An Analysis of a Duopoly with Incomplete Information

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Abstract. In this paper we analyze the case of two firms A and B, each competing with the other under conditions of incomplete information. Thre analysed cases are: company A assume one of the following roles: either ignore the firm B or leading production or leading price acting as if B knows its intention and firm B acts either ignoring company A, or as satellite of production or satellite of price acting as if A knows its intention.

Keywords: duopoly; Cournot; Stackelberg; leader

1. Introduction

The duopoly is a market situation where there are two suppliers of a good unsubstituted and a sufficient number of consumers.

Considering below, two competitors A and B which produce the same normal good, we propose analyzing their activity in response to the work of each other company.

Each of them, when it set the production level and the selling price will cover the production and price of the other company. If one of the two firms will set price or quantity produced first, the other adjusting for it, it will be called price leader or leader of production respectively, the second company called the satellite price, or satellite production respectively.

The approach of the following considerations will be that company A will assume a role: either ignore the firm B or leading production or leading price acting as if B knows its intention and firm B acts either ignoring company A, or as satellite of production or satellite of price acting as if A knows its intention.

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Let therefore two producers, the demand inverse function: p(Q)=a-bQ, a,b>0, total costs of A and B being $TC_A(Q)=\alpha Q+\gamma$, $TC_B(Q)=\beta Q+\delta$, $\alpha,\beta,\gamma,\delta\geq 0$ where Q is the production. We have now: $MC_A=\alpha$ and $MC_B=\beta$ - marginal costs of each firm.

2. The Case of Cournot Equilibrium

In this case, both companies A and B act independently of each other and at the same time.

Considering the profits of both firms, we have:

$$\Pi_A(Q_A) = [a - b(Q_A + Q_B)]Q_A - (\alpha Q_A + \gamma) = -bQ_A^2 + (a - bQ_B - \alpha)Q_A - \gamma$$

$$\Pi_B(Q_B) = [a - b(Q_A + Q_B)]Q_B - (\beta Q_B + \delta) = -bQ_B^2 + (a - bQ_A - \beta)Q_B - \delta$$

In order to maximize the profits, we must have:

$$\frac{\partial \Pi_A(Q_A)}{\partial Q_A} = -2bQ_A + a - bQ_B - \alpha = 0$$

$$\frac{\partial \Pi_B(Q_B)}{\partial Q_B} = -2bQ_B + a - bQ_A - \beta = 0$$

from where (noting with C from Cournot):

$$Q_A^{(C)} = \frac{a - 2\alpha + \beta}{3b}, \quad Q_B^{(C)} = \frac{a + \alpha - 2\beta}{3b}$$

The price will be:

$$p^{(C)}\left(Q_A^{(C)} + Q_B^{(C)}\right) = a - b\left(Q_A^{(C)} + Q_B^{(C)}\right) = \frac{a + \alpha + \beta}{3}$$

and the profits:

$$\Pi_A^{(C)}\left(Q_A^{(C)}\right) = -bQ_A^{(C)2} + \left(a - bQ_B^{(C)} - \alpha\right)Q_A^{(C)} - \gamma = \frac{(a - 2\alpha + \beta)^2}{9b} - \gamma$$

$$\Pi_B^{(C)}\left(Q_B^{(C)}\right) = -bQ_B^{(C)2} + \left(a - bQ_A^{(C)} - \beta\right)Q_B^{(C)} - \delta = \frac{(a + \alpha - 2\beta)^2}{9b} - \delta$$

We have now (noting $D^{(C)} = \Pi_A^{(C)}\left(Q_A^{(C)}\right) - \Pi_B^{(C)}\left(Q_B^{(C)}\right)$ - the difference between two profits):

$$D^{(C)} = \Pi_A^{(C)}\left(Q_A^{(C)}\right) - \Pi_B^{(C)}\left(Q_B^{(C)}\right) = \frac{(\alpha-\beta)(-2a+\alpha+\beta)}{3b} - \gamma + \delta$$

3. The Case of Stackelberg Production Leader and Satellite in Terms of Production

In this case, the company A assumes the role of production leader and company B recognizes this.

Because the company A is a leader of production, it will produce Q_A units of a good. The company B will adjust production after A, in response to its leadership, producing $Q_B=f(Q_A)$ units of good (f - the reaction function). Because the selling price depends on the total quantity of products reached the market, we have $p=p(Q_A+Q_B)$ – the price per unit of good.

On the other hand, the company A must establish a level of production depending on the reaction of firm B, because it will determine through the production realized the selling price of the product. Similarly, the company B will adjust its production levels according to A, because at a higher or lower level, the price will change and therefore profit of the company ([1]).

Let therefore, the profit of the production leader:

$$\Pi_A(Q_A) = [a - b(Q_A + Q_B)]Q_A - (\alpha Q_A + \gamma) = -bQ_A^2 + (a - bQ_B - \alpha)Q_A - \gamma$$

Since $Q_B=f(Q_A)$ we have:

$$\Pi_A(Q_A) = -bQ_A^2 + (a - bf(Q_A) - \alpha)Q_A - \gamma$$

Consider also the profit of the satellite:

$$\Pi_B(Q_B) = [a - b(Q_A + Q_B)]Q_B - (\beta Q_B + \delta) = -bQ_B^2 + (a - bQ_A - \beta)Q_B - \delta$$

The extreme condition for the profit of B is:

$$\frac{\partial \Pi_B(Q_B)}{\partial Q_B} = -2bQ_B + a - bQ_A - \beta = 0$$

therefore:

$$Q_B = f(Q_A) = \frac{a-\beta}{2b} - \frac{Q_A}{2}$$

Now the profit of A will become:

$$\Pi_A(Q_A) = -\frac{bQ_A^2}{2} + \frac{a-2\alpha+\beta}{2}Q_A - \gamma$$

and the condition for maximizing:

$$\frac{\partial \Pi_A(Q_A)}{\partial Q_A} = -bQ_A + \frac{a-2\alpha+\beta}{2} = 0$$

therefore (noting with PL from production leader):

$$Q_A^{(PL)} = \frac{a-2\alpha+\beta}{2b},\ Q_B^{(PL)} = f\left(Q_A^{(PL)}\right) = \frac{a+2\alpha-3\beta}{4b}$$

and the price will be:

$$p^{(PL)}\left(Q_A^{(PL)} + Q_B^{(PL)}\right) = a - b\left(Q_A^{(PL)} + Q_B^{(PL)}\right) = \frac{a+2\alpha+\beta}{4}$$

The profits of both firms A and B are:

$$\Pi_A^{(PL)}\left(Q_A^{(PL)}\right) = -bQ_A^{(PL)2} + \left(a - bQ_B^{(PL)} - \alpha\right)Q_A^{(PL)} - \gamma = \frac{(a-2\alpha+\beta)^2}{8b} - \gamma$$

respectively:

$$\Pi_B^{(PL)}\left(Q_B^{(PL)}\right) = -bQ_B^{(PL)2} + \left(a - bQ_A^{(PL)} - \beta\right)Q_B^{(PL)} - \delta = \frac{(a+2\alpha-3\beta)^2}{16b} - \delta$$

Because A assumes that it is leader of production, we must have: $Q_A^{(PL)} > Q_B^{(PL)}$ that is: $\frac{a-2\alpha+\beta}{2b} > \frac{a+2\alpha-3\beta}{4b} \Leftrightarrow a > 6\alpha - 5\beta$.

We have now (noting $D^{(PL)} = \Pi_A^{(PL)}\left(Q_A^{(PL)}\right) - \Pi_B^{(PL)}\left(Q_B^{(PL)}\right)$ - the difference between two profits):

$$D^{(PL)} = \Pi_A^{(PL)}\left(Q_A^{(PL)}\right) - \Pi_B^{(PL)}\left(Q_B^{(PL)}\right) = \frac{a^2 - 2(6\alpha-5\beta)a + 4\alpha^2 + 4\alpha\beta - 7\beta^2}{16b} - \gamma + \delta$$

4. The Case of Stackelberg Price Leader and Satellite in Terms of Price

In this case, the company A assumes the role of price leader and company B recognizes this.

It is obvious that, regardless of the satellite firm behavior, the final sale price will be the same for the two companies, otherwise the demand being moving to the company with the lowest price.

Let the price being p>0 – fixed in the final.

The profit functions of the two companies are therefore:

$$\Pi_A(Q_A) = pQ_A - (\alpha Q_A + \gamma) = (p-\alpha)Q_A - \gamma$$

$$\Pi_B(Q_B) = pQ_B - (\beta Q_B + \delta) = (p-\beta)Q_B - \delta$$

From these formulas we have the necessary condition that $p>\alpha$, $p>\beta$ (otherwise $\Pi_A(Q_A)<0$ or $\Pi_B(Q_B)<0$).

Because both profit functions are of first degree in Q and $p-\alpha>0$, $p-\beta>0$ they reach the maximum at infinity.

In this case (with linear total cost of B) the production of B will be, for the moment, indeterminate, let say $Q_B=q>0$.

Meanwhile, the company leadership is aware that setting a selling price p will lead a production Q_B of the satellite firm, so in terms of a demand curve $Q=Q(p)$ its offer will be restricted to $Q_A(p)=Q-Q_B=Q(p)-q=\frac{a-p}{b}-q=\frac{a-bq}{b}-\frac{1}{b}p$. Because $Q_A(p)>0$ we must have: $q<\frac{a-p}{b}$. If $q\geq\frac{a-p}{b}$ the production of A: $Q_A(p)\leq 0$, therefore A gives up at the price leader role.

Its profit function becomes:

$$\Pi_A(p) = (p-\alpha)Q_A(p) - \gamma = (p-\alpha)\left(\frac{a-bq}{b} - \frac{1}{b}p\right) - \gamma =$$

$$-\frac{1}{b}p^2 + \frac{a-bq+\alpha}{b}p - \frac{\alpha(a-bq)+b\gamma}{b}$$

The profit maximization condition of A is therefore:

$$\frac{\partial \Pi_A(p)}{\partial p} = -\frac{2}{b}p + \frac{a-bq+\alpha}{b} = 0$$

from where (noting with PP from production price):

$$p^{(PP)} = \frac{a-bq+\alpha}{2}$$

From this relation, because $q < \frac{a - p^{(PP)}}{b}$ we have $p^{(PP)} = \frac{a - bq + \alpha}{2} > \frac{a - b\frac{a - p^{(PP)}}{b} + \alpha}{2} = \frac{p^{(PP)} + \alpha}{2}$ that is: $p^{(PP)} > \alpha$ which satisfies the initial condition on the price.

Finally:

$$Q_A^{(PP)} = \frac{a - bq}{b} - \frac{1}{b} p^{(PP)} = \frac{a - bq - \alpha}{2b}, \; Q_B^{(PP)} = q$$

The profits of both firms A and B are:

$$\Pi_A^{(PP)}\left(Q_A^{(PP)}\right) = \left(p^{(PP)} - \alpha\right) Q_A^{(PP)} - \gamma = \frac{(a - bq - \alpha)^2}{4b} - \gamma$$

respectively:

$$\Pi_B^{(PP)}\left(Q_B^{(PP)}\right) = \Pi_B^{(PP)}(q) = \left(p^{(PP)} - \beta\right) Q_B^{(PP)} - \delta = -\frac{b}{2}\left(q - \frac{a + \alpha - 2\beta}{2b}\right)^2 + \frac{(a + \alpha - 2\beta)^2 - 8b\delta}{8b}$$

Because $\Pi_B^{(PP)}$ is a second degree function in q we have now the following cases:

- $(a + \alpha - 2\beta)^2 < 8b\delta$ implies that $\Pi_B^{(PP)}(q) < 0$ $\forall q \in \mathbf{R}$ therefore the company B will lose, therefore it not accept the role of price satellite
- $(a + \alpha - 2\beta)^2 = 8b\delta$ implies that the maximum of the profit of B will be $\Pi_B^{(PP)}(q) = 0$ for $q = \frac{a + \alpha - 2\beta}{2b}$
- $(a + \alpha - 2\beta)^2 > 8b\delta$ implies that $\Pi_B^{(PP)}$ has two real roots q_1, $q_2 \in \mathbf{R}$. Because $q_1 q_2 = \frac{2\delta}{b} > 0$ we have that $q_1, q_2 < 0$ or $q_1, q_2 > 0$. If $q_1, q_2 < 0 \Leftrightarrow a + \alpha - 2\beta < 0$, the parabola having negative dominant coefficient, we find that for $q > 0$: $\Pi_B^{(PP)}(q) < 0$.

If $q_1, q_2 > 0 \Leftrightarrow a + \alpha - 2\beta > 0$ the maximum of the second degree function is reached for: $q = \frac{a + \alpha - 2\beta}{2b}$ and is $\Pi_B^{(PP)}\left(Q_B^{(PP)}\right) = \frac{(a + \alpha - 2\beta)^2}{8b} - \delta$.

Also, in this case: $\Pi_A^{(PP)}\left(Q_A^{(PP)}\right) = \frac{(a - 3\alpha + 2\beta)^2}{16b} - \gamma$ and it must satisfy also: $(a - 3\alpha + 2\beta)^2 > 16b\gamma$

The productions will be: $Q_A^{(PP)} = \frac{a - 3\alpha + 2\beta}{4b}$, $Q_B^{(PP)} = \frac{a + \alpha - 2\beta}{2b}$ and the price: $p^{(PP)} = \frac{a + \alpha + 2\beta}{4}$.

Like a conclusion, for B in order to recognize a role of price leader to A and also A to assume this role, they must satisfy the conditions:

- $(a + \alpha - 2\beta)^2 > 8b\delta$
- $a + \alpha - 2\beta > 0$
- $a - 3\alpha + 2\beta > 0$
- $(a - 3\alpha + 2\beta)^2 > 16b\gamma$

We have now (noting $D^{(PP)} = \Pi_A^{(PP)}\left(Q_A^{(PP)}\right) - \Pi_B^{(PP)}\left(Q_B^{(PP)}\right)$ - the difference between two profits):

$$D^{(PP)} = \Pi_A^{(PP)}\left(Q_A^{(PP)}\right) - \Pi_B^{(PP)}\left(Q_B^{(PP)}\right) = \frac{-a^2 - 2(5\alpha - 6\beta)a + 7\alpha^2 + 4\alpha\beta + 4\beta^2}{16b} - \gamma + \delta$$

5. The Case When A Ignores B, but B Considers A as Production Leader

In this case from sections 2 and 3, we have (noting with C,PL from Cournot and production leader):

$$Q_A^{(C,PL)} = \frac{a-2\alpha+\beta}{3b},\ Q_B^{(C,PL)} = \frac{a+2\alpha-3\beta}{4b}$$

The real price will be:

$$p^{(C,PL)}\left(Q_A^{(C,PL)} + Q_B^{(C,PL)}\right) = a - b\left(Q_A^{(C,PL)} + Q_B^{(C,PL)}\right) = \frac{5a+2\alpha+5\beta}{12}$$

and the profits:

$$\Pi_A^{(C,PL)}\left(Q_A^{(C,PL)}\right) = -bQ_A^{(C,PL)2} + \left(a - bQ_B^{(C,PL)} - \alpha\right)Q_A^{(C,PL)} - \gamma = \frac{5(a-2\alpha+\beta)^2}{36b} - \gamma$$

respectively:

$$\Pi_B^{(C,PL)}\left(Q_B^{(C,PL)}\right) = -bQ_B^{(C,PL)2} + \left(a - bQ_A^{(C,PL)} - \beta\right)Q_B^{(C,PL)} - \delta = \frac{(a+2\alpha-3\beta)(5a+2\alpha-7\beta)}{48b} - \delta$$

We have now (noting $D^{(C,PL)} = \Pi_A^{(C,PL)}\left(Q_A^{(C,PL)}\right) - \Pi_B^{(C,PL)}\left(Q_B^{(C,PL)}\right)$ - the difference between two profits):

$$D^{(C,PL)} = \Pi_A^{(C,PL)}\left(Q_A^{(C,PL)}\right) - \Pi_B^{(C,PL)}\left(Q_B^{(C,PL)}\right) = \frac{5a^2 - 2(58\alpha - 53\beta)a + 68\alpha^2 - 20\alpha\beta - 43\beta^2}{144b} - \gamma + \delta$$

6. The Case when A Ignores B, but B Considers A as Price Leader

In this case from sections 2 and 4 we have (noting with C,PP from Cournot and production price):

$$Q_A^{(C,PP)} = \frac{a-2\alpha+\beta}{3b},\ Q_B^{(C,PP)} = \frac{a+2\alpha-3\beta}{4b}$$

The real price will be:

$$p^{(C,PP)}\left(Q_A^{(C,PP)} + Q_B^{(C,PP)}\right) = a - b\left(Q_A^{(C,PP)} + Q_B^{(C,PP)}\right) = \frac{a+\alpha+4\beta}{6}$$

and the profits:

$$\Pi_A^{(C,PP)}\left(Q_A^{(C,PP)}\right) = -bQ_A^{(C,PP)2} + \left(a - bQ_B^{(C,PP)} - \alpha\right)Q_A^{(C,PP)} - \gamma = \frac{(a - 5\alpha + 4\beta)(a - 2\alpha + \beta)}{18b} - \gamma$$

respectively:

$$\Pi_B^{(C,PP)}\left(Q_B^{(C,PP)}\right) = -bQ_B^{(C,PP)2} + \left(a - bQ_A^{(C,PP)} - \beta\right)Q_B^{(C,PP)} - \delta = \frac{(a + \alpha - 2\beta)^2}{12b} - \delta$$

We have now (noting $D^{(C,PP)} = \Pi_A^{(C,PP)}\left(Q_A^{(C,PP)}\right) - \Pi_B^{(C,PP)}\left(Q_B^{(C,PP)}\right)$ - the difference between two profits):

$$D^{(C,PP)} = \Pi_A^{(C,PP)}\left(Q_A^{(C,PP)}\right) - \Pi_B^{(C,PP)}\left(Q_B^{(C,PP)}\right) = \frac{-a^2 - 2(10\alpha - 11\beta)a + 17\alpha^2 - 14\alpha\beta - 4\beta^2}{36b} - \gamma + \delta$$

7. The Case when A is A Production Leader, but B Ignores A

In this case from section 2 and 3 we have (noting with PL,C from production leader and Cournot):

$$Q_A^{(PL,C)} = \frac{a - 2\alpha + \beta}{2b},\ Q_B^{(PL,C)} = \frac{a + \alpha - 2\beta}{3b}$$

The real price will be:

$$p^{(PL,C)}\left(Q_A^{(PL,C)} + Q_B^{(PL,C)}\right) = a - b\left(Q_A^{(PL,C)} + Q_B^{(PL,C)}\right) = \frac{a + 4\alpha + \beta}{6}$$

and the profits:

$$\Pi_A^{(PL,C)}\left(Q_A^{(PL,C)}\right) = -bQ_A^{(PL,C)2} + \left(a - bQ_B^{(PL,C)} - \alpha\right)Q_A^{(PL,C)} - \gamma = \frac{(a - 2\alpha + \beta)^2}{12b} - \gamma$$

respectively:

$$\Pi_B^{(PL,C)}\left(Q_B^{(PL,C)}\right) = -bQ_B^{(PL,C)2} + \left(a - bQ_A^{(PL,C)} - \beta\right)Q_B^{(PL,C)} - \delta = \frac{(a + 4\alpha - 5\beta)(a + \alpha - 2\beta)}{18b} - \delta$$

We have now (noting $D^{(PL,C)} = \Pi_A^{(PL,C)}\left(Q_A^{(PL,C)}\right) - \Pi_B^{(PL,C)}\left(Q_B^{(PL,C)}\right)$ - the difference between two profits):

$$D^{(PL,C)} = \Pi_A^{(PL,C)}\left(Q_A^{(PL,C)}\right) - \Pi_B^{(PL,C)}\left(Q_B^{(PL,C)}\right) = \frac{a^2 - 2(11\alpha - 10\beta)a + 4\alpha^2 + 14\alpha\beta - 17\beta^2}{36b} - \gamma + \delta$$

8. The Case when A is a Production Leader, but B Considers A as Price Leader

In this case from section 2, we have (noting with PL,PP from production leader and production price):

$$Q_A^{(PL,PP)} = \frac{a - 2\alpha + \beta}{2b},\ Q_B^{(PL,PP)} = \frac{a + \alpha - 2\beta}{2b}$$

The real price will be:

$$p^{(PL,PP)}\left(Q_A^{(PL,PP)} + Q_B^{(PL,PP)}\right) = a - b\left(Q_A^{(PL,PP)} + Q_B^{(PL,PP)}\right) = \frac{\alpha + \beta}{2}$$

and the profits:

$$\Pi_A^{(PL,PP)}\left(Q_A^{(PL,PP)}\right) = -bQ_A^{(PL,PP)2} + \left(a - bQ_B^{(PL,PP)} - \alpha\right)Q_A^{(PL,PP)} - \gamma = \frac{\beta - \alpha}{2} - \gamma$$

respectively:

$$\Pi_B^{(PL,PP)}\left(Q_B^{(PL,PP)}\right) = -bQ_B^{(PL,PP)2} + \left(a - bQ_A^{(PL,PP)} - \beta\right)Q_B^{(PL,PP)} - \delta = \frac{\alpha - \beta}{2} - \delta$$

We have now (noting $D^{(PL,PP)} = \Pi_A^{(PL,PP)}\left(Q_A^{(PL,PP)}\right) - \Pi_B^{(PL,PP)}\left(Q_B^{(PL,PP)}\right)$ - the difference between two profits):

$$D^{(PL,PP)} = \Pi_A^{(PL,PP)}\left(Q_A^{(PL,PP)}\right) - \Pi_B^{(PL,PP)}\left(Q_B^{(PL,PP)}\right) = \beta - \alpha - \gamma + \delta$$

9. The Case when A is a Price Leader, but B Ignores A

In this case from section 2, we have (noting with PP,C from production price and Cournot):

$$Q_A^{(PP,C)} = \frac{a - 3\alpha + 2\beta}{4b},\ Q_B^{(PP,C)} = \frac{a + \alpha - 2\beta}{3b}$$

The real price will be:

$$p^{(PP,C)}\left(Q_A^{(PP,C)}+Q_B^{(PP,C)}\right)=a-b\left(Q_A^{(PP,C)}+Q_B^{(PP,C)}\right)=\frac{5a+5\alpha+2\beta}{12}$$

and the profits:

$$\Pi_A^{(PP,C)}\left(Q_A^{(PP,C)}\right)=-bQ_A^{(PP,C)2}+\left(a-bQ_B^{(PP,C)}-\alpha\right)Q_A^{(PP,C)}-\gamma=\frac{(5a-7\alpha+2\beta)(a-3\alpha+2\beta)}{48b}-\gamma$$

respectively:

$$\Pi_B^{(PP,C)}\left(Q_B^{(PP,C)}\right)=-bQ_B^{(PP,C)2}+\left(a-bQ_A^{(PP,C)}-\beta\right)Q_B^{(PP,C)}-\delta=\frac{5(a+\alpha-2\beta)^2}{36b}-\delta$$

We have now (noting $D^{(PP,C)}=\Pi_A^{(PP,C)}\left(Q_A^{(PP,C)}\right)-\Pi_B^{(PP,C)}\left(Q_B^{(PP,C)}\right)$ - the difference between two profits):

$$D^{(PP,C)}=\Pi_A^{(PP,C)}\left(Q_A^{(PP,C)}\right)-\Pi_B^{(PP,C)}\left(Q_B^{(PP,C)}\right)=\frac{-5a^2+2(-53\alpha+58\beta)a+43\alpha^2+20\alpha\beta-68\beta^2}{144b}-\gamma+\delta$$

10. The Case When A is A Price Leader, but B Considers A as Production Leader

In this case from section 2, we have (noting with PP,PL from production price and production leader):

$$Q_A^{(PP,PL)}=\frac{a-3\alpha+2\beta}{4b},\ Q_B^{(PP,PL)}=\frac{a+2\alpha-3\beta}{4b}$$

The real price will be:

$$p^{(PP,PL)}\left(Q_A^{(PP,PL)}+Q_B^{(PP,PL)}\right)=a-b\left(Q_A^{(PP,PL)}+Q_B^{(PP,PL)}\right)=\frac{2a+\alpha+\beta}{4}$$

and the profits:

$$\Pi_A^{(PP,PL)}\left(Q_A^{(PP,PL)}\right)=-bQ_A^{(PP,PL)2}+\left(a-bQ_B^{(PP,PL)}-\alpha\right)Q_A^{(PP,PL)}-\gamma=\frac{(2a-3\alpha+\beta)(a-3\alpha+2\beta)}{16b}-\gamma$$

respectively:

$$\Pi_B^{(PP,PL)}\left(Q_B^{(PP,PL)}\right) = -bQ_B^{(PP,PL)2} + \left(a - bQ_A^{(PP,PL)} - \beta\right)Q_B^{(PP,PL)} - \delta = \frac{(2a+\alpha-3\beta)(a+2\alpha-3\beta)}{16b} - \delta$$

We have now (noting $D^{(PP,PL)} = \Pi_A^{(PP,PL)}\left(Q_A^{(PP,PL)}\right) - \Pi_B^{(PP,PL)}\left(Q_B^{(PP,PL)}\right)$ - the difference between two profits):

$$D^{(PP,PL)} = \Pi_A^{(PP,PL)}\left(Q_A^{(PP,PL)}\right) - \Pi_B^{(PP,PL)}\left(Q_B^{(PP,PL)}\right) = \frac{7(\alpha-\beta)(-2a+\alpha+\beta)}{16b} - \gamma + \delta$$

11. Theory of Games in the Case of Duopoly

Of the above, we have nine cases representing the situation in which are the two companies. The question is the decision of each of them not knowing the competitor's decision, in order to obtain a great difference of the profits.

Let therefore, the following zero sum game:

Table 1

A/B	Cournot	production satellite	price satellite	min
Cournot	$D^{(C)}$	$D^{(C,PL)}$	$D^{(C,PP)}$	$v_1=\min\left(D^{(C)}, D^{(C,PL)}, D^{(C,PP)}\right)$
production leader	$D^{(PL,C)}$	$D^{(PL)}$	$D^{(PL,PP)}$	$v_2=\min\left(D^{(PL,C)}, D^{(PL)}, D^{(PL,PP)}\right)$
price leader	$D^{(PP,C)}$	$D^{(PP,PL)}$	$D^{(PP)}$	$v_3=\min\left(D^{(PP,C)}, D^{(PP,PL)}, D^{(PP)}\right)$
max	$w_1=\max\left(D^{(C)}, D^{(PL,C)}, D^{(PP,C)}\right)$	$w_2=\max\left(D^{(C,PL)}, D^{(PL)}, D^{(PP,PL)}\right)$	$w_3=\max\left(D^{(C,PP)}, D^{(PL,PP)}, D^{(PP)}\right)$	W=min $\{w_1,w_2,w_3\}$ /max $\{v_1.v_2,v_3\}$=V

Applying the Wald criterion we obtain the best choice of A corresponding to the value V and the best choice of B corresponding to the value W.

Example

Let two producers, the demand inverse function: p(Q)=100-Q, total costs of A and B being $TC_A(Q)=\alpha Q+\gamma$, $TC_B(Q)=\beta Q+\gamma$, $\alpha,\beta,\gamma\geq 0$ where Q is the production.

We have the following game:

Table 2

A/B	Cournot	production satellite	price satellite	min
Cournot	$\frac{(\alpha-10)(\alpha-190)}{3}$	$\frac{\alpha(\alpha-290)+4825}{4}$	$\frac{7}{16}\alpha^2-65\alpha+100$	v_1
production leader	$\frac{\alpha(17\alpha-2950)+37925}{36}$	$\frac{\alpha(17\alpha-2140)+11600}{36}$	$\frac{\alpha(\alpha-515)+7075}{9}$	v_2
price leader	$10-\alpha$	$\frac{43}{144}\alpha^2-\frac{650}{9}\alpha+\frac{3700}{9}$	$\frac{7(\alpha-10)(\alpha-190)}{16}$	v_3
max	w_1	w_2	w_3	W / V

Using the computer for simulating in the cases: $\beta=10$, $\alpha\in[0,20]$ with step 1 we obtain:

- $\alpha=0$ - A – Cournot – V=322.2, B – price satellite – W=322.2
- $\alpha=1$ - A – Cournot – V=263.3, B – price satellite – W=263.3
- $\alpha=2$ - A – Cournot – V=205.2, B – price satellite – W=205.2
- $\alpha=3$ - A – Cournot – V=148.1, B – price satellite – W=148.1
- $\alpha=4$ - A – Cournot – V=92, B – price satellite – W=92
- $\alpha=5$ - A – Cournot – V=36.81, B – price satellite – W=36.81
- $\alpha=6$ - A – production leader – V=4, B – price satellite – W=4
- $\alpha=7$ - A – production leader – V=3, B – price satellite – W=3
- $\alpha=8$ - A – production leader – V=2, B – price satellite – W=2
- $\alpha=9$ - A – production leader – V=1, B – price satellite – W=1
- $\alpha=10$ - A – production leader – V=0, B – price satellite – W=0
- $\alpha=11$ - A – production leader – V=-1, B – price satellite – W=-1
- $\alpha=12$ - A – production leader – V=-2, B – price satellite – W=-2
- $\alpha=13$ - A – production leader – V=-3, B – price satellite – W=-3

- α=14 - A – production leader – V=-4, B – price satellite – W=-4
- α=15 - A – production leader – V=-47.22, B – Cournot – W=-47.22
- α=16 - A – production leader – V=-101, B – Cournot – W=-101
- α=17 - A – production leader – V=-154.6, B – Cournot – W=-154.6
- α=18 - A – production leader – V=-207.9, B – Cournot – W=-207.9
- α=19 - A – production leader – V=-261, B – Cournot – W=-261
- α=20 - A – production leader – V=-313.9, B – Cournot – W=-313.9

We can see that for a marginal price of A it will have an advantage in front of B, but in order to have a maximal difference it must ignore for $\alpha \le 5$ what B will do and for $\alpha \ge 6$ it will adopt a position of production leader. For $\alpha > 10$, because its marginal cost is greater than that of B it will lose but in order to minimize the difference between it and B it must adopt also the position of production leader.

12. References

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