The Determination of Spatial Interdependencies in the European Union

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Abstract. The article deals with neighborhood ties to European Union countries in terms of graph theory. It is determined the minimum distance between states - the number of links in a graph and then their degree of connectivity to the Union. It also studied the link between the degree of connectivity with low GDP per capita considering that the development of the relatively isolated states can not grow without the development of communications infrastructure.

Keywords: graph; globalization; Bellman

1. About Globalization

The concept of globalization emerged in a more serious form somewhere at the end of World War I, is one that arises heated arguments for it or against it. The current period governed by the existence of the Internet (with all that it implies – e-mail, social networking sites, sites with statistical data more than enough etc.), a ready means of locomotion, a television increasingly more aggressive, a culture of increasingly standardized, but more pervasive, all of this born discussions leading to hopes or fear.

Benefits of globalization consists, according to the authors, in the faster spread of scientific achievements, faster implementation of new technologies and, especially, the rapid exchange of information. If until the 9th decade of the last century the information circulated through personal and scientific journals, symposia, exhibitions, media (as they addressed the public), since that time, the Internet has produced it almost instantaneous propagation.

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As disadvantages, globalization brings, primarily uniformity. In terms of cultural, influence of powerful nations is to the full felt. Importing a way of life, often a foreign to the receptor, leads to paradoxical situations and sometimes frictions which generating pain. The best example in this respect are American cinema (with superheroes who struggle alone with armies, preferably Russian ...), Spanish (the passion for a beautiful lady lead to violent crimes) or Indian (in which, this time, the noble evil is defeated by a pair of beautiful singing and waving their hands gracefully behind a tree).

The Anglo-Saxon rhythms are already indisputable masters of modern music, despite symptomatic poverty or feelings expressed lyrics. It can thus continue to analyze other areas of arts (literature phenomena of Harry Potter or Dan Brown's writings, sculpture, painting, etc.). Also, information flows faster, a bit too quickly, the planet's population no longer having time to think, taking it in pure form, being at the same time led by superficiality and, not incidentally, to manipulation.

On the other hand, globalization is a very effectiveness phenomena especially for educated man. Information that is rapidly distributed nourishes him with data and/or allowing new ideas, in turn, disseminate their own results.

A problem that the authors face it is that if the European Union (obviously this study could easily extend to the entire earth) globalization extends through modern methods or classics that include the displacement of populations between regions and, once with them, the transmission of information.

To study this, the authors turned to the graph theory.

2. Bellman-Kalaba Algorithm

Considering a graph whose set of nodes is $A=\{x_1,...,x_n\}$, $n\geq 2$ and arches - U subset of Cartesian product $A\times A$, assign each arc $(x_i, x_j)\in U$ the effective distance from x_i to x_j.

Let $D_1=(d_{ij})\in M_n(\mathbf{R})$ the matrix where, if there isn't an arc between x_i and x_j (or edge in non-orientate graphs) are considered $d_{ij}=\infty$ (on implementation on computer, a great value), and $d_{ii}=0$ $\forall i=\overline{1,n}$.

The problem will be to determine the minimum length of roads from one node to all other nodes fixed x_k.

Bellman-Kalaba algorithm consists of several steps:

Step 1: We note with $v^{(i)}\in\mathbf{R}^n$ the vector containing the minimum lengths of roads from x_k nodes $\{x_1,...,x_n\}$ at most "i" arcs. How D_1 column matrix "k" contains lengths of roads formed with a single arc of the x_k to x_i, $i=\overline{1,n}$, follows that $v^{(1)}$ will have the components of column k of the matrix D_1. Step 2: Assume that were determined $v^{(i)}$, $i=\overline{1,s}$ with $s\geq 1$ and the matrix $D_s=\left(\tilde{d}_{ij}\right) \in M_n(\mathbf{R})$ where $c = d_{ij} + v_j^{(s)}$, $i,j=\overline{1,n}$ is constructed. Practically we add the distance from the node x_i to any node x_j with at most "s" arcs with those from x_j to the reference node x_k. The amount $\tilde{d}_{ij}$ represents the minimum length of the road with most (s+1) arches from the x_i to x_k, necessarily passing through x_j. It is then determined, $v_i^{(s+1)} = \min_{j=\overline{1,n}} \tilde{d}_{ij}$, $i=\overline{1,n}$ which represents the minimum length of the roads with most than "s+1" arcs from x_i the node reference x_k, thus generating the vector $v^{(s+1)}$.

Step 3: The algorithm is repeated until for $t\geq 1$: $v^{(t+1)}=v^{(t)}$ that is the minimum length of not more than "t" arcs may not decrease at the addition of an additional arc. What should be noted is that the actual determination of the minimum length of the road is quite difficult to obtain (but not impossible) the road itself, but that did not influence the present approach.

3. Determination of Minimum Length of Roads Between EU Countries

In this section we will determine the minimum lengths of the roads between EU countries for the purposes of considering only the existence arcs (actually the edges, since this is an undirected graph) between them, and not the actual distance (which would involve determining the "center" of a country - otherwise a complicated endeavor even if mathematically it is possible). So either graph links between countries:

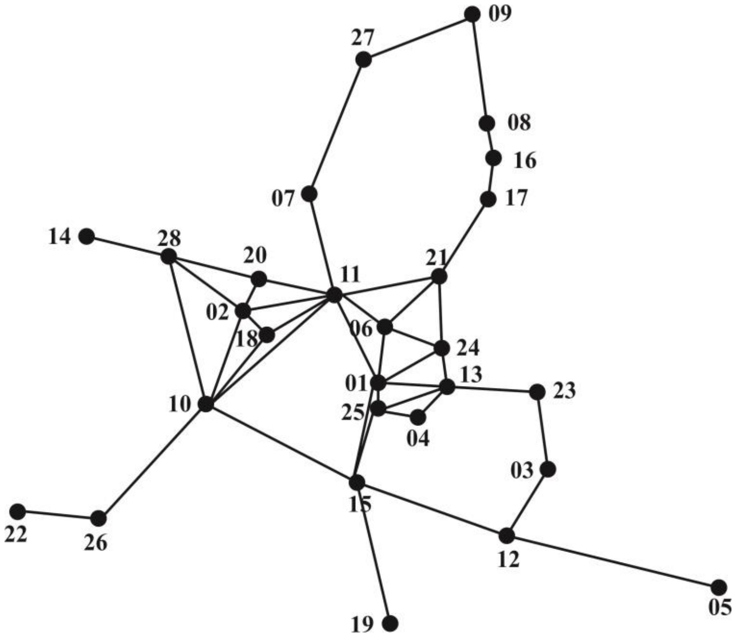

Figure 1.

Note: 01 – Austria, 02 – Belgium, 03 – Bulgaria, 04 – Croatia, 05 – Cyprus, 06 - Czech Republic;

07 – Denmark, 08 – Estonia, 09 – Finland, 10 – France, 11 – Germany, 12 – Greece, 13 – Hungary;

14 – Ireland, 15 – Italy, 16 – Latvia, 17 – Lithuania, 18 – Luxembourg, 19 – Malta, 20 – Netherlands;

21 – Poland, 22 – Portugal, 23 – Romania, 24 – Slovakia, 25 – Slovenia, 26 – Spain, 27 – Sweden;

28 - United Kingdom.

Edges between nodes (figure 1) indicates the existence of common borders between countries (usually on land, the only exceptions being made for linkages United Kingdom, Malta, Cyprus with other neighboring countries - because of their insularity and pairs Denmark- Sweden, Finland-Estonia, Greece-Italy to nearby maritime each other). The length of each edge is unitary. The matrix of the graph in figure 1 (symmetrical because it is non-oriented) where in the rows and columns there are countries with relevant coding above:

Table 1.

Country	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
01	0	∞	∞	∞	∞	1	∞	∞	∞	∞	1	∞	1	∞	1	∞	∞	∞	∞	∞	∞	∞	∞	1	1	∞	∞	∞
02	∞	0	∞	∞	∞	∞	∞	∞	∞	1	1	∞	∞	∞	∞	∞	∞	1	∞	1	∞	∞	∞	∞	∞	∞	∞	1
03	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	∞	1	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	1	∞	∞	∞	∞	∞
04	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	∞	1	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	1	∞	∞	∞
05	∞	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	1	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
06	1	∞	∞	∞	∞	0	∞	∞	∞	∞	1	∞	∞	∞	∞	∞	∞	∞	∞	∞	1	∞	∞	1	∞	∞	∞	∞
07	∞	∞	∞	∞	∞	∞	0	∞	∞	∞	1	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	1	∞
08	∞	∞	∞	∞	∞	∞	∞	0	1	∞	∞	∞	∞	∞	∞	1	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
09	∞	∞	∞	∞	∞	∞	∞	1	0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	1	∞
10	∞	1	∞	∞	∞	∞	∞	∞	∞	0	1	∞	∞	∞	1	∞	∞	1	∞	∞	∞	∞	∞	∞	∞	1	∞	1
11	1	1	∞	∞	∞	1	1	∞	∞	1	0	∞	∞	∞	∞	∞	∞	1	∞	1	1	∞	∞	∞	∞	∞	∞	∞
12	∞	∞	1	∞	1	∞	∞	∞	∞	∞	∞	0	∞	∞	1	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
13	1	∞	∞	1	∞	∞	∞	∞	∞	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	∞	∞	1	1	1	∞	∞	∞
14	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	1
15	1	∞	∞	∞	∞	∞	∞	∞	∞	1	∞	1	∞	∞	0	∞	∞	∞	1	∞	∞	∞	∞	∞	1	∞	∞	∞
16	∞	∞	∞	∞	∞	∞	∞	1	∞	∞	∞	∞	∞	∞	∞	0	1	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
17	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	1	0	∞	∞	∞	1	∞	∞	∞	∞	∞	∞	∞
18	∞	1	∞	∞	∞	∞	∞	∞	∞	1	1	∞	∞	∞	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
19	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	1	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	∞	∞
20	∞	1	∞	∞	∞	∞	∞	∞	∞	∞	1	∞	∞	∞	∞	∞	∞	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	1
21	∞	∞	∞	∞	∞	1	∞	∞	∞	∞	1	∞	∞	∞	∞	∞	1	∞	∞	∞	0	∞	∞	1	∞	∞	∞	∞

22	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	0	∞	∞	∞	1	∞	∞
23	∞	∞	1	∞	∞	∞	∞	∞	∞	∞	∞	∞	1	∞	∞	∞	∞	∞	∞	∞	∞	∞	0	∞	∞	∞	∞	∞
24	1	∞	∞	∞	∞	1	∞	∞	∞	∞	∞	∞	1	∞	∞	∞	∞	∞	∞	∞	1	∞	∞	0	∞	∞	∞	∞
25	1	∞	∞	1	∞	∞	∞	∞	∞	∞	∞	∞	1	∞	1	∞	∞	∞	∞	∞	∞	∞	∞	∞	0	∞	∞	∞
26	∞	∞	∞	∞	∞	∞	∞	∞	∞	1	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	1	∞	∞	∞	0	∞	∞
27	∞	∞	∞	∞	∞	∞	1	∞	1	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	0	∞
28	∞	1	∞	∞	∞	∞	∞	∞	∞	1	∞	∞	∞	1	∞	∞	∞	∞	∞	1	∞	∞	∞	∞	∞	∞	∞	0

Applying the Bellman-Kalaba algorithm, finally gives the matrix of minimum distances between countries:

Table 2.

Country	**01**	**02**	**03**	**04**	**05**	**06**	**07**	**08**	**09**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**
01	0	2	3	2	3	1	2	5	4	2	1	2	1	4	1	4	3	2	2	2	2	4	2	1	1	3	3	3
02	2	0	4	4	4	2	2	5	4	1	1	3	3	2	2	4	3	1	3	1	2	3	4	3	3	2	3	1
03	3	4	0	3	2	4	5	7	7	3	4	1	2	5	2	6	5	4	3	5	4	5	1	3	3	4	6	4
04	2	4	3	0	4	3	4	6	6	3	3	3	1	5	2	5	4	4	3	4	3	5	2	2	1	4	5	4
05	3	4	2	4	0	4	5	8	7	3	4	1	4	5	2	7	6	4	3	5	5	5	3	4	3	4	6	4
06	1	2	4	3	4	0	2	4	4	2	1	3	2	4	2	3	2	2	3	2	1	4	3	1	2	3	3	3
07	2	2	5	4	5	2	0	3	2	2	1	4	3	4	3	4	3	2	4	2	2	4	4	3	3	3	1	3
08	5	5	7	6	8	4	3	0	1	5	4	7	5	7	6	1	2	5	7	5	3	7	6	4	6	6	2	6
09	4	4	7	6	7	4	2	1	0	4	3	6	5	6	5	2	3	4	6	4	4	6	6	5	5	5	1	5
10	2	1	3	3	3	2	2	5	4	0	1	2	3	2	1	4	3	1	2	2	2	2	4	3	2	1	3	1
11	1	1	4	3	4	1	1	4	3	1	0	3	2	3	2	3	2	1	3	1	1	3	3	2	2	2	2	2
12	2	3	1	3	1	3	4	7	6	2	3	0	3	4	1	6	5	3	2	4	4	4	2	3	2	3	5	3
13	1	3	2	1	4	2	3	5	5	3	2	3	0	5	2	4	3	3	3	3	2	5	1	1	1	4	4	4

14	4	2	5	5	5	4	4	7	6	2	3	4	5	0	3	6	5	3	4	2	4	4	6	5	4	3	5	1
15	1	2	2	2	2	2	3	6	5	1	2	1	2	3	0	5	4	2	1	3	3	3	3	2	1	2	4	2
16	4	4	6	5	7	3	4	1	2	4	3	6	4	6	5	0	1	4	6	4	2	6	5	3	5	5	3	5
17	3	3	5	4	6	2	3	2	3	3	2	5	3	5	4	1	0	3	5	3	1	5	4	2	4	4	4	4
18	2	1	4	4	4	2	2	5	4	1	1	3	3	3	2	4	3	0	3	2	2	3	4	3	3	2	3	2
19	2	3	3	3	3	3	4	7	6	2	3	2	3	4	1	6	5	3	0	4	4	4	4	3	2	3	5	3
20	2	1	5	4	5	2	2	5	4	2	1	4	3	2	3	4	3	2	4	0	2	4	4	3	3	3	3	1
21	2	2	4	3	5	1	2	3	4	2	1	4	2	4	3	2	1	2	4	2	0	4	3	1	3	3	3	3
22	4	3	5	5	5	4	4	7	6	2	3	4	5	4	3	6	5	3	4	4	4	0	6	5	4	1	5	3
23	2	4	1	2	3	3	4	6	6	4	3	2	1	6	3	5	4	4	4	4	3	6	0	2	2	5	5	5
24	1	3	3	2	4	1	3	4	5	3	2	3	1	5	2	3	2	3	3	3	1	5	2	0	2	4	4	4
25	1	3	3	1	3	2	3	6	5	2	2	2	1	4	1	5	4	3	2	3	3	4	2	2	0	3	4	3
26	3	2	4	4	4	3	3	6	5	1	2	3	4	3	2	5	4	2	3	3	3	1	5	4	3	0	4	2
27	3	3	6	5	6	3	1	2	1	3	2	5	4	5	4	3	4	3	5	3	3	5	5	4	4	4	0	4
28	3	1	4	4	4	3	3	6	5	1	2	3	4	1	2	5	4	2	3	1	3	3	5	4	3	2	4	0

The data in Table 2 shows the minimum number of arcs necessary for the transition from one country to another. For example, on line 1 and column 9 we have a value of 4 which means that the shortest path (not necessarily unique) from Austria to Finland passes through four countries (except the one starting): Germany-Denmark-Sweden-Finland .

Because the effect of one state against another is inversely proportional to the distance (reasonable assumption), we shall reverse matrix values above (replacing 1/0 with 1 – because we shall assume that the effect of one country on itself is maximum). So we get:

Table 3

Country	01	02	03	04	05	06	07	08	09	10	11	12
01	1	0.5	0.333	0.5	0.333	1	0.5	0.2	0.25	0.5	1	0.5
02	0.5	1	0.25	0.25	0.25	0.5	0.5	0.2	0.25	1	1	0.333
03	0.333	0.25	1	0.333	0.5	0.25	0.2	0.143	0.143	0.333	0.25	1
04	0.5	0.25	0.333	1	0.25	0.333	0.25	0.167	0.167	0.333	0.333	0.333
05	0.333	0.25	0.5	0.25	1	0.25	0.2	0.125	0.143	0.333	0.25	1
06	1	0.5	0.25	0.333	0.25	1	0.5	0.25	0.25	0.5	1	0.333
07	0.5	0.5	0.2	0.25	0.2	0.5	1	0.333	0.5	0.5	1	0.25
08	0.2	0.2	0.143	0.167	0.125	0.25	0.333	1	1	0.2	0.25	0.143
09	0.25	0.25	0.143	0.167	0.143	0.25	0.5	1	1	0.25	0.333	0.167
10	0.5	1	0.333	0.333	0.333	0.5	0.5	0.2	0.25	1	1	0.5
11	1	1	0.25	0.333	0.25	1	1	0.25	0.333	1	1	0.333
12	0.5	0.333	1	0.333	1	0.333	0.25	0.143	0.167	0.5	0.333	1
13	1	0.333	0.5	1	0.25	0.5	0.333	0.2	0.2	0.333	0.5	0.333
14	0.25	0.5	0.2	0.2	0.2	0.25	0.25	0.143	0.167	0.5	0.333	0.25
15	1	0.5	0.5	0.5	0.5	0.5	0.333	0.167	0.2	1	0.5	1
16	0.25	0.25	0.167	0.2	0.143	0.333	0.25	1	0.5	0.25	0.333	0.167
17	0.333	0.333	0.2	0.25	0.167	0.5	0.333	0.5	0.333	0.333	0.5	0.2
18	0.5	1	0.25	0.25	0.25	0.5	0.5	0.2	0.25	1	1	0.333
19	0.5	0.333	0.333	0.333	0.333	0.333	0.25	0.143	0.167	0.5	0.333	0.5
20	0.5	1	0.2	0.25	0.2	0.5	0.5	0.2	0.25	0.5	1	0.25
21	0.5	0.5	0.25	0.333	0.2	1	0.5	0.333	0.25	0.5	1	0.25
22	0.25	0.333	0.2	0.2	0.2	0.25	0.25	0.143	0.167	0.5	0.333	0.25
23	0.5	0.25	1	0.5	0.333	0.333	0.25	0.167	0.167	0.25	0.333	0.5
24	1	0.333	0.333	0.5	0.25	1	0.333	0.25	0.2	0.333	0.5	0.333
25	1	0.333	0.333	1	0.333	0.5	0.333	0.167	0.2	0.5	0.5	0.5
26	0.333	0.5	0.25	0.25	0.25	0.333	0.333	0.167	0.2	1	0.5	0.333
27	0.333	0.333	0.167	0.2	0.167	0.333	1	0.5	1	0.333	0.5	0.2
28	0.333	1	0.25	0.25	0.25	0.333	0.333	0.167	0.2	1	0.5	0.333

26	25	24	23	22	21	20	19	18	17	16	15	14	13
0.333	1	1	0.5	0.25	0.5	0.5	0.5	0.5	0.333	0.25	1	0.25	1
0.5	0.333	0.333	0.25	0.333	0.5	1	0.333	1	0.333	0.25	0.5	0.5	0.333
0.25	0.333	0.333	1	0.2	0.25	0.2	0.333	0.25	0.2	0.167	0.5	0.2	0.5
0.25	1	0.5	0.5	0.2	0.333	0.25	0.333	0.25	0.25	0.2	0.5	0.2	1
0.25	0.333	0.25	0.333	0.2	0.2	0.2	0.333	0.25	0.167	0.143	0.5	0.2	0.25
0.333	0.5	1	0.333	0.25	1	0.5	0.333	0.5	0.5	0.333	0.5	0.25	0.5
0.333	0.333	0.333	0.25	0.25	0.5	0.5	0.25	0.5	0.333	0.25	0.333	0.25	0.333
0.167	0.167	0.25	0.167	0.143	0.333	0.2	0.143	0.2	0.5	1	0.167	0.143	0.2
0.2	0.2	0.2	0.167	0.167	0.25	0.25	0.167	0.25	0.333	0.5	0.2	0.167	0.2
1	0.5	0.333	0.25	0.5	0.5	0.5	0.5	1	0.333	0.25	1	0.5	0.333
0.5	0.5	0.5	0.333	0.333	1	1	0.333	1	0.5	0.333	0.5	0.333	0.5
0.333	0.5	0.333	0.5	0.25	0.25	0.25	0.5	0.333	0.2	0.167	1	0.25	0.333
0.25	1	1	1	0.2	0.5	0.333	0.333	0.333	0.333	0.25	0.5	0.2	1
0.333	0.25	0.2	0.167	0.25	0.25	0.5	0.25	0.333	0.2	0.167	0.333	1	0.2
0.5	1	0.5	0.333	0.333	0.333	0.333	1	0.5	0.25	0.2	1	0.333	0.5
0.2	0.2	0.333	0.2	0.167	0.5	0.25	0.167	0.25	1	1	0.2	0.167	0.25
0.25	0.25	0.5	0.25	0.2	1	0.333	0.2	0.333	1	1	0.25	0.2	0.333
0.5	0.333	0.333	0.25	0.333	0.5	0.5	0.333	1	0.333	0.25	0.5	0.333	0.333
0.333	0.5	0.333	0.25	0.25	0.25	0.25	1	0.333	0.2	0.167	1	0.25	0.333
0.333	0.333	0.333	0.25	0.25	0.5	1	0.25	0.5	0.333	0.25	0.333	0.5	0.333
0.333	0.333	1	0.333	0.25	1	0.5	0.25	0.5	1	0.5	0.333	0.25	0.5
1	0.25	0.2	0.167	1	0.25	0.25	0.25	0.333	0.2	0.167	0.333	0.25	0.2
0.2	0.5	0.5	1	0.167	0.333	0.25	0.25	0.25	0.25	0.2	0.333	0.167	1
0.25	0.5	1	0.5	0.2	1	0.333	0.333	0.333	0.5	0.333	0.5	0.2	1
0.333	1	0.5	0.5	0.25	0.333	0.333	0.5	0.333	0.25	0.2	1	0.25	1
1	0.333	0.25	0.2	1	0.333	0.333	0.333	0.5	0.25	0.2	0.5	0.333	0.25
0.25	0.25	0.25	0.2	0.2	0.333	0.333	0.2	0.333	0.25	0.333	0.25	0.2	0.25
0.5	0.333	0.25	0.2	0.333	0.333	1	0.333	0.5	0.25	0.2	0.5	1	0.25

27	0.333	0.333	0.167	0.2	0.167	0.333	1	0.5	1	0.333	0.5	0.2	0.25	0.2	0.25	0.333	0.25	0.333	0.2	0.333	0.333	0.2	0.2	0.25	0.25	0.25	1	0.25
28	0.333	1	0.25	0.25	0.25	0.333	0.333	0.167	0.2	1	0.5	0.333	0.25	1	0.5	0.2	0.25	0.5	0.333	1	0.333	0.333	0.2	0.25	0.333	0.5	0.25	1

In table 3, how much will be small the distance between two states, a greater value will be allocated.

The data obtained in Table 3 can not be used in their raw form, benefiting countries form Centre of European Union which, geographically, have the most connections between them. For this reason, we normalize these values by dividing each line distances to the sum of its elements. Following this approach, the normalized matrix is not symmetrical, depending from the links neighboring third countries. So we get in the end:

Table 4. Normalized distances between countries 01-07 and 01-07

Country	**01**	**02**	**03**	**04**	**05**	**06**	**07**
01	0.0658	0.0329	0.0219	0.0329	0.0219	0.0658	0.0329
02	0.0361	0.0721	0.018	0.018	0.018	0.0361	0.0361
03	0.0337	0.0253	0.1013	0.0337	0.0507	0.0253	0.0203
04	0.0478	0.0239	0.0318	0.0956	0.0239	0.0318	0.0239
05	0.0385	0.0289	0.0577	0.0289	0.1155	0.0289	0.0231
06	0.0732	0.0366	0.0183	0.0244	0.0183	0.0732	0.0366
07	0.0423	0.0423	0.0169	0.0212	0.0169	0.0423	0.0846

Table 5. Normalized distances between countries 01-07 and 08-14

Country	**08**	**09**	**10**	**11**	**12**	**13**	**14**
01	0.0132	0.0164	0.0329	0.0658	0.0329	0.0658	0.0164
02	0.0144	0.018	0.0721	0.0721	0.024	0.024	0.0361
03	0.0145	0.0145	0.0337	0.0253	0.1013	0.0507	0.0203

04	0.016	0.016	0.0318	0.0318	0.0318	0.0956	0.0191
05	0.0144	0.0165	0.0385	0.0289	0.1155	0.0289	0.0231
06	0.0183	0.0183	0.0366	0.0732	0.0244	0.0366	0.0183
07	0.0282	0.0423	0.0423	0.0846	0.0212	0.0282	0.0212

Table 6. Normalized distances between countries 01-07 and 15-21

Country	**15**	**16**	**17**	**18**	**19**	**20**	**21**
01	0.0658	0.0164	0.0219	0.0329	0.0329	0.0329	0.0329
02	0.0361	0.018	0.024	0.0721	0.024	0.0721	0.0361
03	0.0507	0.0169	0.0203	0.0253	0.0337	0.0203	0.0253
04	0.0478	0.0191	0.0239	0.0239	0.0318	0.0239	0.0318
05	0.0577	0.0165	0.0193	0.0289	0.0385	0.0231	0.0231
06	0.0366	0.0244	0.0366	0.0366	0.0244	0.0366	0.0732
07	0.0282	0.0212	0.0282	0.0423	0.0212	0.0423	0.0423

Table 7. Normalized distances between countries 01-07 and 22-28

Country	**22**	**23**	**24**	**25**	**26**	**27**	**28**
01	0.0164	0.0329	0.0658	0.0658	0.0219	0.0219	0.0219
02	0.024	0.018	0.024	0.024	0.0361	0.024	0.0721
03	0.0203	0.1013	0.0337	0.0337	0.0253	0.0169	0.0253
04	0.0191	0.0478	0.0478	0.0956	0.0239	0.0191	0.0239
05	0.0231	0.0385	0.0289	0.0385	0.0289	0.0193	0.0289
06	0.0183	0.0244	0.0732	0.0366	0.0244	0.0244	0.0244
07	0.0212	0.0212	0.0282	0.0282	0.0282	0.0846	0.0282

Table 8. Normalized distances between countries 08-14 and 01-07

Country	01	02	03	04	05	06	07
08	0.0236	0.0236	0.0169	0.0197	0.0148	0.0296	0.0394
09	0.0281	0.0281	0.0161	0.0188	0.0161	0.0281	0.0562
10	0.0327	0.0654	0.0218	0.0218	0.0218	0.0327	0.0327
11	0.0609	0.0609	0.0152	0.0203	0.0152	0.0609	0.0609
12	0.043	0.0286	0.086	0.0286	0.086	0.0286	0.0215
13	0.0757	0.0252	0.0378	0.0757	0.0189	0.0378	0.0252
14	0.0282	0.0563	0.0225	0.0225	0.0225	0.0282	0.0282

Table 9. Normalized distances between countries 08-14 and 08-14

Country	08	09	10	11	12	13	14
08	0.1182	0.1182	0.0236	0.0296	0.0169	0.0236	0.0169
09	0.1123	0.1123	0.0281	0.0374	0.0188	0.0225	0.0188
10	0.0131	0.0164	0.0654	0.0654	0.0327	0.0218	0.0327
11	0.0152	0.0203	0.0609	0.0609	0.0203	0.0305	0.0203
12	0.0123	0.0144	0.043	0.0286	0.086	0.0286	0.0215
13	0.0151	0.0151	0.0252	0.0378	0.0252	0.0757	0.0151
14	0.0161	0.0188	0.0563	0.0375	0.0282	0.0225	0.1127

Table 10. Normalized distances between countries 08-14 and 15-21

Country	15	16	17	18	19	20	21
08	0.0197	0.1182	0.0591	0.0236	0.0169	0.0236	0.0394
09	0.0225	0.0562	0.0374	0.0281	0.0188	0.0281	0.0281

10	0.0654	0.0164	0.0218	0.0654	0.0327	0.0327	0.0327
11	0.0305	0.0203	0.0305	0.0609	0.0203	0.0609	0.0609
12	0.086	0.0144	0.0172	0.0286	0.043	0.0215	0.0215
13	0.0378	0.0189	0.0252	0.0252	0.0252	0.0252	0.0378
14	0.0375	0.0188	0.0225	0.0375	0.0282	0.0563	0.0282

Table 11. Normalized distances between countries 08-14 and 22-28

Country	**22**	**23**	**24**	**25**	**26**	**27**	**28**
08	0.0169	0.0197	0.0296	0.0197	0.0197	0.0591	0.0197
09	0.0188	0.0188	0.0225	0.0225	0.0225	0.1123	0.0225
10	0.0327	0.0164	0.0218	0.0327	0.0654	0.0218	0.0654
11	0.0203	0.0203	0.0305	0.0305	0.0305	0.0305	0.0305
12	0.0215	0.043	0.0286	0.043	0.0286	0.0172	0.0286
13	0.0151	0.0757	0.0757	0.0757	0.0189	0.0189	0.0189
14	0.0282	0.0188	0.0225	0.0282	0.0375	0.0225	0.1127

Table 12. Normalized distances between countries 15-21 and 01-07

Country	**01**	**02**	**03**	**04**	**05**	**06**	**07**
15	0.0687	0.0343	0.0343	0.0343	0.0343	0.0343	0.0229
16	0.027	0.027	0.018	0.0216	0.0154	0.036	0.027
17	0.0315	0.0315	0.0189	0.0236	0.0158	0.0473	0.0315
18	0.0394	0.0788	0.0197	0.0197	0.0197	0.0394	0.0394
19	0.0498	0.0332	0.0332	0.0332	0.0332	0.0332	0.0249
20	0.041	0.0821	0.0164	0.0205	0.0164	0.041	0.041
21	0.0374	0.0374	0.0187	0.0249	0.015	0.0748	0.0374

Table 13. Normalized distances between countries 15-21 and 08-14

Country	**08**	**09**	**10**	**11**	**12**	**13**	**14**
15	0.0115	0.0137	0.0687	0.0343	0.0687	0.0343	0.0229
16	0.108	0.054	0.027	0.036	0.018	0.027	0.018
17	0.0473	0.0315	0.0315	0.0473	0.0189	0.0315	0.0189
18	0.0158	0.0197	0.0788	0.0788	0.0262	0.0262	0.0262
19	0.0142	0.0166	0.0498	0.0332	0.0498	0.0332	0.0249
20	0.0164	0.0205	0.041	0.0821	0.0205	0.0273	0.041
21	0.0249	0.0187	0.0374	0.0748	0.0187	0.0374	0.0187

Table 14. Normalized distances between countries 15-21 and 15-21

Country	**15**	**16**	**17**	**18**	**19**	**20**	**21**
15	0.0687	0.0137	0.0172	0.0343	0.0687	0.0229	0.0229
16	0.0216	0.108	0.108	0.027	0.018	0.027	0.054
17	0.0236	0.0945	0.0945	0.0315	0.0189	0.0315	0.0945
18	0.0394	0.0197	0.0262	0.0788	0.0262	0.0394	0.0394
19	0.0996	0.0166	0.0199	0.0332	0.0996	0.0249	0.0249
20	0.0273	0.0205	0.0273	0.041	0.0205	0.0821	0.041
21	0.0249	0.0374	0.0748	0.0374	0.0187	0.0374	0.0748

Table 15. Normalized distances between countries 15-21 and 22-28

Country	**22**	**23**	**24**	**25**	**26**	**27**	**28**
15	0.0229	0.0229	0.0343	0.0687	0.0343	0.0172	0.0343
16	0.018	0.0216	0.036	0.0216	0.0216	0.036	0.0216
17	0.0189	0.0236	0.0473	0.0236	0.0236	0.0236	0.0236

18	0.0262	0.0197	0.0262	0.0262	0.0394	0.0262	0.0394
19	0.0249	0.0249	0.0332	0.0498	0.0332	0.0199	0.0332
20	0.0205	0.0205	0.0273	0.0273	0.0273	0.0273	0.0821
21	0.0187	0.0249	0.0748	0.0249	0.0249	0.0249	0.0249

Table 16. Normalized distances between countries 22-28 and 01-07

Country	**01**	**02**	**03**	**04**	**05**	**06**	**07**
22	0.0296	0.0394	0.0236	0.0236	0.0236	0.0296	0.0296
23	0.0482	0.0241	0.0963	0.0482	0.0321	0.0321	0.0241
24	0.0778	0.0259	0.0259	0.0389	0.0195	0.0778	0.0259
25	0.0765	0.0255	0.0255	0.0765	0.0255	0.0383	0.0255
26	0.0302	0.0454	0.0227	0.0227	0.0227	0.0302	0.0302
27	0.0335	0.0335	0.0168	0.0201	0.0168	0.0335	0.1005
28	0.0273	0.0821	0.0205	0.0205	0.0205	0.0273	0.0273

Table 17. Normalized distances between countries 22-28 and 08-14

Country	**08**	**09**	**10**	**11**	**12**	**13**	**14**
22	0.0169	0.0197	0.0591	0.0394	0.0296	0.0236	0.0296
23	0.0161	0.0161	0.0241	0.0321	0.0482	0.0963	0.0161
24	0.0195	0.0156	0.0259	0.0389	0.0259	0.0778	0.0156
25	0.0128	0.0153	0.0383	0.0383	0.0383	0.0765	0.0191
26	0.0152	0.0182	0.0908	0.0454	0.0302	0.0227	0.0302
27	0.0503	0.1005	0.0335	0.0503	0.0201	0.0251	0.0201
28	0.0137	0.0164	0.0821	0.041	0.0273	0.0205	0.0821

Table 18. Normalized distances between countries 22-28 and 15-21

Country	**15**	**16**	**17**	**18**	**19**	**20**	**21**
22	0.0394	0.0197	0.0236	0.0394	0.0296	0.0296	0.0296
23	0.0321	0.0193	0.0241	0.0241	0.0241	0.0241	0.0321
24	0.0389	0.0259	0.0389	0.0259	0.0259	0.0259	0.0778
25	0.0765	0.0153	0.0191	0.0255	0.0383	0.0255	0.0255
26	0.0454	0.0182	0.0227	0.0454	0.0302	0.0302	0.0302
27	0.0251	0.0335	0.0251	0.0335	0.0201	0.0335	0.0335
28	0.041	0.0164	0.0205	0.041	0.0273	0.0821	0.0273

Table 19. Normalized distances between countries 22-28 and 22-28

Country	**22**	**23**	**24**	**25**	**26**	**27**	**28**
22	0.1182	0.0197	0.0236	0.0296	0.1182	0.0236	0.0394
23	0.0161	0.0963	0.0482	0.0482	0.0193	0.0193	0.0193
24	0.0156	0.0389	0.0778	0.0389	0.0195	0.0195	0.0195
25	0.0191	0.0383	0.0383	0.0765	0.0255	0.0191	0.0255
26	0.0908	0.0182	0.0227	0.0302	0.0908	0.0227	0.0454
27	0.0201	0.0201	0.0251	0.0251	0.0251	0.1005	0.0251
28	0.0273	0.0164	0.0205	0.0273	0.041	0.0205	0.0821

Noting with D – “distances” in the normalized matrix – meaning the matrix of degree links to a specific country with all the others, result that elements of D^2 by multiplying the lines of D with its columns, these would provide the degree of connection of a country to another, passing binding by one of the other EU countries. Similarly, elements of D^n will provide a degree of connection with another country, at least in passing binding “n” EU countries.

Numerical data analysis reveals that (aside from the inherent rounding) elements of B^8 columns differ by less than 0.0001, significant results being accurate to 4 decimal places.

Therefore, we get finally degrees of connection (in terms of distance) of countries in the European Union:

Table 20. Linking grades in the European Union countries

Country	Degree	Country	Degree	Country	Degree
Germany	0.0502	United Kingdom	0.0373	Bulgaria	0.0302
France	0.0467	Netherlands	0.0373	Latvia	0.0283
Austria	0.0465	Denmark	0.0361	Finland	0.0272
Italy	0.0446	Greece	0.0356	Ireland	0.0272
Belgium	0.0424	Spain	0.0337	Cyprus	0.0265
Czech Republic	0.0418	Lithuania	0.0324	Portugal	0.0259
Poland	0.0409	Croatia	0.032	Estonia	0.0259
Hungary	0.0404	Romania	0.0318	Bulgaria	0.0302
Slovenia	0.04	Malta	0.0307		
Slovak Republic	0.0393	Sweden	0.0304		

4. The Relationship between the Degree of Connection and Size of the Countries GDP per Capita

In the following we will investigate the dependence of GDP per capita of EU countries, of which we excluded Luxembourg because the gap is very large compared to the rest of the data (in 2013 it has a GDP/capita of 83,400 euros to the following: Denmark - 44,400 euros) which affects the state diagram in figure 2.

Table 21. PIB/inhabitant (euro) for the European Union countries (except Luxembourg) during 2011-2013

Country	2011	2012	2013
Austria	35.700	36.400	37.000
Belgium	33.600	34.000	34.500
Bulgaria	5.200	5.500	5.500
Croatia	10.300	10.200	10.100
Cyprus	21.000	20.500	19.000
Czech Republic	14.800	14.600	14.200
Denmark	43.200	43.900	44.400
Estonia	12.100	13.000	13.900
Finland	35.000	35.500	35.600
France	30.700	31.100	31.300
Germany	31.900	32.600	33.300
Greece	18.700	17.400	17.400
Hungary	9.900	9.800	9.900
Ireland	35.500	35.700	35.600
Italy	26.000	25.700	25.600
Latvia	9.800	10.900	11.600
Lithuania	10.200	11.000	11.700
Malta	16.100	16.500	17.200
Netherlands	35.900	35.800	35.900
Poland	9.600	9.900	10.100
Portugal	16.100	15.600	15.800
Romania	6.500	6.600	7.100
Slovak Republic	12.800	13.200	13.300
Slovenia	17.600	17.200	17.100
Spain	22.700	22.300	22.300
Sweden	40.800	42.800	43.800
United Kingdom	28.200	30.200	29.600

Source: http://ec.europa.eu/eurostat/web/products-datasets/-/nama_aux_gph

For an accurate graphical representation, we determined the values maximum for each year and we computed the ratio of GDP/capita to the maximum value divided after by 15 (for comparability in absolute degree of connection).

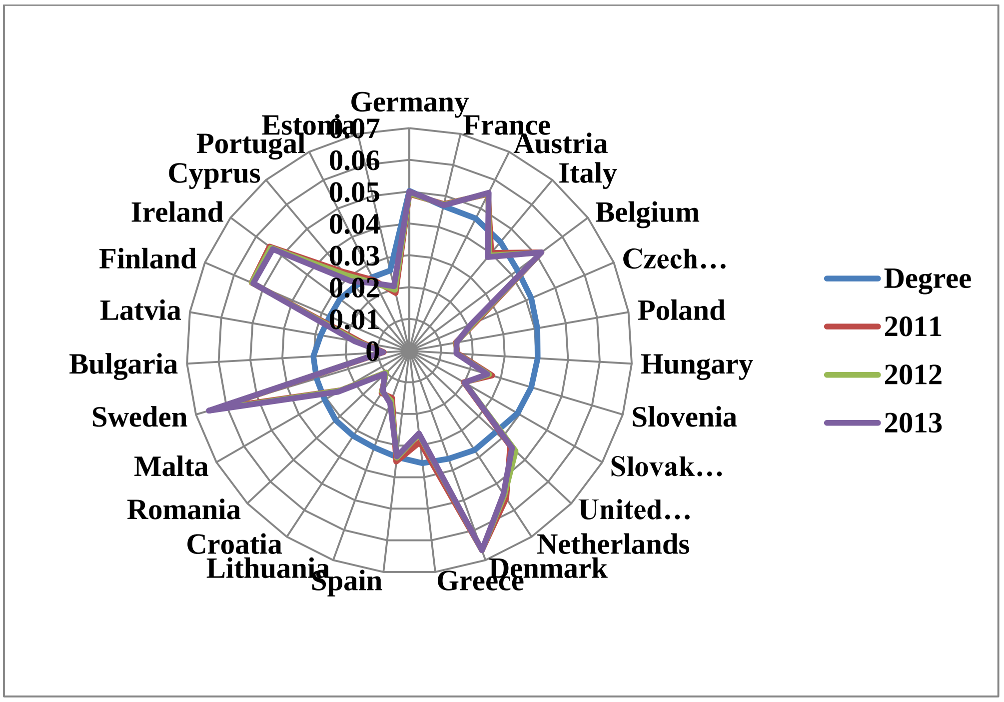

Figure 2

The analysis reveals the following:

- Countries with a high GDP/capita are generally those with a large number of close connections (in terms of neighborhood relations) with European countries: Germany, France, Austria, Italy, Belgium, United Kingdom, Netherlands, Denmark,
- There are a limited number of countries whose GDP/inhabitant is very high: Sweden, Finland and Ireland which are relatively isolated geographically but whose economic policies have overcome the barriers of distance.
- Symptomatic are former socialist countries: Czech Republic, Poland, Hungary, Slovenia, Slovak Republic, despite their geographical placement, pay still errors of the past.
- Countries such as Greece (the situation here is somewhat special in recent years), Lithuania, Croatia, Romania, Bulgaria, Latvia, Estonia have economies resettlement on new bases, but they face barriers metric which requires the development of a strong infrastructure of telecommunications which will blur their relative geographic isolation.

5. **Conclusions**

The above analysis establishes a new approach to economic relations within the European Union in terms of neighborhood relations. Multiple links between states favor the exchange of goods more quickly, the migratory movement of the population - especially in contiguous areas, globalization - as a complex phenomenon can lead to economic developments but which, unfortunately, can deep differences within the Union. European countries were disadvantaged by geographical location as a chance to build economic development and/or amplification of a communications infrastructure that will alleviate the barriers of distance.

6. References

Ioan, C.A. & Ioan, G. (2012). *Methods of mathematical modeling in economics*. Galati: Zigotto Publishers.

Ioan, C.A. (2001). Determining the minimum effective way with Bellman-Kalaba algorithm. *The Annals of Danubius University*, Fascicle I, Economics.