Evaluating Exchange Rate Value at Risks Models for Fourteen African Currencies

Nausheen Jandoo[1], Preethee Nunkoo Gonpot[2]

Abstract: The global foreign exchange market is undoubtedly the world's biggest market with huge trading volume, surpassing other markets including equities and commodities. This study focuses on exchange rate modelling where we perform an empirical study to evaluate models which can be used to identify a common Value at Risk (VaR) model for fourteen African currencies. The descriptive statistics of our data reveal the salient features common to financial time series which are non-normality, high kurtosis, skewness and presence of heteroscedasticity except for one currency, the central African CFA Franc. The latter is excluded from the modelling exercise. We make use of GARCH, GJR-GARCH and FIGARCH to model volatility using four distributions: normal, student-t, GED and skew-t. Unconditional EVT and dynamic GARCH-EVT methodologies are also used for volatility modelling; both with static (S) and rolling windows (R). Results show that static window shows a better performance than rolling window. Unconditional EVT is seen to overpredict VaR and dynamic EVT is not among the best models. The GARCH (33.3%) and GJR-GARCH (38.5%) models produce better forecasts with a dominance for GJR-GARCH models. Despite the data being skewed, the normal distribution gives better forecast. We also observe that GARCH-S-Normal is suitable for Southern African Development Community (SADC) and FIGARCH for East African Community (EAC) countries. A geographical combination reveals the use of GJR-GARCH for Northern and Western African regions and GARCH-S-Normal for South African region. Despite not finding a unique model for all countries, it is interesting to note that different regions/communities can adopt a common Value at Risk model for forecasting purposes. Our results provide a full validation of the models under the different backtesting methods and thus could be implemented at the practitioner's level.

Keywords: volatility; value at risk; exchange rate; Africa; GARCH; backtesting

JEL Classification: C50; C51; C52; C53

[1] Nausheen Jandoo, Department of Mathematics, University of Mauritius, Republic of Mauritius, Address: Reduit, Republic of Mauritius, Corresponding author: nausheensoobratty@hotmail.fr.

[2] Dr Preethee Nunkoo Gonpot, Department of Mathematics, University of Mauritius, Republic of Mauritius, Address: Reduit, Republic of Mauritius, Tel.: 403 7491, E-mail: pgonpot@uom.ac.mu.

1. Introduction

Methods for quantifying risk have gained much prominence in the financial world since investors greatly rely on them before making any major investment. There are several types of risk, however in this paper we shall concentrate on market risk, more specifically exchange rate risk. Exchange rate is not given much attention, especially when trade and other transactions are taking place in the domestic currency. However, an exchange rate is a crucial factor that affect a country's relative external level of price competitiveness and holds an important position in trade, thus its impact to the economy is important. Most major economies adopt a floating exchange rate regime, a system where a currency's value is determined solely by the interaction of market forces of demand and supply instead of by government intervention. In a world of global financial market, a high percentage of assets is owned and traded by non-residents of a country, thus involving the foreign exchange market. It is obvious that the exchange rate is an integral part of the financial system and is thus important for financial stability. With the growing involvement of the African continent in the global trade, there is a higher probability of market participants being exposed to currency rate fluctuations. The aim of this study is thus to provide a tool to the traders, exporters, importers, producers, and investors, so that they can have an idea of the potential loss they may incur in their investment strategies, by proposing VaR forecasting models. In this paper, the US Dollar against 14 currencies of the African countries have been identified for the study since they trade significantly with the United States. We selected developing countries from different regions of the African continent to shun off any potential bias. The following section gives an overview of some of the literature on exchange rate forecasting.

1.1. Literature Review

Brooks & Burke (1998) apply 186 different GARCH models of different orders, ranging from AR(0)-GARCH(0,0) to AR(5)-GARCH(5,5), to three currencies: the Canadian Dollar, German Mark and Japanese Yen. The study concludes that the GARCH(1,1) model is chosen less than 20% of the time. Vilasuso (2002) used GARCH, IGARCH and FIGARCH to forecast the volatility of five major currencies. The results show that FIGARCH is a better model to produce 1- and 10-day-ahead volatility forecasts for the exchange rates than the other two models. Chong, Chun & Ahmad (2002) employ 11 models to the Malaysian Ringgit against Pound Sterling and observe that the GARCH-in-mean models produce better forecasts than the GARCH models. So & Yu (2006) forecast one-step ahead VaR of 12 stock indices and 4 foreign exchange rates using 6 GARCH models and RiskMetrics and conclude that GARCH-student-t model is the best. Degiannakis & al. (2013) compares the performance of the long memory FIGARCH model with that of short memory GARCH specification to forecast VaR and Expected

Shortfall (ES) across 20 stock indices, and find that FIGARCH does not appear to improve VaR and ES forecasting accuracy compared to GARCH models. Johanssen & Sowa (2013) compare VaR forecasts of three markets: commodities, equities and exchange rates using ARCH(1,1), GARCH(1,1) and EGARCH(1,1) and found that ARCH and EGARCH are better options. Bedowska-Sojka (2015) compares the volatility forecasting performance of GARCH, GJR-GARCH, IGARCH, FIGARCH and models for realised volatility (HAR-RV, HAR-RV-J, ARFIMA). The analysis is performed on WIG20 index quoted on the Warsaw Stock Exchange from 2007 to 2011. Models of both classes are seen to give comparable results and memory features as well as asymmetry improve the VaR forecasts. In their study, Kutu & Ngalawa (2017) concludes that EGARCH is a better model than GARCH for model for volatility of South African exchange rate. Petrica & Stancu (2017) apply different specifications of GARCH models with 5 different distributions to the EUR/RON exchange rate. The predominant models were the EGARCH and PARCH, and the best model to estimate daily returns is EGARCH(2,1) with asymmetric order 2 under the assumption of student-t distribution.

The Extreme Value Theory (EVT) has-been widely used in the calculation of Value at Risk. Neftci (2000) applies both the standard method of VaR calculation under normal conditions and the EVT to interest rates and exchange rates. The latter concludes that the statistical theory of extremes provides a more precise approach for risk management. Mc Neil & Frey (2000) combines the GARCH and EVT approaches to consider both volatility clustering effect and tail study and conclude that this approach give better estimates than methods overlooking the fat tails of innovations. Carvalhal & Mendes (2003) show that the extreme value theory applied to the Asian stock market is a more cautious method than traditional methods to calculate VaR. Gilli & Kellezi (2006) apply EVT to 6 stock markets using two approaches, Block-Maxima Model (BMM) and Peak-Over-Threshold (POT), and concludes that the POT approach is superior. Wang et al. (2010) applies extreme value theory (EVT) to estimate the tails of return series of Chinese yuan (CNY) exchange rates. The EVT-based VaR values underestimate the risks of exchange rates such as USD/CNY and HKD/CNY. However, VaR calculated by EVT measure the risk more accurately for the exchange rates of JPY/CNY and EUR/CNY compared to the historical simulation and variance–covariance method. Numerous studies have been done using the EVT and GARCH approach to forecast volatility and VaR for stock indices (Gencay & Selcuk, 2004) Recently, Jesus et al. (2013) tested EVT to estimate the risk of the foreign exchange market with respect to Dollar/Peso for the period 1970 to 2007. The models compared are the Historical Simulation, Delta Normal and EVT. Results reveal that the estimation of VaR by EVT is the best method.

Other than research on model validation, the effects of the financial crisis on assets have also been investigated. Farhat (2016) estimates, forecasts and evaluates the VaR of Karachi Stock Exchange before and after the global financial crisis of 2008. The observed number of VaR violations using the Bayesian method is found close to the expected number of violations. He concluded that these models produce accurate and reliable VaR forecasts. Emenike (2010) reveals evidence of volatility clustering, fat tails and leverage effect in the Nigeria Stock Exchange. The Khartoum stock exchange has also been subject to these investigations with the conclusion that asymmetric models with presence of leverage effect fit the data better than symmetric ones. H.C Huang et al. (2015) uses four different GARCH models along with four mean equations to compare the performance of VaR forecasting to the GARCH(1,1) model when applied to the MSCI World Index in the financial crisis. Findings reveal that GARCH-in-mean model outperforms the other models in terms of number of violations and that the number of violations decrease by using the in-mean or MA(1) mean equations.

It can be seen that so far the crisis effect has been investigated mainly for stock markets. Concerning exchange rate analysis, not much has been done for African countries. The African continent has undergone a period of sustained economic growth since the past two decades and thus there is a need to investigate the exchange rate market and the effect of the 2008 financial crisis on the market.

1.2. Objectives of the Study

This emergence of the African continent as an important contributor in the economic sector has not gone unnoticed by journalists, economists, business people and investors, which can encourage them to invest in these countries, thus the need to investigate the fluctuations of African exchange rates which can help in decision making. This paper thus aims at addressing the issue of:

validating exchange rate models for some African countries;

identifying the possibility of a unique model for the continent/community/region using different econometric approaches in modelling exchange rate behavior;

investigating the effect of the financial crisis of 2008 on the exchange rate market of Africa.

By carrying out our study, we intend to fill the gap in the literature for exchange rate in the African continent.

2. Methodology

2.1. Market Risk Model

The fluctuations in the exchange rate is usually measured by the standard deviation of outcomes (σ). The evaluation of risk, measured by the volatility, has become a dominant matter such that volatility forecasting has gained much significance in modern finance. According to the Basel II accord, financial institutions have to reserve a minimum amount of funds so that they can cover potential losses. The preferred approach to calculate the capital requirement is Value at Risk (VaR).

The VaR computation for parametric distributions is given by equation (1):

$$VaR_{\alpha\%} = \alpha_{z\%}\sigma\sqrt{t} \tag{1}$$

where σ = standard deviation or volatility, α = quantile of the standardised distribution and t is the holding period.

2.2. Univariate Volatility Model

2.2.1. GARCH (p,q) Model

The GARCH (p,q) model from the work of Bollerslev (1986), being a generalization of the ARCH model of Engle (1982) is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p}\alpha_i\sigma_{t-i}^2 + \sum_{j=1}^{q}\beta_j\varepsilon_{t-j}^2 \tag{2}$$

such that $\omega > 0, \alpha_i > 0$ for $i = 1,\dots p$ and $\beta_j > 0$ for $j = 1,\dots,q.$ α measures the shock reaction, i.e. the extent to which a volatility shock enters the next period while β is a determinant of the degree of persistence such that a large β implies shocks to conditional variance take a long time to die out. For stationarity to be maintained:

$$\sum_{i=1}^{p}\alpha_i + \sum_{j=1}^{q}\beta_j < 1 \tag{3}$$

2.2.2. GJR-GARCH Model

To consider the asymmetric relation between returns and volatility known as the leverage effect, the Glosten, Jagannathan and Runkle (GJR-GARCH) model (1993) is suitable. The choice of model for the asymmetric effect went upon the GJR instead of the exponential GARCH, based on the study of Engle and Ng (1993) who argue that the variability of the conditional variance of EGARCH is too high and that GJR is the best one. The dynamics of the model evolves according to

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \beta_j \varepsilon_{t-j}^2 + \sum_{k=1}^{r} \gamma_k I(\varepsilon_{t-k} < 0) \varepsilon_{t-k}^2 \tag{4}$$

where I(.) is an indicator taking value 1 if $\varepsilon_t < 0$, and 0 otherwise. To ensure positivity:

$$\begin{aligned} &\alpha_i > 0 \text{ for } i = 1, \ldots, p, \\ &\beta_j > 0 \text{ for } j = 1, \ldots, q, \\ &\alpha_i + \gamma_k > 0 \text{ for } i = 1, \ldots, p \text{ and } k = 1, \ldots, r \end{aligned} \tag{5}$$

Furthermore, the model is stationary if

$$\alpha + \beta + \frac{\gamma}{2} < 1 \tag{6}$$

2.2.3. Fractionally Integrated GARCH (FIGARCH)

Baillie et al. (1996) introduced the FIGARCH (p,d,q) model which has the possibility of taking into account the long memory characteristic of financial market volatility. The conditional volatility of a FIGARCH (1,d,1) model evolves as follows:

$$h_t = \omega + \left[1 - \beta_1 L - (1 - \phi_1 L)(1 - L)^d\right] \varepsilon_t^2 + \beta_1 h_{t-1} \tag{7}$$

where $0 < d < 1$. $(1-L)^d$ is known as the fractional differencing operator, and its value depends on the decay rate of a shock to conditional volatility. The fractional differencing operator is expressed in terms of the hypergeometric function:

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(d+1)}{\Gamma(k+1)\Gamma(d-k+1)} L^k \tag{8}$$

For the conditional variance of the FIGARCH(1,d,1) to be positive for all t, $\omega > 0,\ \beta_1 - d \le \phi_1 \le \frac{2-d}{2}$ and $d\left(\phi_1 - \frac{1-d}{2}\right) \le \beta_1(\phi_1 - \beta_1 + d)$.

2.3. Distributions

In our study, we use four different distributions for the random variable, namely the Normal, Student's-t, Generalised Error Distribution (GED) and Skewed-t distribution (

Table 5).

2.4. Extreme Value Theory

To capture extreme returns in financial data, we make use of the extreme value theory where the Peak-Over-Threshold (POT) approach is used. This method analyses the behaviour of large observations that exceed a high threshold. Given a high threshold u and taking into account all the exceedances of u, the distribution of excess values of x over the threshold u, $(x-u)$ is given by

$$F_u(y) = \Pr\{X - u \le y | X > u\} = \frac{F(u+y) - F(u)}{1 - F(u)}, y > 0 \tag{9}$$

Balkema and de Haan (1974) and Pickands (1975) states that for a sufficiently high threshold, u the distribution of the exceedances may be approximated to the Generalised Pareto distribution (GPD).

$$F_u(y) \to G_{\xi,\sigma}(y) \tag{10}$$

The distribution function of the GPD is given by

$$G_{\xi,\sigma}(y) = \begin{cases} 1 - \left(1 + \dfrac{\xi y}{\sigma}\right)^{-\frac{1}{\xi}}, \xi \ne 0 \\ 1 - e^{-y/\sigma}, \xi = 0 \end{cases} \tag{11}$$

where ξ is the shape parameter. Since $x = y + u$ for $X > u$, the distribution of excess values takes the form of

$$F(x) = [1 - F(u)]F_u(y) + F(u) \tag{12}$$

Combining equations (10) and (12), we obtain

$$F(x) = [1 - F(u)]G_{\xi,\beta,u}(x - u) + F(u) \tag{13}$$

Table 5. Probability distributions for the random variable.

Distribution	Probability Density Function
Normal	$f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$.
Student-t	$f(x)=\frac{\Gamma((k+1)/2)}{\Gamma(k/2)}\frac{1}{\sqrt{(k\pi)}}\frac{1}{\left(1+\left(\frac{x^2}{k}\right)\right)^{\frac{(k+1)}{2}}}$
Generalised Error Distribution (GED)	$f(x)=\frac{k}{\lambda\left(2^{1+\frac{1}{k}}\right)\Gamma\left(\frac{1}{k}\right)}e^{\left(-\frac{1}{2}\left\|\frac{x}{\lambda}\right\|^k\right)}$ $\lambda=\left[2^{\left(\frac{-2}{v}\right)}\frac{\Gamma\left(\frac{1}{v}\right)}{\Gamma\left(\frac{3}{v}\right)}\right]^{\frac{1}{2}}$.
Skewed-t distribution	$f(x)=\begin{cases} bc\left(1+\frac{1}{v-2}\left(\frac{bx+a}{1-\lambda}\right)^2\right)^{-\frac{v+1}{2}} & \text{if } x<-\frac{a}{b} \\ bc\left(1+\frac{1}{v-2}\left(\frac{bx+a}{1+\lambda}\right)^2\right)^{-\frac{v+1}{2}} & \text{if } x\geq-\frac{a}{b}\end{cases}$ $a=4\lambda c\frac{v-2}{v-1},\ b^2=1+3\lambda^2-a^2,\ c=\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)}\Gamma\left(\frac{v}{2}\right)}$.

For a high threshold u, $F(u)$ can be determined by the empirical estimator $\frac{(n-N_u)}{n}$ where N_u is the number of exceedances and *n* is the sample size. The tail estimator is then given by

$$\hat{F}(x)=1-\frac{N_u}{n}\left(1+\hat{\xi}\frac{x-u}{\hat{\sigma}}\right)^{-1/\hat{\xi}} \tag{14}$$

where $\hat{\xi}$ and $\hat{\sigma}$ are maximum likelihood estimates of the GPD parameters. Inverting the tail estimator, we obtain a percentile $\hat{x}_p$ at the tail for a probability $p>F(p)$, known as the Value at Risk:

$$\hat{x}_p=u+\frac{\hat{\sigma}}{\hat{\xi}}\left(\left(\frac{n}{N_u}(1-p)\right)^{-\hat{\xi}}-1\right) \tag{15}$$

2.4.1. Threshold Determination

The Mean Excess plot and Hill Plot are used to determine the threshold.

Mean Excess Function (MEF)

This function is the sum of the excesses over the threshold u divided by the number of data points that exceeds the threshold u. It is defined by

$$e_n(u) = \frac{\sum_{t=1}^{n}(X_t - u)}{\sum_{t=1}^{n} I_{(X_t > u)}} \quad \text{where } I = \begin{cases} 1 \text{ if } X_t > u \\ 0, \text{ otherwise} \end{cases} \tag{16}$$

If the plot of the mean excess function follows a straight line where the gradient is positive above a certain value of the threshold u, then this indicates that the excesses over this value follow a GPD with positive shape parameter, ξ, implying that the GDP is heavily-tailed.

Hill Plot

The Hill Plot is another useful method to determine the threshold u. An estimator for ξ was proposed by Hill (1975):

$$\hat{\xi} = \frac{1}{k-1}\sum_{t=1}^{k-1} \ln X_{t,N} - \ln X_{k,N}, \text{ for } k \geq 2 \tag{17}$$

where k is the number of exceedances, N is the sample size and $\alpha = \frac{1}{\xi}$ is the tail index. We choose a threshold from the plot where the shape parameter ξ is fairly stable.

2.5. Dynamic EVT

To take into account volatility clustering, we combine the EVT method with GARCH to calculate VaR, thus giving rise to the GARCH-EVT approach introduced by Mc Neil & Frey (2000). This procedure follows a two-step methodology.

A GARCH model is first fitted to the return series by maximum likelihood estimation. Which provides the residuals for the second step as well as the one-step ahead prediction of μ_{t+1} and σ_{t+1}.

The peak over threshold (POT) method is then applied to the residuals for a constant choice of threshold u to estimate the daily VaR as given by:

$$VaR_q = \mu_{t+1} + \sigma_{t+1} VaR(Z_q) \quad (18)$$

2.6. Backtesting

Unconditional coverage (Frequency of exceptions and Kupiec POF test) and conditional coverage (Chistoffersen and Mixed Kupiec test) methods are used to backtest our models.

Under the frequency of exception test, at a 99% confidence interval, using 250 forecasts, we would expect to have (0.01 x 250) = 2.5 exceptions on average. The test statistic for the Kupiec test (1995, cited in Nieppola, 2009) is

$$LR_{POF} = -2\ln\left(\frac{(1-p)^{N-x} p^x}{\left[1-\left(\frac{x}{N}\right)\right]^{N-x}\left(\frac{x}{N}\right)^x}\right) \quad (19)$$

This test is asymptotically chi-squared (χ^2) distributed with one degree of freedom (critical value 6.6349). The null hypothesis is rejected if

$$LR_{POF} > \chi^2_{1-\alpha} \quad (20)$$

Christoffersen's Interval Forecast test (1998) expands the unconditional Kupiec test by including a separate test statistic for the independence of violations. The structure of the test is as follows:

A variable I_t, indicating the occurrence of a violation is first set up.

$$I_t = \begin{cases} 1 & \text{if a violation occurs} \\ 0 & \text{if no violation occurs} \end{cases} \quad (21)$$

We define n_{ij} as the number of days such that status j occurred on one day while it was status i on the previous day. Moreover, let π_i be the probability of detecting an exception conditional on status *i* on the prior day. Then

$$\pi_0 = \frac{n_{01}}{n_{00}+n_{01}},$$
$$\pi_1 = \frac{n_{11}}{n_{10}+n_{11}}, \quad (22)$$
$$\pi = \frac{n_{01}+n_{11}}{n_{00}+n_{01}+n_{10}+n_{11}}.$$

An accurate model would be one in which the occurrence of an exception today is independent of the happenings of the previous day. Our test will take the form of:

H_o: The model is accurate with equal probabilities of π_0 and π_1.

H_a: The model is inaccurate with unequal probabilities of π_0 and π_1.

Test Statistic for independence is given by LR_{ind}:

$$LR_{ind} = -2\ln\left[\frac{[(1-\pi)^{n_{00}+n_{10}}\pi^{n_{01}+n_{11}}]}{[(1-\pi_0)^{n_{00}}\pi_0^{n_{01}}(1-\pi_1)^{n_{10}}\pi_1^{n_{11}}]}\right]. \quad (23)$$

which is then merged with the Kupiec-test giving the test statistic for conditional coverage as

$$LR_{CC} = LR_{POF} + LR_{ind} \quad (24)$$

This is asymptotically χ^2 distributed with two degrees of freedom (critical value 9.2103). The null hypothesis is rejected if

$$LR_{CC} > \chi^2_{1-\alpha} \quad (25)$$

The Mixed Kupiec-test considers the time between exceptions rather than monitoring the outcome of the previous day provided that the current day produces a violation. The test statistic for each exception is given by

$$LR_i = -2\ln\left(\frac{p(1-p)^{v_i-1}}{\left(\frac{1}{v_i}\right)\left(1-\frac{1}{v_i}\right)^{v_i-1}}\right) \quad (26)$$

where v_i is the time between exceptions i and $i-1$. After calculating the test statistic for each exception, the conditional coverage mixed Kupiec-test takes the form of:

H_o: The exceptions are independent of each other and the failure rate is 0.01.

H_a: The exceptions are not independent of each other and the failure rate is not 0.01.

The test statistic for independence is given by LR_{ind}:

$$LR_{ind} = \sum_{i=2}^{n}\left[-2\ln\left(\frac{p(1-p)^{v_i-1}}{\left(\frac{1}{v_i}\right)\left(1-\frac{1}{v_i}\right)^{v_i-1}}\right)\right] - 2\ln\left(\frac{p(1-p)^{v-1}}{\left(\frac{1}{v}\right)\left(1-\frac{1}{v}\right)^{v-1}}\right) \quad (27)$$

The test statistic for joint test is thus expressed as $LR_{mix} = LR_{POF} + LR_{ind}$.

Critical region: If the number of exceptions is *n*, this test is χ^2 distributed with $n+1$ degrees of freedom. The null hypothesis is rejected if.

$$LR_{mix} > \chi^2_{1-\alpha} \tag{28}$$

2.7. Loss Functions

In addition to statistical tests, we make use of loss functions (Lopez, 1999) to classify our models. A correct model producing the least score is viewed as the most appropriate. Table 6 represents two loss functions implemented in this study as per the paper of Chung & Gonpot (2016).

Table 6. Loss functions used to compare VaR models

Loss function	if $r_t < VaR_t$	Otherwise
Asymmetric Linear Loss (ASL)	$(\alpha-1)(r_t - VaR_t)$	$\alpha(r_t - VaR_t)$
Quantile Loss (QL)	$(r_t - VaR_t)^2$	$(R - VaR_t)^2$

R is the 100c percentile of the returns data available at time *t*-1.

3. Results and Discussions

The US Dollar against 14 currencies of the following countries are under study: Algeria, Egypt, Ethiopia, Kenya, Morocco, Mauritania, Mauritius, Namibia, Nigeria, South Africa, Tanzania, Tunisia, the Western and Central African countries using the CFA Franc. The common forecasting models for all the countries are identified followed by models for country groupings SADC, COMESA and EAC and for groups based on the United Nations Country groupings.

3.1. Data Analysis

Our data spans from 1st January 2000 to 30th June 2012, consisting of 4565 observations. Using the software Matlab, we use GARCH-based models with different distributions to model our data and forecast VaR estimates which will be compared with the returns. Furthermore, we investigate the use of static and rolling sample to generate the VaR forecasts and thus verify which one produces better results. A sample of T returns, is divided into two sub-samples: estimation sample with vector $(r_1, r_2, \cdots, r_{T-n})'$ and evaluation sample with vector $(r_{T-n+1}, r_{T-n+2}, \cdots, r_T)'$. In the case of the static sample, we obtain the GARCH parameters with the estimation sample and use the same parameters each time to produce the forecasts. However, in the case of the rolling sample, we carry out a first estimation with $(r_1, r_2, \cdots, r_{T-n})'$ and generate the first VaR forecast, VaR_1. Then we omit the first return, i.e. we use vector $(r_2, r_3, \cdots, r_{T-n+1})'$ and obtain VaR_2 and so on. We implement our three GARCH models with orders *p, q* and r taking the value of 1.

Three different periods are considered for analysis in this study. These are the whole period (2000-2012), pre-crisis period (2000-2007) and the post-crisis period (2008-2012). This will help to determine whether the same GARCH models can produce reliable VaR forecasts for different time periods and whether the crisis affects the models selected. Table 7 presents the exchange rate regimes adopted in the countries under investigation as well as the abbreviations associated with each country. Table 8 presents the descriptive statistics of the returns for each currency. which reveals zero mean, positive skewness and high kurtosis indicating that the currency returns are highly peaked and do not follow the normal distribution. This hypothesis is further confirmed by the Jarque-Bera normality test where the p-value is zero. The ADF unit root test indicates stationarity of the series while the ARCH test at 5% confidence level rejects the null hypothesis of absence of heteroscedasticity. Since ARCH effects have been detected in our data, we can thus proceed with GARCH estimation to produce the VaR forecasts. However, there are no ARCH effects for USD/XAF. Thus, no further investigation is carried out for this exchange rate.

The combination of the three GARCH type models under the four distributions with static and rolling windows for each, combined with unconditional EVT, dynamic GARCH/EVT lead to the estimation of 37 models for each exchange rate and each period. Table 9 presents the parameters for the GARCH type models for all the currencies.

Table 7. Exchange Rate Regimes of each country

Country	Exchange Rate Regimes	Abbreviation
Algeria	Managed floating with no pre-determined path for the exchange rate	DZD
Egypt	April 1999: pegged	EGP
	2001: crawling within horizontal bands	
	2003: Managed floating with no pre-determined path for the exchange rate	
Ethiopia	Managed floating with no pre-determined path for the exchange rate	ETB
Kenya	Managed floating with no pre-determined path for the exchange rate	KES
Morocco	Conventional fixed peg arrangements: against a composite	MAD
Mauritania	Managed floating with no pre-determined path for the exchange rate	MRO
Mauritius	Managed floating with no pre-determined path for the exchange rate	MUR
Namibia	conventional fixed peg arrangements: against a single currency	NAD
Nigeria	Managed floating with no pre-determined path for the exchange rate	NGN
South Africa	Independently floating	ZAR
Tunisia	2000-2001: managed floating	TND

	2002-2004: crawling peg 2005- managed floating	
Tanzania	Independently floating	TZS
Western CFA	Exchange arrangements with no separate legal tender: fixed exchange rate to the euro	CFA

Table 8. Descriptive statistics of exchange rates

	Mean	Median	Max.	Min.	Std. Dev.	Skewness	Kurtosis	JB p-value	ADF p-value	ARCH p-value
DZD	2.94E-05	0.0000	0.0445	-0.0564	0.0053	-0.0406	15.0121	0.0000	0.0000	0.0000
EGP	1.25E-04	0.0000	0.1551	-0.0979	0.0048	8.1263	323.9632	0.0000	0.0000	0.0000
ETB	1.75E-04	0.0000	0.1831	-0.1600	0.0068	4.5797	299.2917	0.0000	0.0000	0.0130
KES	3.10E-05	0.0000	0.0479	-0.0541	0.0044	0.1281	25.9160	0.0000	0.0000	0.0000
MAD	-2.80E-05	0.0000	0.0516	-0.0267	0.0042	0.5461	10.8312	0.0000	0.0001	0.0000
MRO	5.48E-05	0.0000	0.1117	-0.1079	0.0064	0.9164	75.6638	0.0000	0.0000	0.0000
MUR	4.34E-05	0.0000	0.0582	-0.0660	0.0049	0.2455	22.4191	0.0000	0.0000	0.0000
NAD	6.88E-05	0.0000	0.1598	-0.1097	0.0099	0.8382	32.3622	0.0000	0.0000	0.0000
NGN	1.07E-04	0.0000	0.1412	-0.1542	0.0087	-0.3905	69.1116	0.0000	0.0000	0.0000
TND	5.24E-05	0.0000	0.1507	-0.1539	0.0186	-0.0849	28.1978	0.0000	0.0000	0.0024
TZS	1.49E-04	0.0000	0.0581	-0.0580	0.0056	0.0190	20.2373	0.0000	0.0000	0.0000
XAF	-4.59E-05	0.0000	0.0820	-0.0719	0.0072	-0.0339	21.6340	0.0000	0.0000	0.4257
XOF	1.03E-04	0.0000	0.0334	-0.0387	0.0060	-0.0217	7.0471	0.0000	0.0000	0.0000
ZAR	6.52E-05	0.0000	0.1107	-0.1090	0.0089	0.5007	18.4732	0.0000	0.0000	0.0000

Note: DZD: Algerian Dinar, EGP: Egyptian Pound, ETB: Ethiopian Birr, KES: Kenyan Shilling, MAD: Moroccan Dirham, MRO: Mauritanian Ouguiya, MUR: Mauritian Rupee, NAD: Namibian Dollar, NGN: Nigerian Naira, TND: Tunisian Dinar, TZS: Tanzanian Shilling, XOF: Western African CFA Franc, ZAR: South African Rand

3.2. Backtesting Results

The backtesting results are presented for the exchange rates in terms of number of violations, the statistical tests and the loss functions. The tests are performed at a significance level of 1%. When there is no violation, the tests cannot be computed and the corresponding models are automatically rejected. If a model is not rejected by the Kupiec and Christoffersen test but rejected by the mixed Kupiec test, we do not accept the model since this test takes into account both the number of exceptions and the independence of exceptions. We first assess the models according to the number of violations produced which should be 2.5 ideally. The risk models are then ranked based on the least scores of the loss functions. Note that during the discussions S stands for static window while R stands for rolling window. R1 refers to the model ranked first, R2 refers to model ranked second and R3 refers to the model ranked third[1]. The detailed statistics are not presented but the first three best models chosen for each country in the three different periods under analysis are presented in Table 10, 7 and 8.

3.2.1. Choice of Models for all Countries

Whole period

In the whole period, it is noted that Mauritius, Morocco and Nigeria end up selecting same models (TGARCH-S-Normal and TGARCH-S-GED in same order). Morocco has a strong tourist industry focused on the country's coast as well as Mauritius. This can account for the choice of the models. Algeria also chooses TGARCH-S-Normal as one of best model. Algeria and Nigeria are known among the largest oil exporters in the world. Indeed, in Africa the top oil producer in 2013 was Nigeria followed by Algeria. Imports and exports affect greatly the fluctuations in exchange rates.

No models fit the Tunisian exchange rate. This can be accounted by the fact that in 2011, the country faced a lot of political problem. The economy of the country, considered as one of the most robust performers in Africa, suffered a lot, hence having a direct impact on the exchange rate. The Namibian Dollar is pegged to the South African Rand at a rate of 1:1. Since Namibia imports goods and services from South Africa, this arrangement remains a benefit with the elimination of uncertainty associated with exchange rate variability. With this system, we expect Namibian exchange rate to have almost similar results as South Africa (GARCH-S-Normal for both countries).

Table 9. GARCH, GJR-GARCH and FIGARCH parameters for all currencies for the whole period

DZD- Algerian Dinar, EGP - Egyptian Pound, ETB - Ethiopian Birr, KES - Kenyan Shilling, MAD - Moroccan Dirham, MRO - Mauritanian Ouguiya, NAD - Namibian Dollar, NGN - Nigerian Naira, TND - Tunisian Dinar, XOF - Western African Franc, ZAR - South African Rand

[1] In the tables * indicates that the null hypothesis is not rejected.

	DZD	EGP	ETB	KES	MAD	MRO	NAD	NGN	TND	TZS	XOF	ZAR
GARCH												
μ	2.9432e-05	1.2514e-04	1.7546e-04	3.0977e-05	-2.8006e-05	5.4799e-05	6.8820e-05	1.0693e-04	5.2352e-05	-1.4888e-04	-4.5921e-05	6.5225e-05
ω	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
α	0.0827	0.1146	0.0907	0.1658	0.0241	0.0584	0.0333	0.1319	0.1105	0.1197	0.0438	0.0598
β	0.9116	0.8852	0.6979	0.8197	0.9742	0.9252	0.9477	0.8657	0.8893	0.8801	0.9212	0.9332
GJR-GARCH												
ω	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
α	0.0869	0.0321	0.0000	0.1292	0.0215	0.0618	0.0387	0.1673	0.0882	0.1234	0.0609	0.0757
γ	-0.0081	0.2009	0.1439	0.0678	0.0060	-0.0070	-0.0146	-0.0757	0.0418	-0.0065	-0.0338	-0.0408
β	0.9113	0.8673	0.7538	0.8243	0.9740	0.9253	0.9488	0.8680	0.8907	0.8797	0.9201	0.9369
FIGARCH												
ω	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ϕ	0.3187	0.1250	0.4881	0.1795	0.3478	0.0000	0.4295	0.0000	0.0000	0.1421	0.1255	0.0522
d	0.3626	0.7437	0.0238	0.6409	0.3044	1.0000	0.1409	1.0000	1.0000	0.5749	0.0766	0.2094
β	0.5073	0.8037	0.4157	0.5949	0.6111	0.9184	0.5075	0.8650	0.8892	0.5979	0.0000	0.0000

Table 10. Ranking of models for whole period

Country	Mauritius	Algeria	Egypt	Ethiopia	Kenya	Morocco	Mauritania
R1	GJR-GARCH-S-Normal	FIGARCH-R-Normal	FIGARCH-S-GED	GARCH-S-Student-t	EVT	GJR-GARCH-S-Normal	GARCH-S-GED
R2	GJR-GARCH-S-GED	FIGARCH-S-Normal	GJR-GARCH-S-GED	GARCH-S-Skewt	FIGARCH-S-Normal	GJR-GARCH-S-GED	GARCH-R-GED
R3	GARCH-S-Normal	GJR-GARCH-S-Normal	GARCH-S-GED	GJR-GARCH-S-Student-t	FIGARCH-R-Normal	GJR-GARCH-S-Student-t	GJR-GARCH-R-GED
Country	**Namibia**	**Nigeria**	**Tunisia**	**Tanzania**	**West African Countries**	**South Africa**	
R1	GARCH-S-GED	GJR-GARCH-S-Normal	-	FIGARCH-R-GED	GARCH-S-Skewt	GARCH-S-Normal	
R2	GARCH-S-Normal	GJR-GARCH-S-GED	-	FIGARCH-R-Skewt	GJR-GARCH-S-Skewt	GARCH-R-Normal	

Namibia, Mauritania and Egypt opt for the GARCH-S-GED model as one of the best. The Namibian economy is highly dependent on the extraction and processing of minerals for export. Similarly, the Mauritanian economy also depends on mining, with deposits of iron ore contributing to almost 50% of total exports. Furthermore, Egypt with a rather stable mixed economy has also an active mining sector. The mineral commodities produced in Egypt include aluminium, iron, gold, manganese, marble, and copper, among others. This could be the key factor which can account for the similar models.

Algeria and Kenya end up selecting two similar models which are FIGARCH-R-Normal and FIGARCH-S-Normal. This is quite surprising given the differences in their economies. In fact, agriculture contributes to 8% of GDP in Algeria and 23% of GDP in Kenya. Algeria is very rich in minerals while Kenya has no significant mineral endowment. Concerning tourism, only 1% contributes to GDP in Algeria while in Kenya, this is a developed sector making up 63% of the GDP of the country.

Ethiopia and the Western African countries opt for GARCH-S-Student-t and GARCH-S-Skew-t models among the best models. It is known that the economy of Ethiopia is greatly based on agriculture, similar to most of the countries forming part of the Western African CFA zone.

Pre-crisis and Post-crisis results

The aim of looking at VaR models in the pre-crisis (2000-2007) and post-crisis (2008-2012) is to analyse the effects of the financial crisis on the choice of the models. Thus, with respect to this section, we will concentrate only on the models selection rather than the economic explanation behind the choice of the model. Table 13 presents the groups of countries selecting the same 2 or 3 best models in the pre-crisis and post-crisis periods. Among the best models selected for the groups of countries, we find the three GARCH-S-Normal, GARCH-R-Normal and GJR-GARCH-R-Normal, being selected in the two different periods. In the pre-crisis period, we observe the groups composed of Tanzania, South Africa selecting same three models while in the post-crisis period, we have Algeria, Egypt, Namibia and South Africa. No common models are selecting like models are selected in the pre-crisis and post-crisis period, implying the financial crisis of 2008 did affect the exchange rate movements, leading to high volatility during that period. Thus, the same group of countries are not selecting the same models in the two periods.

After scrutinising the results it is obvious that it is not possible to have a unique model for the selected countries representing the African continent. Being given the diversity of these countries there are however some factors which do bring them under the same umbrella, African groupings.

Table 11. Ranking of models for pre-crisis period

Country	Mauritius	Algeria	Egypt	Ethiopia	Kenya	Morocco	Mauritania
R1	GARCH-EVT-Normal	FIGARCH-R-Skewt	FIGARCH-S-Student-t	GJR-GARCH-S-GED	EVT	GJR-GARCH-T-Normal	GARCH-EVT-Skewt
R2	GARCH-EVT-Student-t	FIGARCH-R-Student-t	FIGARCH-S-GED	GJR-GARCH-R-GED	FIGARCH-S-Normal	GARCH-S-Normal	GARCH-EVT-Normal
R3	GARCH-EVT-Skewt	FIGARCH-S-Skewt	GJR-GARCH-R-GED	GARCH-R-GED	GARCH-EVT-Normal	GJR-GARCH-R-GED	GJR-GARCH-EVT-Normal
Country	**Namibia**	**Nigeria**	**Tunisia**	**Tanzania**	**West African Countries**	**South Africa**	
R1	GJR-GARCH-S-GED	FIGARCH-S-Student-t	GARCH-R-Normal	GARCH-R-Normal	GARCH-R-Skewt	GJR-GARCH-R-Normal	
R2	FIGARCH-R-Normal	FIGARCH-S-GED	GJR-GARCH-R-Normal	GJR-GARCH-R-Normal	GJR-GARCH-R-Skewt	GARCH-S-Normal	
R3	-	FIGARCH-EVT-GED	GJR-GARCH-GED	GARCH-S-Normal	GARCH-R-Student-t	GARCH-R-Normal	

Table 12. Ranking of models for post-crisis period

Country	*Mauritius*	*Algeria*	*Egypt*	*Ethiopia*	*Kenya*	*Morocco*	*Mauritania*
R1	GARCH-S-Normal	GARCH-R-Normal	GJR-GARCH-R-Normal	GARCH-S-GED	GJR-GARCH-EVT-Normal	GJR-GARCH-S-Normal	GJR-GARCH-S-Normal
R2	GJR-GARCH-R-Normal	GJR-GARCH-R-Normal	GARCH-R-Normal	GARCH-R-GED	GARCH-R-Normal	GJR-GARCH-S-Student-t	GARCH-S-Normal
R3	GJR-GARCH-S-Normal	GARCH-S-Normal	GARCH-S-Normal	GJR-GARCH-S-GED	GJR-GARCH-R-Normal	GJR-GARCH-S-Skew-t	GJR-GARCH-S-GED
Country	**Namibia**	**Nigeria**	**Tunisia**	**Tanzania**	**West African Countries**	**South Africa**	
R1	GARCH-S-Normal	GJR-GARCH-S-Normal	GJR-GARCH-S-Normal	GJR-GARCH-R-Skewt	FIGARCH-R-Student-t	GARCH-R-Normal	
R2	GARCH-R-Normal	GARCH-S-Normal	GARCH-R-Normal	GARCH-R-Skewt	FIGARCH-R-Skew-t	GARCH-S-Normal	
R3	GJR-GARCH-R-Normal	GJR-GARCH-R-Normal	GJR-GARCH-R-Normal	GARCH-S-Skewt	FIGARCH-R-GED	GJR-GARCH-R-Normal	

3.2.2. Analysis based on African Groupings

The African countries have been grouped to help them in mutual economic development. Several organisations have been created to cater for each bloc in the continent with the aim of creating free trade areas, common central bank, customs union, a single market and a common currency. The aim is to finally establish a common economic and monetary union. The three African groupings to be considered in this section are SADC, COMESA and EAC, which have the the common goal of regional integration in Africa.

SADC was formed in 1980 by the head of states of nine countries from southern part of Africa. The leaders of the countries recognised that they were not able to raise people standards' of living and that working together could help them achieve their objectives which are to accomplish development and economic growth and alleviate poverty in the region. COMESA was formed in 1994 after Preferential Trade Area (PTA) was disbanded with the main objective of forming a wide economic area that has less economic barriers as faced by single member state. The EAC dates since 1967 with the aim of strengthening and improving the cooperation of the member countries through things as transport, communication, free immigration process, security, trade and industries. We will try to find out if a common model can be found for these countries when considered not as an individual entity but as a member of an organisation**.**

***SADC* (South African Development Community)**

Among the countries under investigation, Mauritius, Namibia, South Africa and Tanzania are members of SADC. Namibia and South Africa yield same model (GARCH-S-Normal), most probably because the Namibian Dollar is pegged to the South African Rand at a rate of 1:1. It is found that Mauritius also choose the same model in the third position. Out of four countries, we have three choosing the same model. Thus, we can use the GARCH-S-Normal to forecast the VaR of the SADC countries' exchange rates.

COMESA (Common Market for Eastern and Southern Africa)

Members of COMESA include Egypt, Ethiopia, Kenya and Mauritius. Only Mauritius and Egypt have the TGARCH-S-GED in common.

EAC (East African Community)

Kenya and Tanzania are members of the EAC. A look at Table 10 clearly indicates that the two countries select dissimilar models if the window and distribution are taken into account. However, we perceive that these two countries have a tendency to select FIGARCH as the best models, irrespective of the window and distribution.

Therefore, if we consider these three groups, we can say that SADC countries can use the GARCH-S-Normal model to forecast their exchange rates VaR and EAC can make use of the FIGARCH model. However for COMESA, we fail to identify a similar model for all countries. In the next section, we will regroup our data by geographical region based on the United Nation Country Grouping with the same aim of identifying common models for each region. Our objective is to find out if we can find a common model for the different regions. The different groups according to the United Nation Country Grouping are presented in.

Table 13. Groups of countries selecting same models in the pre-crisis and post-crisis period

Pre-crisis period		Post-crisis period	
Groups	**Models selected**	**Groups**	**Models selected**
Tanzania South Africa	GARCH-S-Normal GARCH-R-Normal GJR-GARCH-R-Normal	Algeria Egypt Namibia South Africa	GARCH-R-Normal GARCH-S-Normal GJR-GARCH-R-Normal
Tunisia Tanzania South Africa	GARCH-R-Normal GJR-GARCH-R-Normal	Algeria Egypt Mauritius Namibia Nigeria South Africa	GARCH-S-Normal GJR-GARCH-R-Normal
Morocco Tanzania South Africa	GARCH-S-Normal GJR-GARCH-R-Normal		

Table 14. Models selected for each grouping under investigation

SADC	COMESA	EAC
GARCH-S-Normal for Mauritius, Namibia and South Africa	TGARCH-S-GED selected for Mauritius and Egypt only	FIGARCH model for Kenya and Tanzania

3.2.3. Analysis based on Geographical Region

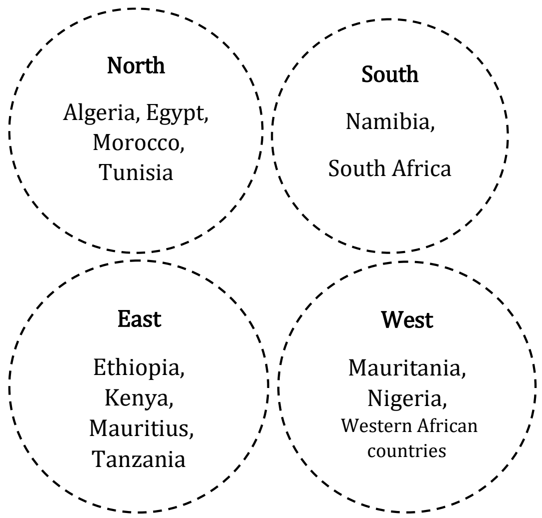

Figure 1. Geographical Groupings of countries under investigation

Our results are presented in

Table **15**. The analysis based on regional groupings shows that similar model can be found only for countries in the southern region (Namibia and South Africa). In the north, two pairs of countries are able to produce similar models. Algeria and Morocco select the GJR-GARCH-S Normal model while Egypt and Morocco both choose the GJR-GARCH-S-GED. If the distribution is not taken into account, Algeria, Egypt and Morocco all three opt for the GJR-GARCH model. Mauritania, Nigeria and the Western African countries falling in the west region all selects at least one GJR-GARCH model among the top 3 model. Thus GJR-GARCH seems to be an appropriate model for both North and West regions.

Table 15. Results based on geographical groupings

Northern Africa	***Southern Africa***	***Eastern Africa***	***Western Africa***
Tunisia: No model GJR-GARCH-S-Normal for Algeria and Morocco GJR-GARCH-S-GED for Egypt and Morocco	GARCH-S-Normal for Namibia and South Africa	Dissimilar model for all countries	GJR-GARCH-R-GED for Mauritania and Nigeria GJR-GARCH-S-Skewt for West African countries

4. Conclusion

We produce VaR forecasts for thirteen currencies in three different periods; whole period, pre-crisis period and post-crisis period. We present the three best models chosen according to the asymmetric and quantile loss functions. In the whole period, 33.3% of GARCH models, 38.5% of GJR-GARCH models, 20.5% of FIGARCH models and 2.5% of unconditional EVT models are chosen. Furthermore, static window dominates rolling window with 69.2% models chosen. Normal and GED are the two best distributions being selected 13 times. In the pre-crisis period, GARCH, GJR-GARCH, FIGARCH and EVT are all chosen with almost the same percentage. Rolling window shows a slightly better performance than static window in this period and the two favourite distributions remains Normal and GED.

In the post-crisis period, GJR-GARCH models (19 out of 39) has a slight advantage over GARCH (17 out of 26) models. In this period, we note almost the same number of models with static (18) and rolling (20) windows. As for the distributions, normal and GED clearly are better than the others. Unconditional EVT demonstrates a poor performance for exchange rates. Only in the case of Kenya, it shows a good performance while for the other exchange rates, this model produces no exceptions. It is also noticed that the dynamic EVT models tend to produce high loss function scores implying that these models are over predicting VaR. If the three periods are considered, GJR-GARCH comes out first with 45 models being selected followed by GARCH with 39 models. The normal distribution remains the best distribution for predicting VaR of exchange rates and static window is chosen more often than the rolling window. Our investigation reveals that there is no single GARCH model that can predict VaR accurately for all currencies. There are several factors that can account for the differences. Over the whole period, Tunisia fails to select any model due to political instability in the country. The Namibian Dinar and South African Rand as expected opt for same models since they are pegged to each other. Mauritius and Morocco, two countries whose economy depend greatly on tourism and Ethiopia and West African countries, whose economy is dependent on agriculture mainly choose same models respectively. Nigeria and Algeria, being the largest oil exporters in Africa both select the TGARCH-S-Normal model. Then, we investigate if same models are selected in the pre-crisis and post-crisis periods for the countries. Except for South Africa, we find that the other exchange rates opt for different models in the two periods.

An analysis for countries forming part of the African regional groupings SADC, COMESA and EAC reveal that SADC countries can use the GARCH-S-Normal model to predict their daily VaR while the EAC can rely on FIGARCH models for this task. For COMESA, we fail to find like models. Finally, based on the United Nation Country grouping, results showed the choice of like models for all the African regions except the Eastern African region. This paper studies GARCH models and tries to find the best model for chosen exchange rates and thus has important implications with respect to VaR estimation. Further research can concentrate on multivariate GARCH modelling in view of determining whether exchange rate volatility is being transmitted between African countries.

5. References

Baillie, R.T. & Bollerslev, T. (1991). lntra day and inter market volatility in foreign exchange rates, *Review of Economic Studies,* 58, pp. 565-585.

Balkema, A.A. & De Haan, L. (1974). Residual life time at great age. *The Annals of Probability,* 2(5), pp. 792–804.

Bedowska-Sojka, B. (2015). Daily VaR Forecasts with Realized Volatility and GARCH Models. *Argumenta Oeconomica,* 34, 1, pp. 157-173.

Bollerslev, T. (1986). Generalised Autoregressive Conditional Heteroscedasticity. *Journal of Econometrics,* 31, pp. 307-327.

Bollerslev, T.; Engle, R.F. & Woolridge, J.M. (1988). A capital asset pricing model with time varying covariances. *Journal of Political Economy*, 96, pp. 116–131.

Brooks, C. & Burke, S.P. (1998). Forecasting exchange rate volatility using conditional variance models selected by information criteria. *Economics Letters,* 61, pp. 273-278.

Carvalhal, A. & Mendes, B.V.M. (2003). Value-at-Risk and Extreme Returns in Asian Stock Markets. *International Journal of Business*, 8, 1, Available at SSRN: https://ssrn.com/abstract=420266 or http://dx.doi.org/10.2139/ssrn.420266.

Chong, C.W.; Chun, L.S. & Ahmad, M.I. (2002). Modelling the Volatility of Currency Exchange Rate Using GARCH Model. *Pertanika Journal of Social Science & Humanities*, 10 (2), pp. 85-95.

Christoffersen, P. (1998). Evaluating Interval Forecasts. *International Economic Review,* 39, pp. 841-862.

Jesus, R.; Ortiz, E. & Cabello, A. (2013). *Long run peso/dollar exchange rates and extreme value behavior: Value at Risk modeling. The North American Journal of Economics and Finance*, 24, pp. 139-152.

Degiannakis, S.; Floros, C. & Dent, P. (2013). Forecasting value at risk and expected shortfall using fractionally integrated models of conditional volatility: International evidence. *International Review of Financial Analysis,* 27, pp. 21-23.

Emenike, K.O. (2010). Modelling Stock Returns Volatility In Nigeria Using GARCH Models. *MPRA Paper 22723, University Library of Munich*, Germany. Retrieved from https://mpra.ub.uni-muenchen.de/id/eprint/22723.

Farhat I. (2016). Risk Forecasting of Karachi Stock Exchange: A Comparison of Classical and Bayesian GARCH models. *Pakistan Journal of Statistics and Operation Research*, 12, 3, pp. 453-465.

Fernandez, V. (2003). Extreme Value Theory and Value at Risk. *Revista de Análisis Económico*, 18 (1), pp. 57-85.

Gencay, R. & Selcuk, F. (2004). Extreme value theory and value-at-risk: relative performance in emerging markets. *International Journal of Forecasting*, 20 (2), pp. 287-303.

Gilli, M. & Kellezi, E. (2006). An Application of Extreme Value Theory for Measuring Financial Risk. *Computational Economics*, 27 (2-3), pp. 207-228.

Glosten, L.R.; Jagannathan, R. & Runkle, D.E. (1993). On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *Journal of Finance*, 48 (5), pp. 1779-1801.

Gonpot, P.N. & Chung, J.P. (2016). Risk model validation for BRICS countries: a value-at-risk, expected shortfall and extreme value theory approach. *Journal of Risk Model Validation*, 9(3), pp. 1-22.

Hill, B. (1975). A simple general approach to inference about the tail of a distribution. *The Annals of Statistics*, 3, pp. 1163–1174.

Huang, H.C.; Su, Y.C. & TSUI, J.T. (2015). Asymmetric GARCH Value-at-Risk over MSCI in Financial Crisis. *International Journal of Economics and Financial Issues*, 5(2), pp. 390-398.

Johanssen, A. & Sowa, V. (2013). A comparison of GARCH models for VaR estimation in three different markets. *Thesis*. Uppsala University .Retrieved from http://www.diva-portal.org/smash/record.jsf?pid=diva2%3A630568&dswid=-4971, date: 04.09.2018.

Jorion, P. (2002). *Value at Risk: The New Benchmark for managing Financial Risk*. 2nd Ed. Singapore: Mc Graw Hill.

Kutu, A.A & Ngalawa, H. (2017). Modelling Exchange Rate Volatility and Global Shocks in South Africa. *Acta Universitatis Danibius*, 13, 3, pp. 178-193.

Lopez, J.A. (1999). Methods for Evaluating Value-at-Risk Estimates. *Federal Reserve Bank of New York, Economic Policy Review*, 2, 3-39. Retrieved from https://www.newyorkfed.org/medialibrary/media/research/staff_reports/research_papers/9802.pdf, date: 04.09.2018.

Mabrouk, S. (2016). Forecasting Financial Assets Volatility Using Integrated GARCH-Type Models: International Evidence. *Journal of Finance and Economics,* 4 (2), pp. 54 - 62.

Mc Neil, A.J. & Frey, R. (2000). Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. *Journal of Empirical Finance,* 7(3–4), pp. 271-300.

Neftci, S.N. (2000). Value at risk calculations, extreme events, and tail estimation. *Journal of Derivatives,* 7, pp. 23–38.

Nieppola, O. (2009). Backtesting Value-at-Risk Models. *Thesis (PhD*). Helsinki School of Economics. Retrieved from:

https://aaltodoc.aalto.fi/bitstream/handle/123456789/181/hse_ethesis_12049.pdf?sequence=1&isAllowed=y, date: on 03.09.2018.

Petrica, A.C. & Stancu, S. (2017). Empirical Results of Modeling EUR/RON Exchange Rate using ARCH, GARCH, EGARCH, TARCH and PARCH models. *Romanian Statistical Review* nr. 1.

Pickands, J. (1975). Statistical inference using extreme order statistics. *Annals of Statistics,* 3, pp. 119–131.

So, M.K.P. & Yu, P.L.H. (2005). Empirical Analysis of GARCH models in value at risk estimation. *Journal of International Financial Markets, Institutions and Money*, 16(2), pp. 180-197. Abstract only.

Tsay, R.S. (2002). *Analysis of Financial Time Series*, 2nd ed. New Jersey: John Wiley & Sons INC., Publication.

Vilasuso, J. (2002). Forecasting exchange rate volatility, *Economics Letters*, 76, pp. 59-64.

Wang, Z.R.; Wu, W.T.; Chen, C. & Zhou, Y.J. (2010). The exchange rate risk of Chinese yuan: Using VaR and ES based on extreme value theory. .*Journal of Applied Statistics*, 37, pp. 265-282.