Mathematical and Quantative Methods

Analysis of the Evolution of the Gross Domestic Product by Means of Cyclic Regressions

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Abstract: In this article, we will carry out an analysis on the regularity of the Gross Domestic Product of a country, in our case the United States. The method of analysis is based on a new method of analysis – the cyclic regressions based on the Fourier series of a function. Another point of view is that of considering instead the growth rate of GDP the speed of variation of this rate, computed as a numerical derivative. The obtained results show a cycle for this indicator for 71 years, the mean square error being 0.93%. The method described allows an prognosis on short-term trends in GDP.

Keywords: GDP; cycle; Fourier; regression

JEL Classification: E17; C25; C65

1. Introduction

In the literature, the economic cycle designate the fluctuations which accompany the evolution of a nation or, sometimes, it simply is associated with the increasing and decreasing of an economy. Throughout history, many states were faced and have experienced economic fluctuations, most tested being the United States.

Given the complexity of economic phenomena, in practice there are as many types of economic cycles or economic fluctuations. We can say that almost any segment of the economic life is subject to the fluctuations that, sometimes, may include periods of more than a year.

Throughout history, the world economy, unfortunately, has experienced difficult periods of recession or depression during which economic activity was marked by unemployment, contractions of the monetary, financial markets, stock exchanges and other imbalances.

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According to literature, the theoretical economic cycle is linked on the one hand, by changes in aggregate demand with all components (public consumers, private consumers, investors) or, on the other hand, of the change in supply aggregates (changes in production costs).

A more comprehensive approach to the problem of the economic cycle requires knowledge of all aspects of the market economy.

Regardless of the factors that have influenced and favored economic cycles, their approach involves different points of view.

The first analysis of the economic cycle through the prism of the phenomenon of recurrence is due to the French economist Clement Juglar, who has studied the fluctuations of the interest rate and price and on the basis of which was discovered in 1860 an economic cycle with alternate periods of prosperity and depression for 8-11 years.

Economists who have a thorough analysis of Clement Juglar's cycle and, in particular Joseph Schumpeter, have concluded that in it there are four phases: the expansion, the crisis, recession and the renascence.

Several years later, in 1878, William Stanley Jevons, in the "Commercial crises and Sun Spots" examines the phenomenon of cyclicity, trying as Clement Juglar explaining the periodicity of the economic activity. Jevons believed that such phenomena are random and crises on the basis of statistical studies, the author is of the opinion that there is a link between them and some extrinsic random variable in the economy ([2]).

At the beginning of the 20th century, another English engineer named Joseph Kitchin based on analyses of interest rates and other variables (the analysis being performed on the economies of the United States of America and United Kingdom) discovers a short economic cycle, approximately 40 months. Discovered by Joseph Kitchin the cycle has two phases: expansion and economic downturn, the transition from the phase of expansion to the slowdown by without the appearance of any crisis.

After the Great Depression in the years 1929-1933, the economists have focused much more on macroeconomic phenomena that determine the appearance of the economic cycle, looking for patterns of prediction.

Thus, in the "The Major Economic Cycles", which appeared in 1925, the Russian Economist Nikolai Kondratieff mark out an economic cycle much longer, about 50-60 years. On the basis of statistical researchs on long-term fluctuations in prices (the analysis being performed on the same economies of the United States of America and United Kingdom), Kondratieff observed periods of accelerated growth of branches of Economics, alternate with slower growth. Within this cycle, Kondratieff identified the expansion phase, the phase of stagnation and recession phase. Without finding a universally accepted explanation, he believes that the basis of these cycles long stay technological progress, confirmed later by Schumpeter, which considers "the bunch of related innovations" that generates each cycle.

Other analysis devoted to the economic cycle have been made by Wesley Clair Mitchell in the work "Business Cycle" (1913) and "Measuring Business Cycles" (1927) in which the author discusses some methods of determination and analysis of economic cycle. Mitchell puts emphasis on the differences between the capitalist societies and the pre-capitalist, considering that a course of business would not be possible in a society pre-capitalist, but can occur in one capitalist ([1]).

John Maynard Keynes - the economist of the Great Depression, lay the groundwork for a new economic theory which reveals a close connection between consumption and investment. According to the keynesian theory and its adherents, any additional expenditure (consumption) generates an income a few times higher than the expenditure itself. This relationship between consumption and investment, known as the investment multiplier, can not produce, considered Keynes, cyclical movements in the economy, but it can lead to an upward trend.

Russian research economist Simon Kuznet, in 1930 put the bases of a cycle lasting on average, over a period of 15-20 years, called "demographic cycle" or "the cycle of investment in infrastructure". Kuznet considers that a factor that influence the emergence and evolution of an economic cycle is the demographic processes, in particular the phenomenon of migration having disturbing effects in the buildings sector.

The Austrian School sees the economic cycle through its representatives, notably to Ludwig von Mises, as a natural consequence of the massive growth of bank credit, an inappropriate monetary policy conducive to relaxing the conditions of crediting and finally the accumulation of toxic assets. Growth of loans generates, in turn, a rise in prices and a fall in interest rates below the optimum level, and the crisis occurs when manufacturers can't sell the production because of the very high prices. In the same stream of thought, Friedrich Hayek considers the phenomenon of over-investment as a factor determining the onset of a new economic cycle, while Joseph Schumpeter considers that the emergence and the onset of the economic cycle is based on the existence of investments with high efficiency carried out in a short period and a low demand for new products.

After attempts at explanation of the economic cycle from the early 1970's of Milton Friedman and Robert Lucas, the work of Finn E. Kydland and Edward C. Prescott "Time to Build And Aggregate Fluctuations" ([3]) launches real business cycle theory, the economic cycles being determined by the fluctuations in the rate of growth of total productivity of factors of production.

Over time, many economists have attempted, through analysis of available statistical data, to develop specific models of foresights of changes taking place in the economy to come to the aid of the decision-makers to act according to actual economic conditions.

2. Cyclic Regressions

Let a function f:**R**→**R**, with f and f' piecewise continuous on **R** and periodic of period T, so f(x+T)=f(x) ∀x∈**R**.

Considering the Fourier series associated with the function f:

$F(x)=\frac{a_0}{2}+\sum_{k=1}^{\infty}\left(a_k\cos\frac{2k\pi x}{T}+b_k\sin\frac{2k\pi x}{T}\right)$ where: $a_k=\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}f(x)\cos\frac{2k\pi x}{T}dx$, $k\geq 0$,

$b_k=\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}f(x)\sin\frac{2k\pi x}{T}dx$, $k\geq 1$ is observed that F(x+T)=F(x) ∀x∈**R** so S it is also a periodic function of period T

The Dirichlet's theorem (Spiegel, 1974) states that in the above conditions, the Fourier series converges to f in every point of continuity of it and to $\frac{f(x+0)+f(x-0)}{2}$ in the other points.

Considering the partial sum of order n, corresponding to the series of function F, we obtain the Fourier polynomials of order n:

$$F_n(x)=\frac{a_0}{2}+\sum_{k=1}^{n}\left(a_k\cos\frac{2k\pi x}{T}+b_k\sin\frac{2k\pi x}{T}\right)$$

It is obvious that $F_n(x)=F_n(x+T)$ ∀x∈**R**.

The Fourier polynomials have the property of approximating the function through one periodical with the observation that the absolute error tends to zero (due to the convergence) with the rise of n.

Due to the existence of an important number of cyclical phenomena in many scientific fields, we intend, below, to approximate their development by means of Fourier polynomials of degree conveniently chosen.

Let therefore a set of data: (x_i,y_i), $i=\overline{1,m}$ and the Fourier polynomial $F_n(x)=\frac{a_0}{2}+\sum_{k=1}^{n}\left(a_k\cos\frac{2k\pi x}{T}+b_k\sin\frac{2k\pi x}{T}\right)$. We shall determine the coefficients a_k, $k=\overline{0,n}$ and b_k, $k=\overline{1,n}$ using the least squares method.

Let therefore:

$$\varepsilon(a_0,a_k,b_k)=\sum_{i=1}^{m}\left(\frac{a_0}{2}+\sum_{k=1}^{n}\left(a_k\cos\frac{2k\pi x_i}{T}+b_k\sin\frac{2k\pi x_i}{T}\right)-y_i\right)^2$$

In order to $\varepsilon(a_0,a_k,b_k)$ take the minimum value, it must:

$$\begin{cases}\frac{\partial\varepsilon}{\partial a_0}=\sum_{i=1}^{m}\left(\frac{a_0}{2}+\sum_{k=1}^{n}\left(a_k\cos\frac{2k\pi x_i}{T}+b_k\sin\frac{2k\pi x_i}{T}\right)-y_i\right)=0\\ \frac{\partial\varepsilon}{\partial a_j}=\sum_{i=1}^{m}\left(\frac{a_0}{2}+\sum_{k=1}^{n}\left(a_k\cos\frac{2k\pi x_i}{T}+b_k\sin\frac{2k\pi x_i}{T}\right)-y_i\right)\cos\frac{2j\pi x_i}{T}=0,\ j=\overline{1,n}\\ \frac{\partial\varepsilon}{\partial b_j}=\sum_{i=1}^{m}\left(\frac{a_0}{2}+\sum_{k=1}^{n}\left(a_k\cos\frac{2k\pi x_i}{T}+b_k\sin\frac{2k\pi x_i}{T}\right)-y_i\right)\sin\frac{2j\pi x_i}{T}=0,\ j=\overline{1,n}\end{cases}$$

Noting: $A_{ik}=\cos\frac{2k\pi x_i}{T}$, $B_{ik}=\sin\frac{2k\pi x_i}{T}$, $i=\overline{1,m}$, $k=\overline{1,n}$, we can write the system in the form:

$$\begin{cases}\frac{n}{2}a_0+\sum_{k=1}^{n}\left(a_k\sum_{i=1}^{m}A_{ik}+b_k\sum_{i=1}^{m}B_{ik}\right)=\sum_{i=1}^{m}y_i\\ \frac{\sum_{i=1}^{m}A_{ij}}{2}a_0+\sum_{k=1}^{n}\left(a_k\sum_{i=1}^{m}A_{ik}A_{ij}+b_k\sum_{i=1}^{m}B_{ik}A_{ij}\right)=\sum_{i=1}^{m}y_iA_{ij},\ j=\overline{1,n}\\ \frac{\sum_{i=1}^{m}B_{ij}}{2}a_0+\sum_{k=1}^{n}\left(a_k\sum_{i=1}^{m}A_{ik}B_{ij}+b_k\sum_{i=1}^{m}B_{ik}B_{ij}\right)=\sum_{i=1}^{m}y_iB_{ij},\ j=\overline{1,n}\end{cases}$$

Let denote now, again, for simplicity:

$$\alpha_k=\sum_{i=1}^{m}A_{ik},\ \beta_k=\sum_{i=1}^{m}B_{ik},\ \gamma_{kj}=\sum_{i=1}^{m}A_{ik}A_{ij},\ \delta_{kj}=\sum_{i=1}^{m}B_{ik}B_{ij},\ \varepsilon_{kj}=\sum_{i=1}^{m}A_{ik}B_{ij},$$

$$\mu=\sum_{i=1}^{m}y_i,\ \nu_j=\sum_{i=1}^{m}y_iA_{ij},\ \lambda_j=\sum_{i=1}^{m}y_iB_{ij},\ k,j=\overline{1,n}.$$

The system becomes:

$$\begin{cases} \frac{n}{2}a_0 + \sum_{k=1}^{n}(\alpha_k a_k + \beta_k b_k) = \mu \\ \frac{\alpha_j}{2}a_0 + \sum_{k=1}^{n}(\gamma_{kj} a_k + \varepsilon_{jk} b_k) = \nu_j,\ j = \overline{1,n} \\ \frac{\beta_j}{2}a_0 + \sum_{k=1}^{n}(\varepsilon_{kj} a_k + \delta_{kj} b_k) = \lambda_j,\ j = \overline{1,n} \end{cases}$$

Considering the system solution a_k, k=$\overline{0,n}$ and b_k, k=$\overline{1,n}$ we have that for a given period F>0 and n≥1, the Fourier polynomial $F_n(x)$= $\frac{a_0}{2} + \sum_{k=1}^{n}\left(a_k \cos\frac{2k\pi x}{T} + b_k \sin\frac{2k\pi x}{T}\right)$ represents the best cycle approximation for the point of view of the method of least squares. We shall call F_n so determined, the cyclic regression of order n and period T.

3. The Analysis of GDP from the Point of Cyclicity

In what follows, we intend to study a possible cycle in the evolution of the Gross Domestic Product of a country.

Considering a period of m consecutive years and GDP_k, k=$\overline{1,m}$ - the real GDP in the period k, let the growth rate of GDP: r_k=$\frac{GDP_k - GDP_{k-1}}{GDP_{k-1}}$. Considering now r_k we have:

$$GDP_k=(1+r_k)GDP_{k-1},\ k=\overline{2,m}$$

The analysis of the growth rate of GDP for the US economy in the period 1793-2010 does not provide, however, relevant results. For this reason, we consider for our analysis the speed of variation of r_k.

Given a function $f:(a,b)\to\mathbf{R}$ and $x_0\in(a,b)$, let h>0 and the points $(x_0-h,f(x_0-h))$, $(x_0,f(x_0))$, $(x_0+h,f(x_0+h))$. The numerical derivative in x_0 (relative to increase h) is, by definition:

$$f'(x_0) = \frac{f'(x_0-0)+f'(x_0+0)}{2} = \frac{\frac{f(x_0)-f(x_0-h)}{h} + \frac{f(x_0+h)-f(x_0)}{h}}{2} = \frac{f(x_0+h)-f(x_0-h)}{2h}$$

We then consider, the speed of variation of r_k as: $v_k=\frac{r_{k+1}-r_{k-1}}{2}$, $k=\overline{3,m-1}$.

As a result of this indicator, we obtain: $r_{k+1}=r_{k-1}+2v_k$ therefore:

$$GDP_{k+1}=(1+r_{k-1}+2v_k)GDP_k,\ k=\overline{3,m-1}$$

Let consider now, for our analysis, the Gross Domestic Product of the U.S. in the period 1792-2010:

Table 1. The Gross Domestic Product of the U.S. in the period 1792-1865

Year	GDP	r_k	v_k	Year	GDP	r_k	v_k
1792	4.58	-	-	1829	20.30	0.03836	0.03908
1793	4.95	0.08079	-	1830	22.16	0.09163	0.02211
1794	5.60	0.13131	-0.00825	1831	23.99	0.08258	-0.01205
1795	5.96	0.06429	-0.04972	1832	25.61	0.06753	-0.02587
1796	6.15	0.03188	-0.02239	1833	26.40	0.03085	-0.02524
1797	6.27	0.01951	0.00559	1834	26.85	0.01705	0.01102
1798	6.54	0.04306	0.02542	1835	28.27	0.05289	0.00633
1799	7.00	0.07034	0.00704	1836	29.11	0.02971	-0.02198
1800	7.40	0.05714	-0.01085	1837	29.37	0.00893	0.00592
1801	7.76	0.04865	-0.01311	1838	30.59	0.04154	0.00829
1802	8.00	0.03093	-0.01558	1839	31.37	0.02550	-0.01934
1803	8.14	0.01750	0.00358	1840	31.46	0.00287	-0.00147
1804	8.45	0.03808	0.01788	1841	32.17	0.02257	0.01442
1805	8.90	0.05325	0.00456	1842	33.19	0.03171	0.01357
1806	9.32	0.04719	-0.02609	1843	34.84	0.04971	0.01256
1807	9.33	0.00107	-0.02253	1844	36.82	0.05683	0.00679
1808	9.35	0.00214	0.03797	1845	39.15	0.06328	0.01220
1809	10.07	0.07701	0.02674	1846	42.33	0.08123	0.00238
1810	10.63	0.05561	-0.01593	1847	45.21	0.06804	-0.02381
1811	11.11	0.04516	-0.00801	1848	46.73	0.03362	-0.02707
1812	11.55	0.03960	0.00599	1849	47.38	0.01391	0.00651
1813	12.21	0.05714	0.00109	1850	49.59	0.04664	0.03328
1814	12.72	0.04177	-0.02464	1851	53.58	0.08046	0.03435
1815	12.82	0.00786	-0.02089	1852	59.76	0.11534	0.00068
1816	12.82	0.00000	0.00777	1853	64.65	0.08183	-0.04043
1817	13.12	0.02340	0.01830	1854	66.88	0.03449	-0.02006

1818	13.60	0.03659	-0.00214	1855	69.67	0.04172	0.00285
1819	13.86	0.01912	0.00155	1856	72.47	0.04019	-0.01831
1820	14.41	0.03968	0.01716	1857	72.84	0.00511	0.00016
1821	15.18	0.05344	-0.00074	1858	75.79	0.04050	0.03367
1822	15.76	0.03821	-0.00864	1859	81.28	0.07244	-0.01515
1823	16.33	0.03617	0.01060	1860	82.11	0.01021	-0.02733
1824	17.30	0.05940	0.00417	1861	83.57	0.01778	0.05700
1825	18.07	0.04451	-0.01199	1862	93.95	0.12421	0.02959
1826	18.71	0.03542	-0.00676	1863	101.18	0.07696	-0.05642
1827	19.29	0.03100	-0.01097	1864	102.33	0.01137	-0.02417
1828	19.55	0.01348	0.00368	1865	105.26	0.02863	-0.02863

* GDP-US $ billion 2005

Source: http://www.usgovernmentrevenue.com

Table 2. The Gross Domestic Product of the U.S. in the period 1866-1938

Year	GDP	r_k	v_k	Year	GDP	r_k	v_k
1866	100.43	-0.04589	-0.00575	1903	481.80	0.02905	-0.04336
1867	102.15	0.01713	0.04243	1904	464.80	-0.03528	0.04185
1868	106.13	0.03896	0.00505	1905	517.20	0.11274	0.03814
1869	109.02	0.02723	-0.00444	1906	538.40	0.04099	-0.04356
1870	112.30	0.03009	0.00999	1907	552.20	0.02563	-0.07455
1871	117.60	0.04720	0.02705	1908	492.50	-0.10811	0.02333
1872	127.50	0.08418	0.01876	1909	528.10	0.07228	0.05945
1873	138.30	0.08471	-0.03305	1910	533.80	0.01079	-0.01994
1874	140.80	0.01808	-0.04307	1911	551.10	0.03241	0.01802
1875	140.60	-0.00142	0.01159	1912	576.90	0.04682	0.00356
1876	146.40	0.04125	0.02564	1913	599.70	0.03952	-0.06177
1877	153.70	0.04986	-0.00469	1914	553.70	-0.07671	-0.00613
1878	158.60	0.03188	0.03340	1915	568.80	0.02727	0.10771
1879	177.10	0.11665	0.02556	1916	647.70	0.13871	-0.02599
1880	191.80	0.08300	0.00424	1917	631.70	-0.02470	-0.02424
1881	215.80	0.12513	-0.01486	1918	688.70	0.09023	0.01635
1882	227.30	0.05329	-0.04893	1919	694.20	0.00799	-0.04980
1883	233.50	0.02728	-0.03478	1920	687.70	-0.00936	-0.01549

1884	229.70	-0.01627	-0.01190	1921	671.90	-0.02298	0.03251
1885	230.50	0.00348	0.04870	1922	709.30	0.05566	0.07726
1886	249.20	0.08113	0.03458	1923	802.60	0.13154	-0.01238
1887	267.30	0.07263	-0.01176	1924	827.40	0.03090	-0.05405
1888	282.70	0.05761	-0.02199	1925	846.80	0.02345	0.01720
1889	290.80	0.02865	0.01986	1926	902.10	0.06530	-0.00691
1890	319.10	0.09732	-0.00853	1927	910.80	0.00964	-0.02689
1891	322.80	0.01160	-0.02310	1928	921.30	0.01153	0.02541
1892	339.30	0.05112	-0.03483	1929	977.00	0.06046	-0.04886
1893	319.60	-0.05806	-0.04919	1930	892.80	-0.08618	-0.06266
1894	304.50	-0.04725	0.08601	1931	834.90	-0.06485	-0.02225
1895	339.20	0.11396	0.01537	1932	725.80	-0.13067	0.02595
1896	333.60	-0.01651	-0.03540	1933	716.40	-0.01295	0.11978
1897	348.00	0.04317	0.06300	1934	794.40	0.10888	0.05091
1898	386.10	0.10948	0.01261	1935	865.00	0.08887	0.01082
1899	412.50	0.06838	-0.04226	1936	977.90	0.13052	-0.01882
1900	422.80	0.02497	-0.00758	1937	1028.00	0.05123	-0.08248
1901	445.30	0.05322	0.01323	1938	992.60	-0.03444	0.01479
1902	468.20	0.05143	-0.01209				

* GDP-US $ billion 2005

Source: http://www.usgovernmentrevenue.com

Table 3. The Gross Domestic Product of the U.S. in the period 1939-2010

Year	GDP	r_k	v_k	Year	GDP	r_k	v_k
1939	1072.80	0.08080	0.06108	1975	4879.50	-0.00213	0.02958
1940	1166.90	0.08771	0.04496	1976	5141.30	0.05365	0.02406
1941	1366.10	0.17071	0.04842	1977	5377.70	0.04598	0.00106
1942	1618.20	0.18454	-0.00350	1978	5677.60	0.05577	-0.00737
1943	1883.10	0.16370	-0.05189	1979	5855.00	0.03125	-0.02925
1944	2035.20	0.08077	-0.08745	1980	5839.00	-0.00273	-0.00294
1945	2012.40	-0.01120	-0.09510	1981	5987.20	0.02538	-0.00835
1946	1792.20	-0.10942	0.00111	1982	5870.90	-0.01942	0.00991
1947	1776.10	-0.00898	0.07670	1983	6136.20	0.04519	0.04564
1948	1854.20	0.04397	0.00193	1984	6577.10	0.07185	-0.00190

1949	1844.70	-0.00512	0.02174	1985	6849.30	0.04139	-0.01861
1950	2006.00	0.08744	0.04122	1986	7086.50	0.03463	-0.00470
1951	2161.10	0.07732	-0.02457	1987	7313.30	0.03200	0.00324
1952	2243.90	0.03831	-0.01564	1988	7613.90	0.04110	0.00186
1953	2347.20	0.04604	-0.02231	1989	7885.90	0.03572	-0.01117
1954	2332.40	-0.00631	0.01298	1990	8033.90	0.01877	-0.01903
1955	2500.30	0.07199	0.01304	1991	8015.10	-0.00234	0.00759
1956	2549.70	0.01976	-0.02592	1992	8287.10	0.03394	0.01543
1957	2601.10	0.02016	-0.01440	1993	8523.40	0.02851	0.00341
1958	2577.60	-0.00903	0.02579	1994	8870.70	0.04075	-0.00169
1959	2762.50	0.07173	0.01690	1995	9093.70	0.02514	-0.00167
1960	2830.90	0.02476	-0.02421	1996	9433.90	0.03741	0.00971
1961	2896.90	0.02331	0.01791	1997	9854.30	0.04456	0.00307
1962	3072.40	0.06058	0.01020	1998	10283.50	0.04355	0.00185
1963	3206.70	0.04371	-0.00135	1999	10779.80	0.04826	-0.00108
1964	3392.30	0.05788	0.01025	2000	11226.00	0.04139	-0.01873
1965	3610.10	0.06420	0.00364	2001	11347.20	0.01080	-0.01163
1966	3845.30	0.06515	-0.01946	2002	11553.00	0.01814	0.00705
1967	3942.50	0.02528	-0.00837	2003	11840.70	0.02490	0.00880
1968	4133.40	0.04842	0.00289	2004	12263.80	0.03573	0.00283
1969	4261.80	0.03106	-0.02326	2005	12638.40	0.03055	-0.00450
1970	4269.90	0.00190	0.00126	2006	12976.20	0.02673	-0.00457
1971	4413.30	0.03358	0.02561	2007	13228.90	0.02142	-0.01118
1972	4647.70	0.05311	0.01218	2008	13228.80	-0.00001	-0.02280
1973	4917.00	0.05794	-0.02931	2009	12880.60	-0.02632	-0.00033
1974	4889.90	-0.00551	-0.03004	2010	13248.20	0.02854	-

* GDP-US $ billion 2005

Source: http://www.usgovernmentrevenue.com

The analysis procedure will be to determine the Fourier regressions of best approximation on the interval [1794, 2009], for the data set (k,v_k). We calculate thus, for each $n=\overline{1,20}$ (number of terms of Fourier development) and $T=\overline{10,100}$ (the development period) the mean square error:

$$\varepsilon_{n,T}=\sqrt{\frac{\sum_{k=1}^{m}\left(v_k-\bar{v}_k\right)^2}{m}}$$

where $\overline{v}_k = F_n(k)$, $k=\overline{1,m}$, determining the period and the number of terms of development, corresponding to the mean square error lower. Finally we will select the period T and n for the lowest $\varepsilon_{n,T}$. In our analysis we found that for n=20, T=71: $\varepsilon_{20,50}=0.009286$ (0.93%) is the lowest mean square error. For a better accuracy of results, we will determine again the Fourier development corresponding to the interval [1939,2009], therefore for a period of 71 years and n=20.

The Fourier coefficients thus determined are:

Table 4. The Fourier coefficients for n=20 and T=71

a_0	$-1.192538\cdot10^{-03}$						
a_1	$-2.233183\cdot10^{-04}$	a_{11}	$3.103659\cdot10^{-03}$	b_1	$2.178501\cdot10^{-04}$	b_{11}	$-8.27152\cdot10^{-03}$
a_2	$4.439536\cdot10^{-04}$	a_{12}	$1.340319\cdot10^{-04}$	b_2	$4.527911\cdot10^{-04}$	b_{12}	$-9.438256\cdot10^{-03}$
a_3	$2.34559\cdot10^{-03}$	a_{13}	$-8.517535\cdot10^{-03}$	b_3	$1.434893\cdot10^{-03}$	b_{13}	$-4.030202\cdot10^{-03}$
a_4	$3.236102\cdot10^{-03}$	a_{14}	$5.9386\cdot10^{-04}$	b_4	$-1.943463\cdot10^{-03}$	b_{14}	$3.670609\cdot10^{-03}$
a_5	$2.601004\cdot10^{-03}$	a_{15}	$-5.273752\cdot10^{-03}$	b_5	$-5.831661\cdot10^{-05}$	b_{15}	$8.056044\cdot10^{-04}$
a_6	$2.778745\cdot10^{-03}$	a_{16}	$-2.599379\cdot10^{-03}$	b_6	$3.83987\cdot10^{-04}$	b_{16}	$2.648987\cdot10^{-05}$
a_7	$-4.762545\cdot10^{-03}$	a_{17}	$-2.808314\cdot10^{-03}$	b_7	$-1.958835\cdot10^{-03}$	b_{17}	$-7.070958\cdot10^{-03}$
a_8	$-9.197807\cdot10^{-03}$	a_{18}	$-1.595883\cdot10^{-03}$	b_8	$-6.166687\cdot10^{-03}$	b_{18}	$1.147\cdot10^{-03}$
a_9	$-2.676535\cdot10^{-3}$	a_{19}	$4.790776\cdot10^{-03}$	b_9	$-7.567441\cdot10^{-03}$	b_{19}	$1.169301\cdot10^{-03}$
a_{10}	$3.218776\cdot10^{-03}$	a_{20}	$-6.507447\cdot10^{-04}$	b_{10}	$-6.535201\cdot10^{-03}$	b_{20}	$1.051339\cdot10^{-02}$

Substituting in the expression of F_{20}, the values $k=\overline{1,71}$ we obtain the new values, by periodicity, of $\overline{v}_k$.

Table 5. The new values for the speed of variation for n=20 and T=71

k	$\overline{v}_k$	k	$\overline{v}_k$	k	$\overline{v}_k$	k	$\overline{v}_k$	k	$\overline{v}_k$
1	0.05714	16	0.02126	31	-0.0132	46	0.00601	61	-0.005
2	0.05809	17	0.003	32	-0.0024	47	-0.0198	62	-0.0136
3	0.03131	18	-0.0258	33	0.02405	48	-0.009	63	-0.0129
4	0.00552	19	-0.0056	34	0.01322	49	0.00841	64	0.0036
5	-0.0452	20	0.02038	35	-0.0288	50	0.0007	65	0.01294
6	-0.1041	21	0.00849	36	-0.0271	51	-0.0154	66	0.00183
7	-0.0842	22	-0.0056	37	0.0204	52	-0.0129	67	-0.0061
8	0.00821	23	0.00344	38	0.03383	53	0.00452	68	-0.004
9	0.05373	24	0.01058	39	0.00024	54	0.01403	69	-0.0143
10	0.02564	25	0.00748	40	-0.0188	55	0.00639	70	-0.021
11	0.01371	26	0.00539	41	-0.0126	56	-0.0034	71	0.0126
12	0.02792	27	-0.0016	42	-0.0138	57	-0.0005		
13	0.00128	28	-0.0113	43	-0.0091	58	0.00706		
14	-0.0375	29	-0.0088	44	0.01978	59	0.00654		
15	-0.0162	30	-0.0061	45	0.03365	60	0.00122		

otherwise having $v_k=v_s$ where $s=k-71\left[\frac{k}{71}\right]$ for k not dividing by 71 and $v_k=v_{71}$ for k multiple of 71, where [a] is the highest integer less than $a\in\mathbf{R}$.
The comparative graphs of the evolution v_k and of the new indicators after Fourier regression are:

The Fourier regression for T=71 between 1939-2009

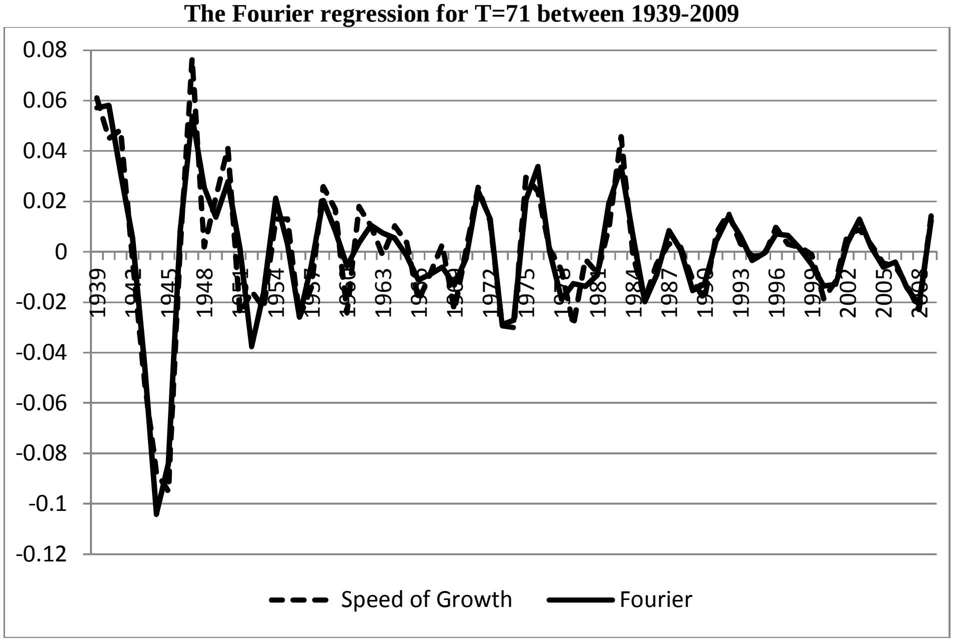

Figure 1

In annual terms, we have: $v_k=\overline{v}_{k-1938}$ for any $k\geq1939$. The GDP's estimate is:

$$GDP_{k+1}=(1+r_{k-1}+2\,\overline{v}_{k-1938})GDP_k$$

In particular:

$GDP_{2010}=(1+r_{2008}+2\,\overline{v}_{71})GDP_{2009}=(1-0.000007+2\cdot0.0126)\cdot12880,60=13205.10$

with a relative error towards the real value of 0.33%.

Conclusions

The method of cyclic regressions used in this article is particularly useful in the situation analysis of periodic phenomena, providing a possible law of evolution. In the present case, the analysis of the evolution of GDP in the light of the speed of variation of the GDP's rate, reveals a periodicity of 71 years, the mean square error recorded being 0.93% which is a very good approximation. The method described allows, on the basis of the conclusions obtained, making forecasts, we appreciate at the short term, due to the occurrence of factors which can change significantly the predicted data.

On the other hand, for greater accuracy of the forecasts, will be recalculated every time the coefficients of Fourier series, for the last 71 years.

It should be noted also that the method is based exclusively on the numeric data without taking account of causal factors. On the other hand, the classical models of cyclicity are based on a series of observations, but does not strictly mathematical the determination of the periodicity.

References

Howard, S. (2001). The Business Cycle Theory of Wesley Mitchel. *Journal of Economic Issues*, March.

Jevons, W.S. (1878). Commercial Crises and Sun-Spots. *Nature*, Volume 19, Issue 472, pp. 33-37.

Kydland F.E., Prescott E.C. (1982). Time to Build and Aggregate Fluctuations. *Econometrica*, Vol.50, Issue 6, pp.1345-1370

Maddison, A. (2007). Fluctuations in the momentum of growth within the capitalist epoch, *Cliometrica.*

Okun A.M. (1970). *The Political Economy of Prosperity*. Norton W.W.&Company Inc.

Samuelson A.P., Nordhaus D.W. (2004). *Economics: An Introductory Analysis*. McGraw-Hill.

Spiegel M.R. (1974). *Fourier Analysis with Applications to Boundary Value Problems*. McGraw-Hill Book Company.