Theoretical Ground over the Cobweb Model

Professor Florin Dan Puşcaciu, PhD
"Danubius" University of Galati

Abstract: We propose a theoretical analysis of the linear Cobweb model and making some simulations with help of informatics product with multiple applications in economical research, which is Maple.

Keywords: Cobweb model, simulation, Maple program

Jel Classification: O40, O41

Presentation of Cobweb Model

On some markets including industrial goods with a long cycle of fabrication, the offer can not extend immediately for a greater growth. This way, in order to obtain crops, first it must be planted, it grows and then it is harvested. This process asks for a certain period of time.

Cobweb model is the one that took into consideration the offer reaction at the modifications of the demand from a certain market, through the presumption that the offered quantity now Q_t^{of} depends on the price from a previous period P_{t-1}, that is Q_t^{of}= f(P_{t-1}), where the basics shows a period of time. The consumer's demands on the same product market Q_t^{cer} depend on the current price, Q_t^{cer}= f(P_t).

In the case of a linear model of the market forces, we will have:

$$Q_t^{cer} = a+bP_t \quad \text{and} \quad Q_t^{of} = c+d\,P_{t-1}$$

Where a, b, c are the specific function parameters of the demand and supply, and the normal goods b is possibly negative.

The balance of the market involves equalization of the demand and supply, which says:

$$Q_t^{cer}=Q_t^{of} \Rightarrow a+bP_t=c+dP_{t-1} \Rightarrow P_t=(\frac{c-a}{b})+\frac{d}{b}P_{t-1}$$

The last relation shows a difference in the equation of first order, because the prices are different with only one time unit.

In legal terms this equation can be generalized as: $x_t=\alpha+\beta x_{t-1}$, where x shows the variable which modifies at a certain time, and α & β are constant measures as: α=(c-a)/b and β=d/b.

The solution of a different equation of first order has two components:

1) **The balance solution**: in Cobweb model it is as the price balance for a long period of time. As the price balance is the same in every period of time, it means that $P_t=P_{t-1}$, that is the balance solution represents a constant measure in connection with variable adjustment which modifies in time.

We designate P* balance price for a long period of time which maintains in every period, so: $P^*=P_t=P_{t-1}$, and substitute in difference equation $P_t=(\frac{c-a}{b})+\frac{d}{b}P_{t-1}$ we will have: $P^*=(\frac{c-a}{b})+\frac{d}{b}P^*$, $P^*=\frac{a-c}{d-b}$, in equal mode and with the balance price, with only one period.

2) **The complementary solution**: name the way which the variable, the price of Cobweb model modifies from the balance solution by the time. The difference equation $P_t=(\frac{c-a}{b})+\frac{d}{b}P_{t-1}$, can be written as $P_t=\frac{d}{b}P_{t-1}$, because the first element is not changing in time. We presume that $P_t=Ak^t$ where A and k are constants; this function applies for all t values, so $P_{t-1}=Ak^{t-1}$, and substituting the prices in difference equation, we obtain: $Ak^t=\frac{d}{b}Ak^{t-1}$. The value of A can be shown by knowing a certain measure of the price from a certain period of time.

This way, the final solution of a difference equation Cobweb model will be:

P_t =balance solution +complementary solution: $P_t=(\frac{a-c}{d-b})+A(\frac{d}{b})^t$

3) Numerical simulations

The final form of the model depends on the value of rapport d/b, which, for values that differ from 0 of A, will create three situations:

a) If $|\frac{d}{b}|<1$, then $(\frac{d}{b})^t \to 0$ as so t $\to\infty$.This situation is registered on a stabile market, as so the deviation from the balance price is becoming smaller. We impose the absolute size of the report because b is negative. See figure 1 and 2.

b) If $|\frac{d}{b}|>1$, then $(\frac{d}{b})^t \to \infty$ as so t $\to\infty$.This situation is registered on an unstable market. During the time, the price will deviate from his balance value, with a bigger size, after a initial deviation. See figure 3 and 4.

c) If $|\frac{d}{b}|=1$,then $|(\frac{d}{b})^t|=1$ as so t $\to\infty$.This situation will be registered on a fluctuant market, the price will change between two levels. See figure 5 and 6.

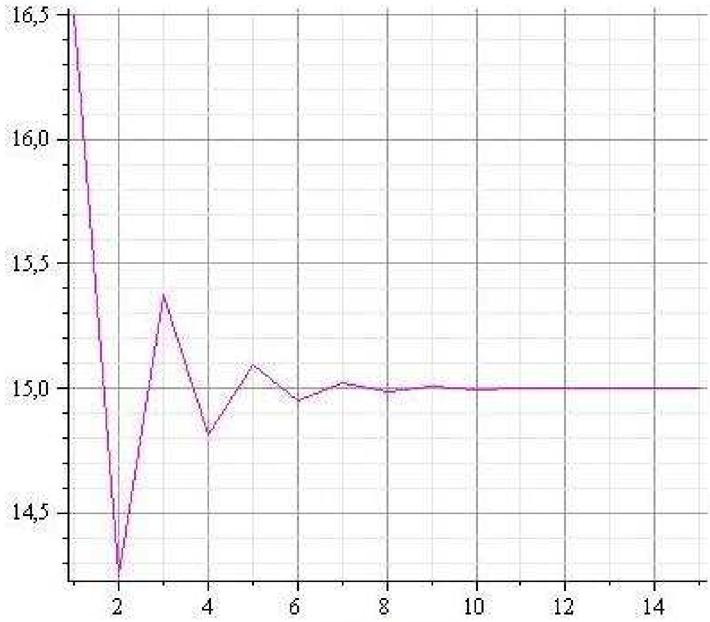

Fig 1 **Cobweb Model**
convergent-type k<1

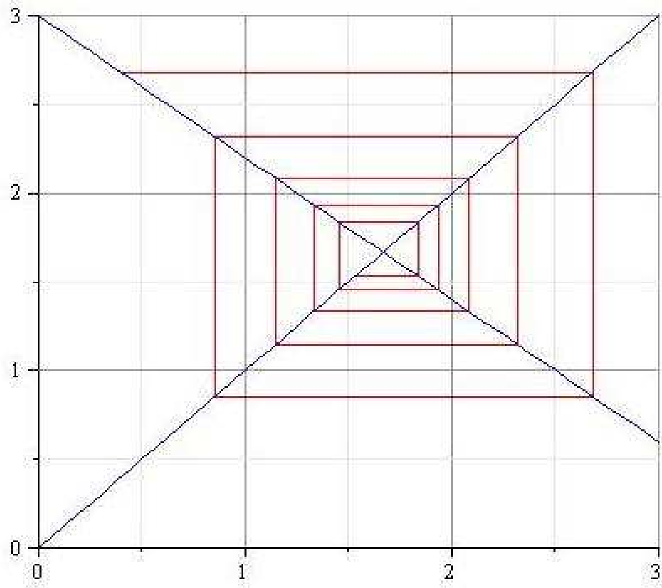

Fig 2 **Cobweb Model**
convergent-type

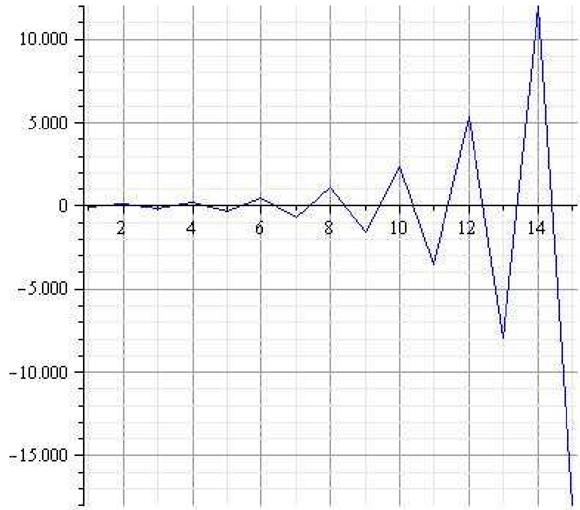

Fig 3 **Cobweb Model divergent type** k>1

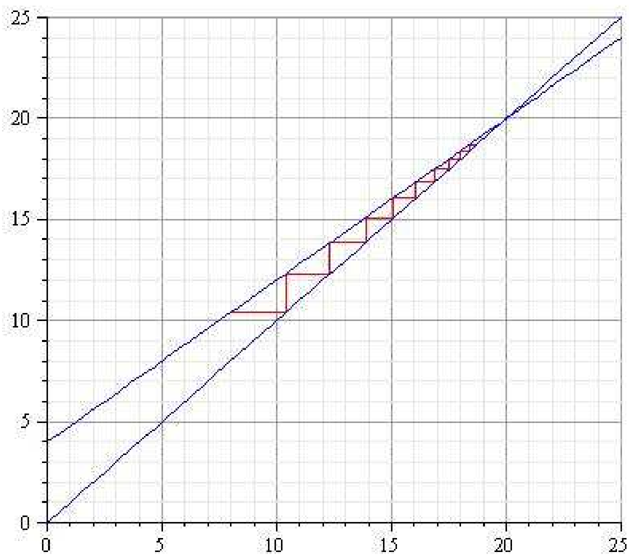

Fig 4 **Cobweb Model divergent type**

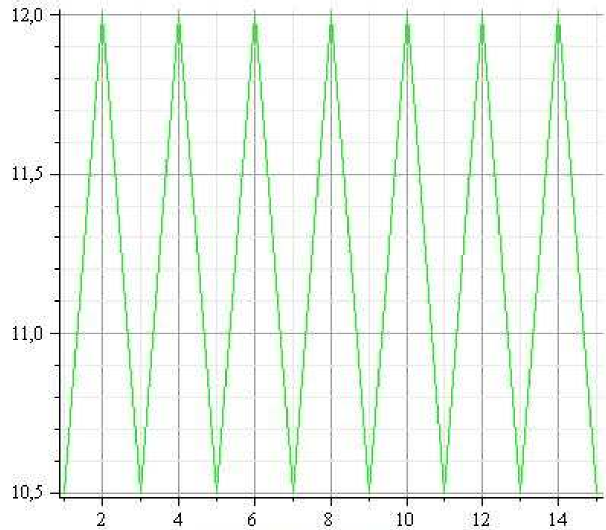

Fig 5 **Cobweb Model fluctuant type** k=1

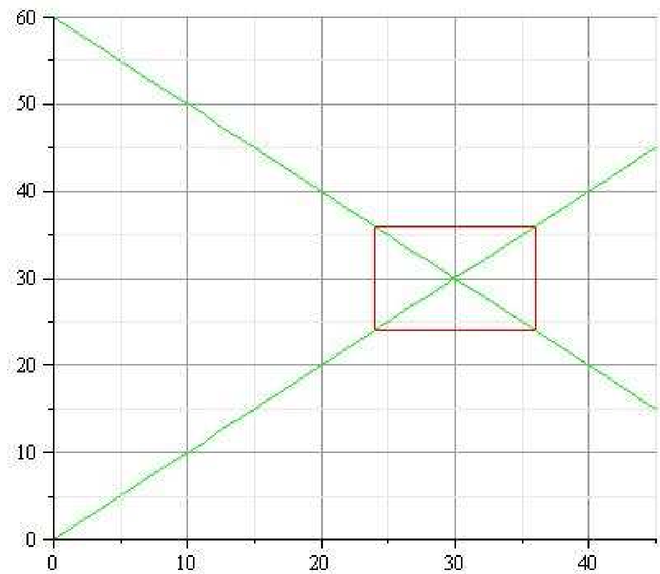

Fig 6 **Cobweb Model fluctuant type**

Refrences

Rosser, Mike. (1996). *Basic Mathematics for Economists*, London: Routledge.

Gandolfo, Giancarlo. (1972). *Mathematical Methods and Model in Economic Dynamics,* Amsterdam: Editura North-Holland Publishing Company.

Lica, Dionis, Teodorescu, Narcisa. (2004). *Sistem electronic de calcule matematice*, București: Matrix Rom.