Fiscal Sustainability and Interdependence of Primary Balance and Public Debt in South Africa

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Abstract: The main goal of this study was to evaluate sustainability of fiscal policy in South Africa, and assess the interdependence of primary balance and public debt, as ratios of gross domestic product, over the sample period 1997q4 to 2016q2. The Vector Error Correction (VEC) model was applied to estimate the fiscal reaction function using EViews program, while the VEC Granger-Causality/Block Exogeneity Wald test, impulse response functions and variance decompositions were applied to test for presence of interdependence between primary balance and public debt. Empirical results show strong evidence of consistency of government fiscal policy with the intertemporal budget constraint and interdependence between primary balance and public debt over the period under review. In implementing corrective fiscal adjustment measures to ensure fiscal sustainability, government should therefore consistently take into consideration the interdependency between primary fiscal balance and public debt profiles.

Keywords: fiscal policy; sustainability; interdependency; primary balance; public debt

JEL Classification: H11; H30; H61; H62; H63

1. Introduction

The manner in which government conducts fiscal policy in an economy plays a fundamental role towards achievement of broad macroeconomic objectives. Since the global financial crisis during 2008, the South African economy has experienced prolonged unpredicted fiscal deterioration which led the country to face economic challenges that have further adversely affected the level and composition of public debt (Magubu, Maisonnave, Chitiga & Decaluwé, 2015). While the total balance of public debt in the domestic bond market remains high, interest payable on public debt remains one of the key items of annual government expenditure in an environment characterised by largely low interest rates (Magubu et al., 2015). The

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domestic bond market provides as the main source of new financing, providing about 70% of the annual requirement (Magubu et al., 2015). National Treasury (2018) notes that the liquid domestic capital market remains as government's main source of borrowing despite the volatile market conditions. The proportion of domestic debt to total public debt was 96.3% in 1994/95, and declined marginally to 90.1% in 2016/17.

Comparatively, the main budget balance as a ratio of output has largely been in the deficit territory over the period 2000/01 to 2014/15. The respective balance reached -6.3% in 2008/09, narrowed to -3.8% in 2016/17, and averaged -4.4% over the period 2010/11 to 2016/17 (National Treasury, 2018). Concurrently, the primary balance-to-output ratio fluctuated between -1.5% in 2010/11 and -0.3% in 2016/17, and averaged -0.9% during 2010/11 to 2015/16. Such fiscal developments suggest the need to constantly monitor fiscal risks in the economy to ensure sustainability of fiscal policy. The International Monetary Fund (IMF, 2009) defines fiscal risks as possible deviations of actual fiscal outturns from outcomes that were expected at the time of tabling the national budget; which is consistent with Stuart and Dlamini (2015) who define fiscal risks as possible adverse events that can substantially affect the probability of government to attain fiscal sustainability.

This paper is organised as follows: Section 1 provides the introduction, while Section 2 presents literature and theoretical framework. Section 3 provides the econometric methodology, Section 4 presents the results, and Section 5 provides concluding remarks.

2. Literature and Theoretical Framework

The majority previous studies on fiscal sustainability largely anchor on concepts of static budget constraint and inter-temporal budget constraint (Abdulla, Mustafa & Dahalan, 2012). The static budget constraint is fulfilled if government can finance its current spending with its revenue and new borrowing, and rolling over its maturing liabilities; while the inter-temporal budget constraint hinges on the solvency criterion and requires present discounted value of future primary balances to be at least equal to the unpaid debt stock value (Hamilton & Flavin, 1986; Chalk & Hemming, 2000; Burnside, 2004; Polito & Wickens, 2005; Kirchgaessner & Prohl, 2006).

Preceding studies that assessed fiscal policy sustainability in South Africa, but did not explicitly evaluate interdependence between primary balance and public debt (as ratios of output) include Tshiswaka-Kashalala (2006), Burger, Stuart, Jooste & Cuevas (2011), Jibao, Schoeman & Naidoo (2011), and Ganyaupfu (2014). The respective studies applied different estimation techniques, which include Vector Error Correction (VEC) models, Ordinary Least Squares (OLS), Threshold

Autoregressive (TAR) models, Vector Autoregressive (VAR) models, State Space modelling, and Linear Smooth Transition Error Correction Model (LSTECM) using Non-linear Least Squares (NLS) method.

Following Bohn (1998), the reaction of primary balance (b_t) to variations in public debt levels in the past period (d_{t-1}), as ratios of output, serves as a strong indicator for fiscal sustainability defined by the function $b_t = \alpha + \beta d_{t-1} + \theta b_{t-1} + \pi Z_t + \mu_t$; where b_t is primary balance, d_{t-1}, is public debt in past period, Z_t is a vector of exogenous variables, μ_t is the Gaussian white noise with variance σ^2.

Given a constant interest rate (i) and growth rate (η), the relationship between primary balance and debt becomes $d_t = (1 + i - \eta)d_{t-1} - b_t$, such that $d_t = (1 + i - \eta - \beta)d_{t-1} - \theta b_{t-1} - \alpha - \pi Z_t - \mu_t$. Realizing that $b_{t-1} = (1 + i - \eta)\, d_{t-2} - d_{t-1}$, d_t becomes $d_t = (1 + i - \eta - \beta + \theta)d_{t-1} - \theta(1 + i - \eta)d_{t-2} - \alpha - \pi Z_t - \mu_t$. The Augmented-Dickey Fuller (ADF) regression function for d_t yields $\Delta d_t = [(i - \eta)(1 - \theta) - \beta]d_{t-1} + \theta(1 + i - \eta)\Delta d_{t-1} - \alpha - \pi Z_t - \mu_t$; such that d_t becomes stationary if $\beta \geq (i - \eta)\ (1 - \theta)$, and stabilises in the long run if primary balance positively responds significantly to a change in the debt level, assuming $i > \eta$. Thus, fiscal policy can be deemed sustainable if $\beta/(1 - \theta)$ exceeds interest rate minus growth rate ($\beta/(1 - \theta) > 1 - \eta$).

In present value budget constraint (PVBC) terms, current and future primary spending in present value terms must not exceed current and future revenue (net of interest payments) formulated as:

$$\sum_{i=0}^{\infty} \frac{PE_{t+i}}{\prod_{h=1}^{i}(1 + r_{t+h})} \leq \sum_{i=0}^{\infty} \frac{GDP_{t+i}}{\prod_{h=1}^{i}(1 + r_{t+h})} - (1 + r_t)D_{t-1} \qquad (2.1)$$

where PE_t is primary expenditure (net of interest payments), GDP is national income, D_t denotes public debt stock at the start of period t-1, and r_t represents the nominal interest rate.

Equation (2.1) demonstrates that regardless of satisfying the solvency condition, liquidity can be deemed to exist when government holds liquid assets and financing instruments sufficient to meet or rollover maturing obligations. In line with this particular condition, fiscal sustainability occurs when the present value budget constraint is satisfied, defined by the function:

$$D_t = \sum_{i=0}^{\infty} \frac{B_{t+i}}{\prod_{h=0}^{i}\left(1+r_{t+h}\right)} = \sum_{i=0}^{\infty} \frac{Z_{t+h}}{\prod_{h=0}^{i}\left(1+r_{t+h}\right)} - \sum_{i=0}^{\infty} \frac{PE_t}{\prod_{h=0}^{i}\left(1+r_{t+h}\right)} \tag{2.2}$$

where D_t represents government debt stock at the start of period t, B_t denotes primary balance, Z_t signifies government total revenue, PE_t represents primary expenditure (total spending less interest payments), and r_t denotes the nominal interest rate.

The condition given by equation (2.2) indicates that current government debt must not exceed, or at most equal, the excess sum of future primary surpluses over primary deficits in present value terms. Therefore, government can experience temporary primary deficits as long as such primary deficits can eventually be offset by the total of future primary surpluses. Expressing variables in equation (2.2) as ratios of output (GDP) yields the PVBC in the functional form formulated as:

$$d_t = \sum_{i=0}^{\infty} \frac{\prod_{j=1}^{i}\left(1+\eta_{t+j}\right)}{\prod_{h=0}^{i}\left(1+r_{t+h}\right)} b_{t+i} = \sum_{i=0}^{\infty} \frac{\prod_{j=1}^{i}\left(1+\eta_{t+j}\right)}{\prod_{h=0}^{i}\left(1+r_{t+h}\right)} b_{t+1} - \sum_{i=0}^{\infty} \frac{\prod_{j=1}^{i}\left(1+\eta_{t+j}\right)}{\prod_{h=0}^{i}\left(1+r_{t+h}\right)} e_{t+i}$$

(2.3

where the lower cases of variables denote the respective variables expressed as ratios of GDP, and η represents the nominal growth of GDP. Since government debt comprises of domestic debt denominated in local currency and external debt denominated in foreign currency, equation (2.3) can be altered to express domestic debt and external debt components defined by the function as:

$$d_t = dd_t + \phi_t ed_t = \sum_{i=0}^{\infty} \left[\frac{\left(1+\mu_{t+i}\right)}{\prod_{h=0}^{i}\left(1+r_{t+h}\right)} + \frac{\mu_{t+i}\,\varepsilon_t \prod_{h=0}^{i}\left(1+\tau_{t+h}\right)}{\prod_{h=0}^{i}\left(1+r^{f}_{t+i}\right)} \right] \prod_{j=1}^{i}\left(1+\eta_{t+j}\right) b_{t+i} \tag{2.4}$$

where dd_t is the initial government domestic debt stock dominated in local currency at period t, ed_t denotes the initial government external debt stock dominated in foreign currency, ϕ_t is the nominal exchange rate, μ_t denotes the rate of appreciation of the nominal exchange rate, and r^f represents the nominal interest rate on external debt.

The public debt function expressed by equation (2.4) indicates that the main determinants of public finance sustainability are government revenue, primary expenditure, domestic, and foreign debt stocks with corresponding nominal interest rates, nominal exchange rate and real GDP growth (Yamauchi, 2004). The exchange rate implicitly impacts fiscal policy sustainability via the amount of domestic currency the country has to pay towards securing the external debt component of the total government debt stock. The variations in growth of nominal gross domestic product, denoted by η, remain critical to ensuring fiscal sustainability particularly in respect of the manner in which government reacts to cyclicality in output.

3. Econometric Methodology

3.1. Data

Timeseries quarterly data for primary balance-to-GDP ratio (B/Y) and debt-to-GDP ratio (D/Y) for the period 1997q4 to 2016q3 was sourced from South African Reserve Bank (SARB, 2017). Data series for exogenous variables gross domestic product (GDP) and central bank policy rate (r) were sourced from International Monetary Fund (IMF, 2017) International Financial Statistics (IFS). The GDP data was used to compute output gap ($\hat{y}_t$) using Hodrick-Prescott filter.

3.2. Stationarity Tests

The Augmented Dickey-Fuller (ADF) method, which performs well also when sample size is small (Dickey & Fuller, 1979), was used to test for presence of unit roots based on the AR(p) process defined as $\Delta X_t = \pi + \beta X_{t-1} - \sum_{i=1}^{p-1} \alpha_i \Delta X_{t-i} + \varepsilon_t$; where ε_t is a white noise error term, and p is a class of autoregression.

3.3. Optimal Lag Order Selection

The optimal lag length was selected based on Likelihood Ratio (LR) statistic, Akaike Information Criterion (AIC), Final Prediction Error (FPE), Schwarz Information Criterion (SIC) and Hannan-Quinn Information Criterion (HQIC) techniques.

3.4. Vector Auto-Regressive (VAR) Model

The unstructured VAR framework, which allows endogenous variables to interact without imposing theoretical structures on estimates, was used to model interrelations of a system of multivariate equations for B/Y and D/Y, and examine the joint dynamic behaviour among such variables given by the matrix:

$$\begin{bmatrix} \left(\frac{B}{Y}\right)_t \\ \left(\frac{D}{Y}\right)_t \end{bmatrix} = \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \end{bmatrix} + \begin{bmatrix} \pi_{12} & \theta_{13} \\ \pi_{22} & \theta_{23} \end{bmatrix} \begin{bmatrix} \left(\frac{B}{Y}\right)_{t-1} \\ \left(\frac{D}{Y}\right)_{t-1} \end{bmatrix} + \begin{bmatrix} \gamma_{12} \\ \gamma_{22} \end{bmatrix} [r_t] + \begin{bmatrix} \delta_{13} \\ \delta_{23} \end{bmatrix} \left[\hat{y}_t \right] + \begin{bmatrix} \varepsilon_{11t} \\ \varepsilon_{21t} \end{bmatrix} \quad (3.1$$

The VAR (p) model (equation 3.2) is a seemingly unrelated regression (SUR) with lagged (B/Y) and (D/Y) as endogenous variables, while (r) and output gap ($\hat{y}_t$) are exogenous variables.

3.5. Cointegration Test

The Johansen's procedure (Johansen, 1988), which applies VAR(p) as a starting point, was used to test for presence of a cointegrating relationship between endogenous variables B/Y and D/Y in form of a vector X_t defined by the function:

$$X_t = \Phi_1 X_{t-1} + \Phi_2 X_{t-2} + \ldots\ldots + \Phi_p X_{t-p} + u_t \quad (3.2)$$

where: X_t is a 2x1 vector of B/Y and D/Y variables that are I(1), u_t is a 2x1 vector of innovations, Φ_1 through Φ_p represents 2x2 coefficient matrices, and the impact matrix Φ denotes the degree of system cointegration. The Johansen's procedure used to detect cointegration between B/Y and D/Y was conducted based on the Maximum Eigenvalue and Trace likelihood ratio (LR) statistics techniques.

3.6. Vector Error Correction (VEC) Model

In order to determine the interdependence between B/Y and D/Y, the government's fiscal reaction function $(B/Y)_t = \alpha + \beta(B/Y)_{t-1} + \tau(D/Y)_{t-1} + \varepsilon_t$ was formulated to first assess whether or not the intertemporal budget constraint (IBC) condition was satisfied. Estimation of the reaction function was conducted using the Vector Error Correction (VEC) model comprising a system of equations:

$$\Delta\left(\frac{B}{Y}\right)_t = \alpha_{11} + \pi_{12}\left[\left(\frac{B}{Y}\right)_{t-1} - \theta_{12}\left(\frac{D}{Y}\right)_{t-1} - \theta_{13}\right] + \varphi_{11}\,\Delta\left(\frac{B}{Y}\right)_{t-1} + \varphi_{12}\,\Delta\left(\frac{D}{Y}\right)_{t-1} + \gamma_4(r_t) + \delta_5\left(\hat{y}_t\right) + \varepsilon_{11t}$$

(3.3)

$$\Delta\left(\frac{D}{Y}\right)_t = \alpha_{21} + \pi_{13}\left[\left(\frac{B}{Y}\right)_{t-1} - \theta_{12}\left(\frac{D}{Y}\right)_{t-1} - \theta_{13}\right] + \varphi_{21}\,\Delta\left(\frac{B}{Y}\right)_{t-1} + \varphi_{22}\,\Delta\left(\frac{D}{Y}\right)_{t-1} + \gamma_4(r_t) + \delta_5\left(\hat{y}_t\right) + \varepsilon_{21t}$$

(3.4)

The one period lagged B/Y in equation (3.3) captures inertia in government behaviour. Parameters $(B/Y)_{t-1} - \theta_{12}(D/Y)_{t-1} - \theta_{13}$ in both equations denote the deviation from the long-run equilibrium. The parameter π_{12} in equation (3.3) denotes the error correction term (ECT), which measures the fiscal (primary balance) reaction to the debt position or deviations from the long-run equilibrium.

3.7. VEC Granger Causality/Block Exogeneity Wald Test

Following assessment of fiscal policy sustainability, endogeneity in government fiscal behaviour was evaluated by testing for interdependence between B/Y and D/Y using VEC Granger causality /Block Exogeneity Wald test approach (Granger, 1969). The respective test was conducted based on the null that all lags of one given variable can be excluded from each equation in the system.

The scalar random variable $(D/Y)_t$ can be deemed not to Granger cause $(B/Y)_t$ if and only if:

$$E\left[\left(\frac{B}{Y}\right)_t \middle| \left(\frac{D}{Y}\right)_{t-1}, \left(\frac{B}{Y}\right)_{t-1}, \ldots\right] = E\left[\left(\frac{B}{Y}\right)_t \middle| \left(\frac{B}{Y}\right)_{t-1}, \ldots\right] \tag{3.5}$$

Similarly, the random variable $(B/Y)_t$ can be deemed not to Granger cause $(D/Y)_t$ if and only if:

$$E\left[\left(\frac{D}{Y}\right)_t \middle| \left(\frac{B}{Y}\right)_{t-1}, \left(\frac{D}{Y}\right)_{t-1}, \ldots\right] = E\left[\left(\frac{D}{Y}\right)_t \middle| \left(\frac{D}{Y}\right)_{t-1}, \ldots\right] \tag{3.6}$$

Therefore, $(D/Y)_t$ does not Granger cause $(B/Y)_t$ if the forecast of $(B/Y)_t$ remains the same whether or not conditioned upon the past values of $(D/Y)_t$; and vice versa. Following Granger (1969), if $(D/Y)_t$ and $(B/Y)_t$ exhibit stationarity in respect of spectral systems, then $(D/Y)_t$ can be expressed in the form $\left(\frac{D}{Y}\right)_t = \int_{-\vartheta}^{\vartheta} e^{it\theta}\, d\Phi_{(D/Y)}(\pi)$, where: $\Phi_{(D/Y)}(\pi)$ denotes a complex random process $E\left[d\Phi_{(D/Y)}(\pi)\overline{d\Phi_{(D/Y)}(\varpi)}\right] = dF_{(D/Y)}(\pi)$, if $\pi = \varpi$; or else $E\left[d\Phi_{(D/Y)}(\pi)\overline{d\Phi_{(D/Y)}(\varpi)}\right] = 0$, where $dF_{(D/Y)}(\pi)$ can be specified as $dF_{(D/Y)}(\pi) = f_{(D/Y)}(\pi)d\pi$. The cross spectrum between $(D/Y)_t$ and $(B/Y)_t$ gets defined by $E\left[d\Phi_{(D/Y)}(\pi)\overline{d\Phi_{(D/Y)}(\pi)}\right] = \Xi(\pi)d\pi$ if $\pi = \varpi$; and covariance is $\mu_t^{db} = E\left[\left(\frac{D}{Y}\right)_t \overline{\left(\frac{B}{Y}\right)_{t-\tau}}\right] = \int_{-\vartheta}^{\vartheta} e^{i\tau\theta}\, \Xi(\pi)d\pi$. Following Enders (2003), the Granger causality/Block Exogeneity Wald test statistic was defined as $(T - 3p - 1)\left(\log\left|\Sigma_{re}\right| - \log\left|\Sigma_{un}\right|\right) \sim \chi^2(2p)$; where T is the number of observations; $\sum_{un}$ denotes variance or covariance matrices of the unrestricted VAR system; $\sum_{re}$

denotes the variance or covariance matrices of the restricted system when the lag of a variable was excluded from the system, p denotes the number of lags of the variable that was excluded from the system.

3.8. Impulse Response Functions

Impulse response functions (IRFs) analysis was conducted to assess the impact of a shock to an endogenous variable (X) on itself and on the other endogenous variable (Y); and the time horizon it took variable (Y) to return to long-run equilibrium path owing to a shock in variable (X). The unstructured VAR was transformed into a vector moving-average (VMA) based on the property that for every stationary VAR (p), there exists an infinite VMA which follows the decomposition:

$$X_t = \varepsilon_t + \zeta_1 \varepsilon_{t-1} + \zeta_2 \varepsilon_{t-2} + \ldots = \sum_{i=0}^{\infty} \zeta_i \varepsilon_{t-i} \qquad (3.7)$$

The matrix ζ_s can be interpreted as $\zeta_s = (\partial X_{t+s}) / (\partial \varepsilon')$. If the first element of ε_t gets changed by ρ_1, second element by ρ_2, third element by ρ_3, and so on, joint effect of vector X_{t+s} can be shown as:

$$\Delta X_{t+s} = \frac{\partial X_{t+s}}{\partial \varepsilon_{1,t}} \rho_1 + \frac{\partial X_{t+s}}{\partial \varepsilon_{2,t}} \rho_2 + \frac{\partial X_{t+s}}{\partial \varepsilon_{3,t}} \rho_3 + \ldots + \frac{\partial X_{t+s}}{\partial \varepsilon_{n,t}} \rho_n = \zeta_s \rho \qquad (3.8)$$

The parameter $\rho = (\rho_1, \rho_2, \rho_3, \ldots\ldots, \rho_n)'$ for which the row i and column j element of ζ_s as a function of s yields the IFR given by $(\partial X_{i,\, t+s}) / (\partial \varepsilon_{j,t})$; which depicts the dynamic multiplier or response of $X_{i,t+s}$ to a one-time previous impulse in ε_{jt}. The matrix of the unstructured VAR given by equation (3.1) was reintroduced as containing merely endogenous variables (B/Y) and (D/Y) and rewritten more compactly into an infinite VMA representation; yielding the function:

$$X_t = H_0 + H_1 X_{t-1} + e_t \;\Rightarrow\; X_t = \frac{H_0}{I - H_1 L} + \frac{e_t}{I - H_1 L} \qquad (3.9)$$

Solving the first component on the RHS of equation (equation 3.9) provides:

$$\frac{H_0}{I-H_1}=\frac{(I-H_1)^a H_0}{|I-H_1|}=\frac{\begin{bmatrix}1-\pi_{12} & -\theta_{13}\\ -\pi_{12} & 1-\theta_{23}\end{bmatrix}H_0}{\begin{vmatrix}1-\pi_{12} & -\theta_{13}\\ -\pi_{22} & 1-\theta_{23}\end{vmatrix}}=\frac{\begin{bmatrix}1-\theta_{23} & \pi_{22}\\ \theta_{13} & 1-\pi_{12}\end{bmatrix}\begin{bmatrix}\alpha_{11}\\ \alpha_{21}\end{bmatrix}}{(1-\pi_{12})(1-\theta_{23})-\pi_{22}\theta_{13}}=\frac{1}{\Delta}\begin{bmatrix}(1-\theta_{23})\alpha_{11}+\pi_{22}\,\alpha_{21}\\ \theta_{13}\,\alpha_{11}+(1-\pi_{12})\,\alpha_{21}\end{bmatrix}=\begin{bmatrix}\overline{\frac{B}{Y}}\\ \overline{\frac{D}{Y}}\end{bmatrix}$$

(3.10)

Following fulfilment of the stability condition, which requires the roots of $I-H_1L$ to exist outside the unit circle, the second component of VMA representation was expressed in the functional form:

$$\frac{e_t}{I-H_1L}=\sum_{i=0}^{\infty}H_1^i\,e_{t-i}=\sum_{i=0}^{\infty}\begin{bmatrix}\pi_{12} & \theta_{13}\\ \pi_{22} & \theta_{23}\end{bmatrix}\begin{bmatrix}e_{11,t-i}\\ e_{21,t-i}\end{bmatrix} \tag{3.11}$$

The VAR system was thus formulated as a VMA with standard VAR's error terms as:

$$\begin{bmatrix}\left(\frac{B}{Y}\right)_t\\ \left(\frac{D}{Y}\right)_t\end{bmatrix}=\begin{bmatrix}\left(\overline{\frac{B}{Y}}\right)\\ \left(\overline{\frac{D}{Y}}\right)\end{bmatrix}+\sum_{i=0}^{\infty}\underbrace{\begin{bmatrix}\pi_{12} & \theta_{13}\\ \pi_{22} & \theta_{23}\end{bmatrix}^i}_{H^i}\begin{bmatrix}e_{11,t-i}\\ e_{21,t-i}\end{bmatrix} \tag{3.12}$$

The VMA error terms (equation 3.12) are composite errors comprising of structural innovations. Following Shin and Pesaran (1998), the impulse response function (IRF) was then defined by the function $IR(m,h,Z_{t-1})=E(y_{t+m}|\varepsilon_t=h,Z_{t-1})-E(y_{t+m}|=Z_{t-1})$; where m denotes time, h ($h_1,\ldots,h_m$) denotes n x 1 vector that signifies the size of shock, Z_{t-1} denotes accumulative information about the economy from the past period up to time period t-1. Following Sim (1980), OIRFs were defined as $IR_{ij}^0(m)=Q_mP\varepsilon_j$ where m = 0, 1, 2, ….., k , and $Q_m=A_1Q_{m-1}+A_2Q_{m-2}+\ldots+A_pQ_{m-p};\ Q_o=I_n$.

3.9. Impact Multipliers

Impact multipliers were computed to measure the impact effect of a one unit change in a structural innovation. The impact effect of $\varepsilon_{(D/Y)t}$ on the $(B/Y)_t$ and $(D/Y)_t$ was, for instance, computed as:

$$\frac{d\left(\frac{B}{Y}\right)_t}{d\varepsilon_{(D/Y),\,t}} = \Omega_{12}(0) \qquad \frac{d\left(\frac{D}{Y}\right)_t}{d\varepsilon_{(D/Y),\,t}} = \Omega_{22}(0) \tag{3.13}$$

The impact effect of one period ahead on $(B/Y)_t$ and $(D/Y)_t$ was computed as:

$$\frac{d\left(\frac{B}{Y}\right)_t}{d\varepsilon_{(D/Y),\,t+1}} = \Omega_{12}(1) \qquad \frac{d\left(\frac{D}{Y}\right)_t}{d\varepsilon_{(D/Y),\,t+1}} = \Omega_{22}(1) \tag{3.14}$$

Concomitantly, the impact effect expressed in equation (3.14) was the same effect on $(B/Y)_t$ and $(D/Y)_t$; and of a structural innovation one period ago calculated as:

$$\frac{d\left(\frac{B}{Y}\right)_t}{d\varepsilon_{(D/Y),\,t-1}} = \Omega_{12}(1) \qquad \frac{d\left(\frac{D}{Y}\right)_t}{d\varepsilon_{(D/Y),\,t-1}} = \Omega_{22}(1) \tag{3.15}$$

The IRF of (B/Y) to a unit change in a shock to (D/Y) was equal to $\Omega_{12}(0), \Omega_{12}(1), \Omega_{13}(2),\ldots$; and the sum of IRFs was computed as $\sum_{i=0}^{\infty}\Omega_{12}(i)$, and the long-run cumulated effect was $\lim_{n\to\infty}\sum_{i=0}^{n}\Omega_{12}(i)$.

3.10. Cholesky Variance Decomposition

Variance decomposition was conducted to measure the amount of change in a given variable owing to its own shock as well as shocks of other variables in the model. Each variable was explained as a linear combination of its own current innovations and lagged innovations of other variable in the dynamic system. Variances of (B/Y)'s and (D/Y)'s n-step ahead forecast errors were computed as:

$$\sigma^2_{(B/Y),n} = \underbrace{\sigma^2_{(B/Y)}\left(\Omega^2_{11,0}+\Omega^2_{11,1}+\ldots+\Omega^2_{11,n-1}\right)}_{\substack{\text{proportion of variance in (B/Y) due to own shock}\\ \downarrow \\ \text{Decreases over time}}} + \underbrace{\sigma^2_{(D/Y),n}\left(\Omega^2_{21,0}+\Omega^2_{21,1}+\ldots+\Omega^2_{21,n-1}\right)}_{\substack{\text{proportion of variance in (B/Y) due to shock in (D/Y)}\\ \downarrow \\ \text{Increases over time}}}$$

(3.16)

$$\sigma^2_{(D/Y),n} = \underbrace{\sigma^2_{(D/Y)}\left(\Omega^2_{21,0}+\Omega^2_{21,1}+...+\Omega^2_{21,n-1}\right)}_{\substack{\text{proportion of variance in (D/Y) due to own shock}\\ \downarrow \\ \text{Decreases over time}}} + \underbrace{\sigma^2_{(B/Y),n}\left(\Omega^2_{11,0}+\Omega^2_{11,1}+...+\Omega^2_{11,n-1}\right)}_{\substack{\text{proportion of variance in (D/Y) due to shock in (B/Y)}\\ \downarrow \\ \text{Increases over time}}}$$

(3.17)

In circumstances where $\varepsilon_{(D/Y)}$ explains none of the forecast error variance of $(B/Y)_t$ over the forecast horizon $\left(\partial\sigma^2_{(B/Y)n}\right) \div \left(\sigma^2_{(D/Y)}\right) \approx 0$, $(B/Y)_t$ is deemed exogenous. However, if $\varepsilon_{(D/Y)}$ explains most of forecast error variance of $(B/Y)_t$ over the forecast horizon $\left(\partial\sigma^2_{(B/Y)n}\right) \div \left(\sigma^2_{(D/Y)}\right) \approx 0.9$, $(B/Y)_t$ is endogenous.

4. Results and Discussion

4.1. Stationarity Tests

Table 4.1. ADF stationarity tests statistics in first differences

Series	**Model**	**Lag length**	**α = 1%**	**α = 5%**	**α = 10%**	**t-statistic** $\tau_c, \tau_{tc}, \tau_n$
B/Y	Constant	6	-3.530	-2.904	-2.589	-3.496^{**}
	Trend and Constant	6	-4.098	-3.477	-3.166	-3.462^{*}
	None	6	-2.599	-1.945	-1.613	-3.449^{***}
D/Y	Constant	7	-3.531	-2.905	-2.590	-1.073
	Trend and Constant	7	-4.100	-3.478	-3.166	-2.211
	None	7	-2.599	-1.945	-1.613	-1.133
r	Constant	11	-3.538	-2.908	-2.591	-4.614^{***}
	Trend and Constant	11	-4.110	-3.482	-3.169	-4.984^{***}
	None	11	-2.602	-1.946	-1.613	-4.110^{***}
ŷ	Constant	8	-3.533	-2.906	-2.590	-3.181^{**}
	Trend and Constant	8	-4.103	-3.479	-3.167	-3.147
	None	8	-2.600	-1.945	-1.613	-3.209^{***}

$^{[***]\,(**)\,*}$ represent significance at 1 percent, (5) percent levels and [10] percent levels; respectively

τ_c, τ_{tc}, τ_n and $Ø_c$, $Ø_{tc}$, $Ø_n$ represent ADF and PP test results computed using constant, trend and constant, and none; respectively

The selections of proper lag lengths of unit root tests were determined automatically by EViews based on the AIC

Results presented Table 4.1 for unit root tests in first differences show that primary balance-to-GDP ratio (B/Y), central bank policy rate (r) and output gap (ŷ) were stationary at 1% level of significance based on the model with no constant. The debt-to-GDP ratio (D/Y) remained non-stationary at 10% significance level, hence second differenced was applied upon which the unit root hypothesis was rejected. The optimal lag length selection results are presented in the appendix.

4.2. Cointegration Test

Table 4.2. Cointegration test with linear deterministic trend and lag interval: 1 to 1

Null hypothesis (H_0) [Alternative hypothesis (H_1)]	r = 0 [r = 1]	r ≤ 1 [r = 2]
Trace statistic Critical value (p-value)	49.656* 15.494 (0.000)	1.073 3.841 (0.300)
Maximum-Eigen statistic Critical value (p-value)	48.583* 14.264 (0.000)	1.073 3.841 (0.3002)

* denotes rejection of the null hypothesis at 5% significance level

The Johansen Trace and Maximum-Eigen test statistics show existence of 1 cointegrating equation at 5% level of significance level based on the computed Trace statistic (= 49.65697) greater than the critical value (= 15.49471; $p < 0.05$) and Max-Eigen statistic (= 48.58367) larger than the analogous computed critical value (= 14.26460; $p < 0.05$). The presence of a cointegrating equation for series B/Y and D/Y validated the rationale to test for fiscal sustainability using the VEC model.

4.3. VAR Representation of VEC Model Estimates

Table 4.3. VAR model – substituted parameters[†]

Primary balance-to-GDP ratio equation

$$d\left(B/Y\right) = \underset{\{-6.095\}}{-0.696} * \left[\left(B/Y\ (-1)\right) + \underset{\{2.441\}}{1.627} * d\ \left(D/Y\ (-1)\right) + 0.854\right] \underset{\{-1.449\}}{-0.176} * d\left(B/Y\ (-1)\right) \underset{\{-4.210\}}{-1.377} * d\ \left(D/Y\ (-1), 2\right) \underset{\{-3.878\}}{-10.133}$$

$$+ \underset{\{3.868\}}{4.744} * r\,(-2) + \underset{\{2.859\}}{0.748} * y\_gap\ (-2) \qquad \text{(eqn 3.3}$$

Public debt-to-GDP ratio equation

$$d\left(D/Y,\ 2\right) = \underset{\{4.982\}}{0.218} * \left[\left(B/Y\ (-1)\right) + \underset{\{2.441\}}{1.627} * d\ \left(D/Y\ (-1)\right) + 0.854\right] \underset{\{-4.272\}}{-0.199} * d\left(B/Y\ (-1)\right) \underset{\{-1.803\}}{-0.226} * d\ \left(D/Y\ (-1), 2\right) + \underset{\{3.091\}}{3.104}$$

$$\underset{\{-3.105\}}{-1.463} * r\ (-2)\ \underset{-1.519}{-0.152} * y\_gap\ (-2) \qquad \text{(eqn 3.4)}$$

[†] Figures in {} represent computed t-statistics for the respective estimated coefficients

The VAR version estimates of VEC model (Table 4.3) reveal evidence of a statistically significant positive relationship between primary balance and public debt (as ratios of output) in the long-run. For every 1% increase in public debt ratio, the primary balance ratio increased by an average of 1.62% over the period 1999q1 to 2016q2. The systematic positive reaction of primary balance ratio to changes in public debt ratio indicate evidence of consistency of government's behaviour with the government inter-temporal budget constraint; hence fiscal policy was sustainable. The short-run dynamics of the of the primary balance ratio equation show that about 0.69% of the transitory deviation from long-run equilibrium relationship between primary balance and public debt was corrected through reductions in the primary balance ratio during the first quarter after occurrence of the deviation.

4.4. VEC Granger Causality/Block Exogeneity Wald Test

The VEC Granger causality/Block Exogeneity test results were computed to determine whether or not some endogeneity exists in the behaviour of government by examining the short run causality between primary balance and public debt, as ratios of output; with results presented in Table 4.4.

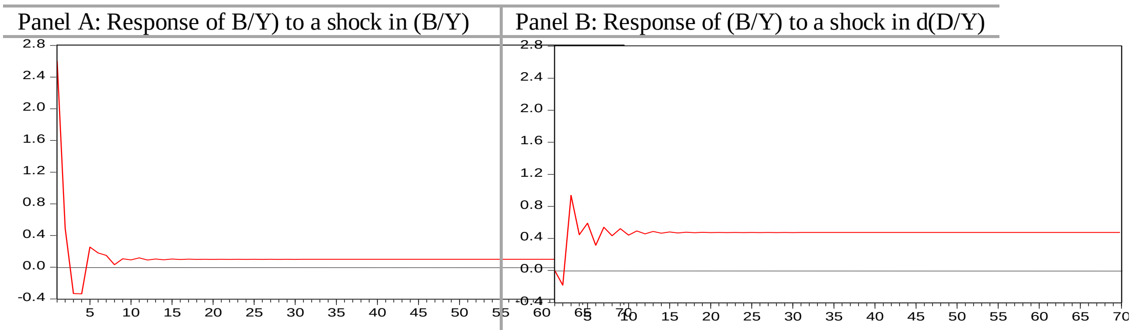
Table 4.4. VEC Granger causality/Block Exogeneity Wald tests

Panel A – Dependent variable: d(B/Y)		Panel B – Dependent variable: d(D/Y, 2)	
Excluded	Chi-square (prob)	Excluded	Chi-square (prob)
d(D/Y, 2)	17.729 (0.000)	d(B/Y)	18.256 (0.000)
All	17.729 (0.000)	All	18.256 (0.000)

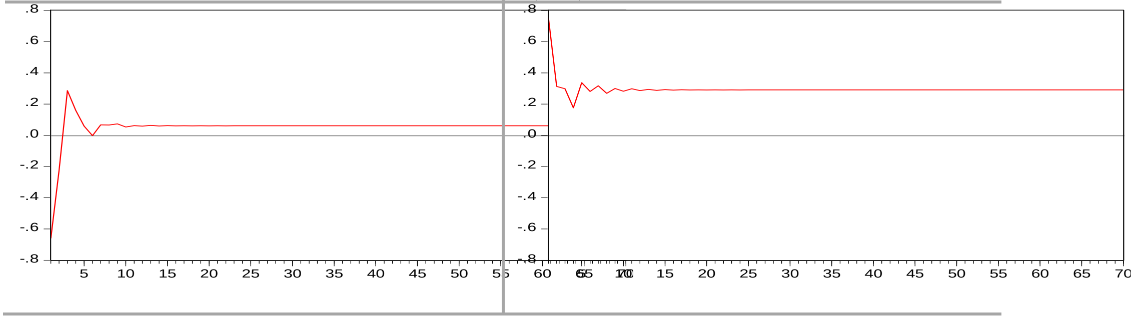
Granger-causality results on joint tests for each of the equation show evidence of endogeneity of primary balance and public debt, as ratios of output. Panel A estimates indicate that the null hypothesis that debt ratio does not Granger cause primary balance ratio was rejected at 1% level of significance. The lagged difference of the debt ratio could thus not be excluded in the estimated differenced primary balance equation. Panel B estimates indicate that null hypothesis that primary balance ratio does not Granger cause the debt ratio was rejected at 1% significance level, thus the lagged difference of primary balance ratio could not be excluded in the differenced debt equation.

4.5. Impulse Response Functions

The impulse response functions and variance decompositions were applied as alternative approaches of characterising the interdependence between primary balance and public debt, as ratios of output.


Panel A: Response of B/Y) to a shock in (B/Y)

Panel B: Response of (B/Y) to a shock in d(D/Y)


Panel C: Response of d(D/Y) to a shock in (B/Y)

Panel D: Response of d(D/Y) to a shock in d(D/Y)

Figure 4.1. Impulse response functions for (B/Y) and d(D/Y) over 70 quarters (1999q1-2016q2)

Figure 4.1 Panel A shows that a negative shock to primary balance ratio had a significant negative effect on its future values, improved to 0.25% in the 5th quarter and remained significantly positive at 0.1% in the long-run. Panel B shows that a negative shock in the debt ratio had a significant positive impact on future primary balance ratio; which fluctuated between 0.31% and 0.58% from 4th quarter through to the 16th quarter, and remained significantly positive and constant at 0.47% from the 17th quarter throughout the long-run. Panel C shows evidence that a shock in the primary balance had a statistically significant positive impact on future path of debt ratio, which reached 0.28% in the 3rd quarter, declined to an insignificant mark of -0.00% in the 6th quarter, rebounded with slight variation between 0.05% and 0.07% through to the 10th quarter, and reverted to the equilibrium where it remained significantly positive and constant at 0.06% over the long-run. Panel D shows that a shock to debt ratio had a profoundly declining positive impact on its future path from 0.75% in the 1st quarter to 0.17% in the 4th quarter, varied between 0.26% and 0.33% in the 5th and 16th quarters, and remained significantly positive and constant at 0.29% in the long-run.

4.6. VAR Cholesky Variance Decompositions

Table 4.5. Variance decompositions of forecast errors

Percentage of forecast error in:	Periods	Explained by shocks to:			
		Order B/Y, d(D/Y)		Order d(D/Y), B/Y	
		B/Y	d(D/Y)	B/Y	d(D/Y)
B/Y	10	74.69	25.31	51.66	48.34
	40	45.41	54.59	56.81	43.19
	70	33.33	66.67	58.91	41.09
d(D/Y)	10	31.36	68.24	29.25	70.75
	40	15.83	84.17	49.33	50.67
	70	11.58	88.42	54.70	45.30

Variance decompositions results (Table 4.5) show that with order (B/Y, d(D/Y)), primary balance shock accounted for 74.7% of variance in itself in the 10th quarter, while the percentage decreased to 45.4% in the 40th quarter, and to 33.3% in the 70th quarter. The percentage increased to 58.9% in the 70th quarter when the reverse order (d(D/Y), B/Y) was used. With order (B/Y, d(D/Y)), the contribution of a shock to debt ratio on variance of primary balance ratio increased from 25.3% in the 10th quarter to 54.6% in the 40th quarter, and 66.7% in the 70th quarter. When the reverse order (d(D/Y), B/Y) was used, the percentage of variance dropped to 41.1% in the 70th quarter.

Concomitantly, the percentage of variance of the forecast error in government debt ratio emanating from a shock to primary balance ratio was 31.4% in the 10th quarter and declined to 11.6% in the 70th quarter with the order (B/Y, d(D/Y)). When the reverse order (d(D/Y), B/Y) was applied, such percentage of variance of forecast error in debt ratio explained by primary balance ratio increased to 54.7% in the 70th quarter. With order (B/Y, d(D/Y)), the percentage of variance in the debt ratio explained by a shock to itself increased from 68.2% in the 10th quarter to 88.4% in the 70th quarter. The percentage however decreased to 45.3% when the reverse order (d(D/Y), B/Y)) was used.

5. Conclusion

The results of the fiscal reaction function reveal strong evidence of a positive relationship between primary balance and public debt, as ratios of output; showing evidence of consistency of the government's behaviour with the government intertemporal budget constraint condition. The Granger-causality, impulse response functions and variance decompositions results all point to evidence of presence of endogeneity and interdependence between primary balance and public debt ratios. Simulations of the impulse response functions provide strong evidence that the macroeconomy can correct itself from transitory deviations in the short-run to the medium term, and return to the long-run equilibrium path after occurrence of a shock.

However, despite evidence of fiscal sustainability, coupled with the fiscal governance framework anchored on sound institutional arrangements, the country's fiscus currently faces potential fiscal risks emanating from contingent and accrued liabilities attributed to government guarantees of funding to a number public enterprises with weak financial positions. Sustained need by public enterprises for financial bailouts to meet operating costs, debt obligations and working capital requirements has heightened the country's fiscal risk of guarantee exposure. Given the pressure to finance social spending programmes, there is need for strong commitment by government to avoid populist spending, implement and consistently monitor fiscal austerity measures across spending priorities in order to maintain fiscal policy sustainability.

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