and finally on measuring the divergence from the normality of these values. If it is needed for the time series to have one differential operation to achieve stationarity, it is a I(1) series. Time series is I(n) in case it is to be differentiated for *n* times to achieve stationarity. Therefore, ARIMA (p,d,q) models are used for the non-stationary time series, specifically the autoregressive integrated average models, where d is the order of differentiation for the series to become stationary. Therefore the ARIMA (p,d,q) model may be rewritten as follows:

$$\phi(L)\ (1-L)^d\ Y_n = \theta(L)\varepsilon_n \quad (4)$$

where L is the lag operator and the order of differentiation is equal to:

$$\Delta^d Y_n = (1-L)^d\ Y_n \quad (5)$$

The identification of modeling the conditional mean value is based on the analysis of estimated autocorrelation and partial autocorrelation function (ACF, PACF). These estimations may be strongly inter-correlated, it is therefore recommended not to insist on unambiguous determination of the model order, but to try more models. We must not forget to carry out the verification, which is based on retrospective review of the assumptions imposed on the random errors. Given that financial data are very often characterized by high volatility, it is necessary to test the model for ARCH effect, i.e. presence of conditional heteroscedasticity. Regarding heteroscedasticity it is therefore a situation where the condition of finite and constant variance of random components is violated. The following model illustrates the conditional heteroscedasticity:

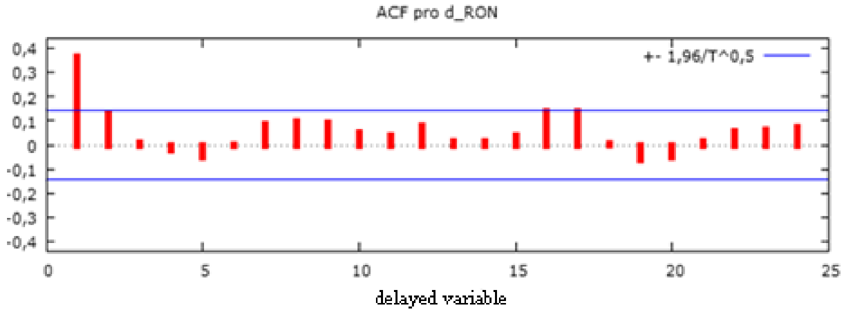
$$(lnX_t - lnX_{t-1})^2 = \alpha + \rho\ (lnX_{t-1} - lnX_{t-2})^2 + ut \quad (6)$$

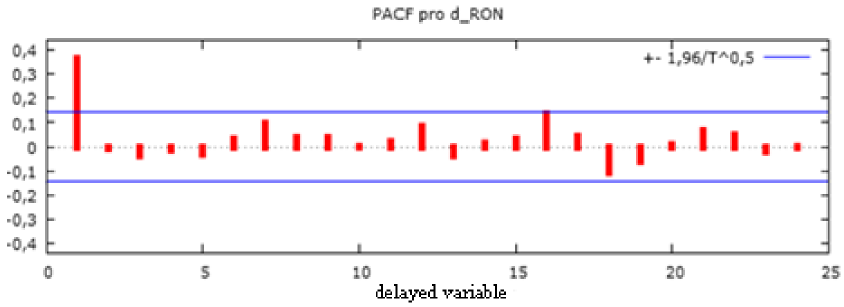
where Xt, Xt- represent values in the time series when time t is changed by one unit. The parameter α is calculated with the use of OLS and u_t is a random component. If the parameter ρ (regressive parameter) is equal to zero, we cannot talk about heteroscedasticity.

3. Results

Based on the priori information about the behavior of the exchange rates, it may be concluded that the specification of the ARIMA (1,1,1) type is an adequate choice. To verify this estimation, we generated the correlograms ACF and PACF which for most of the analyzed currencies confirm the legitimacy of the identification of the data generating process with the use of ARIMA (1,1,1). The exception is Swedish crown and Hungarian forint. Based on comparing the information criteria (AIC, BIC),ARIMA (1,1,1) model without constant was identified for the Swedish currency (SEK) and ARIMA [(1,7),1,(1,7)] model for the Hungarian currency.

Note. The model with a constant was also developed but compared to the significance of the p-value and by comparing the information criteria, it seems optimal to exclude the constant. For illustration, two ACF and PACF correlograms for the first difference for the Romanian Leu and Hungarian forint are shown below.





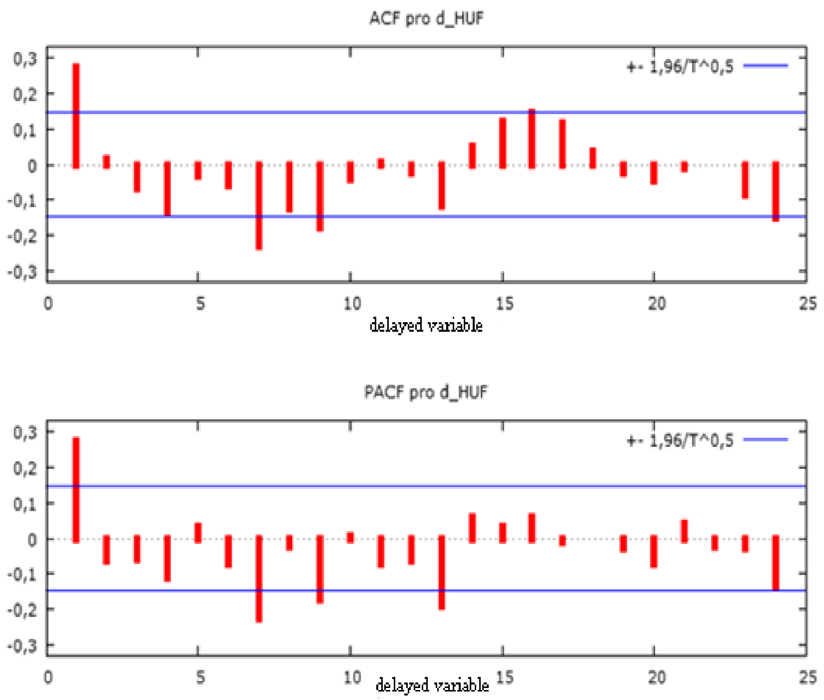


Figure 1. ACF and PACF correlograms for the first difference (RON, HUF)
Source: Author (SW Gretl)

CZK	coefficient	direct. error	z	p-value
phi_1	0,283049	0,0734204	3,855	0,0001***
theta_1	0,850331	0,0606456	14,02	1,15E-44***
SEK	coefficient	direct. error	z	p-value
phi_1	-0,527872	0,101123	-5,22	1,80e-07 ***
theta_1	0,850349	0,0606514	14,02	1,15e-44 ***
GBP	coefficient	direct. error	z	p-value
phi_1	0,186398	0,0736146	2,532	0,0113***
theta_1	-1	0,0233473	-42,83	0**
PLN	coefficient	direct. error	z	p-value
phi_1	0,391535	0,0691388	5,663	0,0000000149***
theta_1	-1	0,0148063	-67,54	0***
HUF	coefficient	direct. error	z	p-value
phi_1	0,342549	0,0850299	4,029	0,0000561***
phi_7	-0,21342	0,0702948	-3,036	0,0024***
theta_1	-1,09445	0,0527216	-20,76	1,02E-95***

theta_7	0,101755	0,0563949	1,804	0,0712*
RON	coefficient	direct. error	z	p-value
phi_1	0,346849	0,0752846	4,607	0,00000408***
theta_1	-0,972696	0,0248644	-39,12	0***

Figure 2. The estimation of the ARIMA for exchange rates with the use of 180 observations for the time period of 1.1999 – 31.12.1999 – 31.12.2013

Source: own calculations

Note. const = constant generated by SW Gretl, phi_1 = regressive coef. of AR processes at the 1. delay, theta_1: regressive coef. of MA processes at the 1. delay, z = test statistics.

From the table, we can conclude that parameters of AR member as well as of MA member are statistically significant at least on the 5 % level for all examined currencies. Then we tested the model for autocorrelation (H0: There is no autocorrelation in the model, H1: There is autocorrelation in the model). The result is the rejection of H1 in favor of null hypothesis, i.e. that there is no autocorrelation in the model, thus the chosen ARIMA (1,1,1) specification is adequate (eventually for HUF ARIMA [(1,7),1,(1,7)] model).

This is followed by testing the stationarity in the data generating process and finding, whether the model is invertible. This verification is based on discovering the absolute values of AR and MA roots.

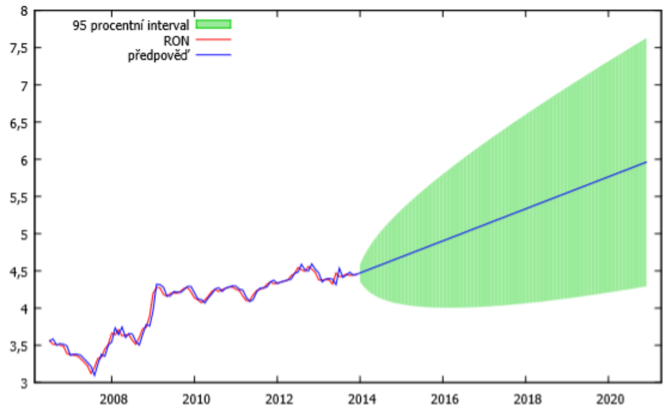
	CZK	SEK	GBP	PLN	HUF	RON
AR: Root 1 – abs. value	3,533	-1,8949	5,3649	2,5541	1,2963	2,8831
MA: Root 1 – absolute value	1,023	-1,176	1,001	1,005	1,0219	1,0281

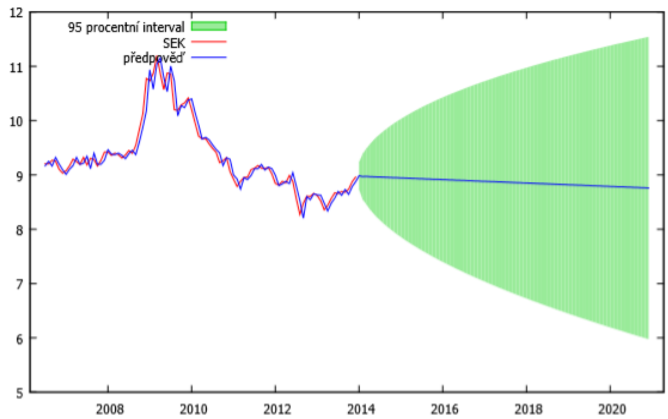
Figure 3. Outputs of AR and MA roots

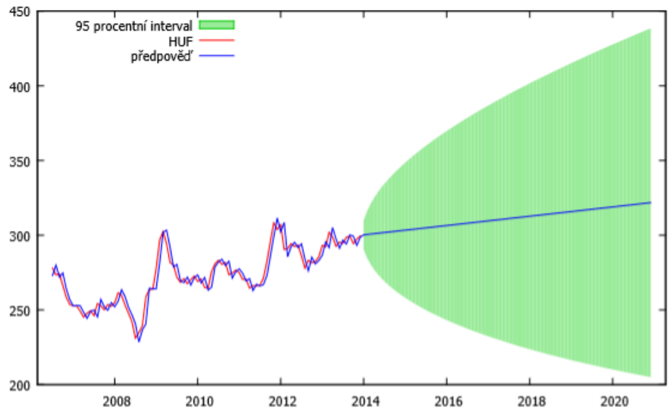
Source: author

Note. Hungary: absolute value of other six roots (in AR and MA) was higher than one.

The absolute value of all roots is higher than one, i.e. the model is stationary and invertible. Based on these results, we developed the prediction of exchange rates up to 2020, which is shown on the following pictures.


95 procentní interval
RON
předpověď
8
7,5
7
6,5
6
5,5
5
4,5
4
3,5
3
2008
2010
2012
2014
2016
2018
2020


95 procentní interval
SEK
předpověď
12
11
10
9
8
7
6
5
2008
2010
2012
2014
2016
2018
2020


95 procentní interval
HUF
předpověď
450
400
350
300
250
200
2008
2010
2012
2014
2016
2018
2020

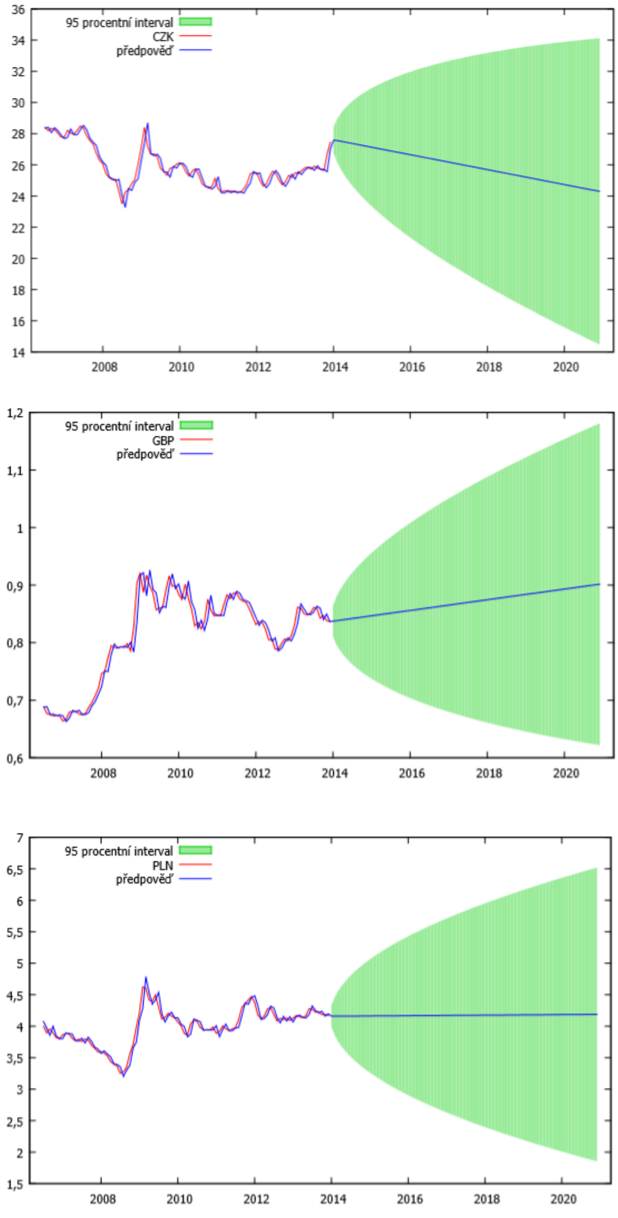


Figure 2. Prediction of exchange rates with the use of ARIMA model (RON, SEK, HUF, CZK, GBP, PLN)

Source: Author (SW Gretl)

Note: blue line = prediction, green line = 95% interval

The prediction shows decreasing trend of the Czech crown. Throughout the years, it should come to the evaluation of CZK against Euro but only slowly as according to the prediction the average exchange rate should be 24,52CZK/EUR in 2020. There is a slight decline also for the Swedish crown. For other currencies, the trend is rising or almost constant (Polish currency). From 2014 to 2020, Romania should register a rapid development of its currency. This rapid depreciation of the exchange rate (from 4,57RON/EUR in 2014 to 5,87RON/EUR in 2020) could cause pressure to increase the export by which the balance of payment deficit could be partly improved as Romania is currently overloaded with import.

Results of these models were subsequently verified with the use of the select autocorrelation function of standardized residues which verified their non-correlation. The final part of the verification was to test the normality of standardized residues and then, with the use of ARCH test, we determined, whether these residues have constant variance, i.e. whether they are conditionally homoscedastic. We tested the null hypothesis, i.e. that there is no ARCH effect present in the residues. If p-value is higher that the importance level of 0,05, we accept this hypothesis.

Currency	p-value	Result of the testing (presence of ARCH effect)
CZK	0,0348396	Residues are conditionally heteroscedastic
SEK	1,32E-05	Residues are conditionally heteroscedastic
GBP	0,00197651	Residues are conditionally heteroscedastic
PLN	0,11284	Residues are conditionally homoscedastic
HUF	0,098173	Residues are conditionally homoscedastic
RON	0,312715	Residues are conditionally homoscedastic

Figure 4. Detection of ARCH effect

Source: author

From the above shown table follows that heteroscedasticity was found at three currencies (i.e. presence of ARCH effect). Because the variances of the random components are not equal, the OLS method has not the optimal properties in this form of generalized linear regressive model, specifically it does not provide substantial estimations, however, these estimations are still impartial and may be used for further research. Polish, Hungarian and Romanian currency shows constant variance of the residues (homoscedasticity).

4. Conclusion

In this study, exchange rates of the six European currencies were predicted with the use of ARIMA (1,1,1) or with the use of ARIMA [(1,7),1,(1,7)]. The results are different for each selected currency – according to the prediction there will be appreciation as well as depreciation of the currency against Euro. It is however necessary to consider the limitations of using the ARIMA models which presented certain problems in estimating and validating the model and which are more effective in rendering the medium-term value (for several months). This long-term prediction should primarily show the future trend of the development of currencies exchange rates and at the same time identify the optimal model of the Box Jenkins models for predicting the European exchange rates. Both of these conditions were fulfilled.

5. References

Bellgard, C. & Goldschmidt, P. (1999). Forecasting across frequencies: Linearity and non-linearity. University of Western Australia Research Paper. *Proceedings of the International Conference on Advanced Technology*. Australia.

Dickey, D. A. & Fuller, W. (1979). Distribution of estimators for autoregressive time series with a unit root. *J. Am. Stat. Assoc*. 74, pp. 427–431.

Dunis, C. & Huang, X. (2002). *Forecasting and trading currency volatility: An application of recurrent neural regression and model combination*. England, Liverpool.

Făt, C.M. & Dezci, E. (2011). *Exchange-rates forecasting: exponential smoothing techniques and Arima models.* [online] Faculty of Economics and Business Administration, Department of Finance, "Babes-Bolyai" University, Cluj-Napoca, Romania, Retrieved from: http://steconomiceuoradea.ro/anale/volume/2011/n1/046.pdf.

Kamruzzaman, J. & Sarker, R. (2003). *Forecasting of Currency Exchange Rates using ANN: A Case Study*, [online] Gippsland School of Computing & IT School of Computer Science, Australia.

Mark, N. (1995). *Exchange rates and fundamentals: evidence on long-horizon predictability. American Economic Review,* pp. 201-218.

Meese, R. & Rogoff, K. (1983). *The out-of-sample failure of empirical exchange rates: sampling error or misspecification?* in FRENKEL, J. *Exchange Rates and International Macroeconomics,* University of Chicago Press, pp. 67-105.

Phillips, P. C. B. & Perron, P. (1988). *Testing for a unit root in time series regression*. Biometrika *75*, pp. 335–346.

Rout, M.; Majhi, B.; Majhi, R. & Ganapati, P. (2014). Forecasting of currency exchange rates using an adaptive ARMA model with differential evolution based training. [online] *Journal of King Saud University – Computer and Information Sciences* (2014) 26, pp. 7–18, Retrieved from http://ac.els-cdn.com/S1319157813000037/1-s2.0-S1319157813000037-main.pdf?_tid=5433f8e6-40a4-11e4-a1d8-00000aab0f6b&acdnat=1411203706_8fa301651e70ee218dbdca66a392f6d3.

Weisang, G. &Yukika, A. (2008). *Vagaries of the Euro: an Introduction to ARIMA Modeling*, [online] Bentley College, USA, Retrieved from http://www.bentley.edu/centers/sites/www.bentley.edu.centers/files/csbigs/weisang.pdf.