An Analysis of the Substitution Effect and of Revenue Effect in the Case of the Consumer's Theory Provided with a CES Utility Function

Catalin Angelo Ioan[1], Gina Ioan[2]

Abstract In the consumer's theory, a crucial problem is to determine the substitution effect and the revenue effect in the case of one good price's modifing. There exists two theories due to John Richard Hicks and Eugen Slutsky which allocates differents shares of the total change of the consumption to these effects. The paper makes an analysis between the two effects, considering the general case of a CES utility function and introduces three indicators which will characterize these shares.

Keywords: CES; substitution; revenue; utility

JEL Classification: D11

1. Introduction

In the consumer's theory, a crucial problem is to determine the substitution effect and the revenue effect in the case of one good price's modifing.

The theory due to John Richard Hicks consider after a modifing of a price, first a new allocation of goods preserving the utility, but modifing the revenue and after taking into account that the revenue is the initial one the changing in allocation due to a different utility.

The theory of Eugen Slutsky consider a combined displacement of the relative consuming obtained a share of the substitution effect or of revenue effect depending only from the parameters of the utility.

The problem is to determine these shares for both methods and to inquire which effect is uppermost.

[1] Associate Professor, PhD, Danubius University of Galati, Faculty of Economic Sciences, Romania, Address: 3 Galati Blvd, Galati, Romania, tel: +40372 361 102, fax: +40372 361 290, Corresponding author: catalin_angelo_ioan@univ-danubius.ro

[2] Assistant Professor, PhD in progress, Danubius University of Galati, Faculty of Economic Sciences, Romania, Address: 3 Galati Blvd, Galati, Romania, tel: +40372 361 102, fax: +40372 361 290, e-mail: gina_ioan@univ-danubius.ro

AUDŒ, Vol 8, no 1, pp. 164-175

2. The Analysis

Let two goods A and B with the initial prices p_A and p_B and an utility function of a CES type U= $T\left(\alpha X^{-\lambda}+\beta Y^{-\lambda}\right)^{-\frac{1}{\lambda}}$, $\alpha,\beta>0$, $\lambda>0$, where X and Y are the consumed quantities in order to obtain an utility U. Let also, at a given time, V – the consumer's revenue.

In order to have the maximum utility for the revenue V it is known that we must have:

$$\begin{cases} \dfrac{U_{mA}}{U_{mB}}=\dfrac{p_A}{p_B} \\ V=p_AX+p_BY \end{cases}$$

where $U_{mA}=\alpha TX^{-\lambda-1}\left(\alpha X^{-\lambda}+\beta Y^{-\lambda}\right)^{-\frac{1}{\lambda}-1}$ and $U_{mB}=\beta TY^{-\lambda-1}\left(\alpha X^{-\lambda}+\beta Y^{-\lambda}\right)^{-\frac{1}{\lambda}-1}$ are the marginal utilities corresponding to the two goods A and B respectively.

We have now:

$$\begin{cases} \dfrac{\alpha X^{-\lambda-1}}{\beta Y^{-\lambda-1}}=\dfrac{p_A}{p_B} \\ V=p_AX+p_BY \end{cases}$$

Let note, in what follows:

$$\varphi=\frac{\alpha}{\beta},\ r_1=\frac{p_A}{p_B}$$

and: $S=r_1^{\frac{\lambda}{\lambda+1}}+\varphi^{-\frac{1}{\lambda+1}}$.

We have therefore:

$$Y=\left(\frac{\alpha p_B}{\beta p_A}\right)^{-\frac{1}{\lambda+1}}X=\varphi^{-\frac{1}{\lambda+1}}r_1^{\frac{1}{\lambda+1}}X$$

$$V=\left(p_A+p_B\varphi^{-\frac{1}{\lambda+1}}r_1^{\frac{1}{\lambda+1}}\right)X=\left(r_1+\varphi^{-\frac{1}{\lambda+1}}r_1^{\frac{1}{\lambda+1}}\right)p_BX$$

We obtain now:

$$X_1=\frac{r_1^{\frac{\lambda}{\lambda+1}}V}{Sp_A},\ Y_1=\frac{\varphi^{-\frac{1}{\lambda+1}}V}{Sp_B}$$

and the corresponding utility is: $U_1=\frac{TV\beta^{-\frac{1}{\lambda}}\varphi^{-\frac{1}{\lambda}}S^{-\frac{\lambda+1}{\lambda}}}{p_B}$.

Let suppose now that it is a change in the price of one of the goods, let say B, from p_B to p'_B, but the revenue V remains constant. Let note now: $r_2=\frac{p'_B}{p_B}$ and, of course: $\frac{p_A}{p'_B}=\frac{r_1}{r_2}$.

Let note, also: $R=r_1^{\frac{\lambda}{\lambda+1}}r_2^{-\frac{\lambda}{\lambda+1}}+\varphi^{-\frac{1}{\lambda+1}}$, $Q=\frac{R}{S}$.

We have, from the upper relations:

$$R-S=r_1^{\frac{\lambda}{\lambda+1}}r_2^{-\frac{\lambda}{\lambda+1}}\left(1-r_2^{\frac{\lambda}{\lambda+1}}\right)$$

$$\varphi^{-\frac{1}{\lambda+1}}=\frac{R-r_2^{-\frac{\lambda}{\lambda+1}}S}{1-r_2^{-\frac{\lambda}{\lambda+1}}}$$

Now:

$$X_3=\frac{r_1^{\frac{\lambda}{\lambda+1}}r_2^{-\frac{\lambda}{\lambda+1}}V}{Rp_A},\ Y_3=\frac{\varphi^{-\frac{1}{\lambda+1}}V}{Rr_2p_B}$$

and the corresponding utility: $U_3=\frac{TV\beta^{-\frac{1}{\lambda}}\varphi^{-\frac{1}{\lambda}}}{r_2p_B}R^{-\frac{\lambda+1}{\lambda}}$.

We shall apply now the Hicks method for our analysis.

At the modify of the price of B, for the same utility:

$U_1=\frac{TV\beta^{-\frac{1}{\lambda}}\varphi^{-\frac{1}{\lambda}}S^{-\frac{\lambda+1}{\lambda}}}{p_B}$ we shall have: $U_1=\frac{TV'\beta^{-\frac{1}{\lambda}}\varphi^{-\frac{1}{\lambda}}}{r_2p_B}R^{-\frac{\lambda+1}{\lambda}}$

therefore:

$$\frac{TV\beta^{-\frac{1}{\lambda}}\varphi^{-\frac{1}{\lambda}}S^{-\frac{\lambda+1}{\lambda}}}{p_B}=\frac{TV'\beta^{-\frac{1}{\lambda}}\varphi^{-\frac{1}{\lambda}}}{r_2p_B}R^{-\frac{\lambda+1}{\lambda}}$$

implies that:

$$V'=\frac{Vr_2S^{-\frac{\lambda+1}{\lambda}}}{R^{-\frac{\lambda+1}{\lambda}}}$$

With the new revenue, we obtain:

$$X_{2H}=\frac{r_1^{\frac{\lambda}{\lambda+1}}r_2^{\frac{1}{\lambda+1}}S^{-\frac{\lambda+1}{\lambda}}V}{R^{-\frac{1}{\lambda}}p_A}$$

$$Y_{2H}=\frac{\varphi^{-\frac{1}{\lambda+1}}S^{-\frac{\lambda+1}{\lambda}}V}{R^{-\frac{1}{\lambda}}p_B}.$$

The substitution effect (which preserves the utility) gives us a difference:

$$\Delta_{1H}X=X_{2H}\text{-}X_1=\frac{r_2^{\frac{1}{\lambda+1}}-Q^{-\frac{1}{\lambda}}}{Q^{-\frac{1}{\lambda}}Sp_A}r_1^{\frac{\lambda}{\lambda+1}}V$$

$$\Delta_{1H}Y=Y_{2H}\text{-}Y_1=\frac{1-Q^{-\frac{1}{\lambda}}}{Q^{-\frac{1}{\lambda}}Sp_B}\varphi^{-\frac{1}{\lambda+1}}V$$

The difference caused by the revenue V instead V’ is therefore:

$$\Delta_{2H}X=X_3\text{-}X_{2H}=\frac{1-r_2Q^{\frac{\lambda+1}{\lambda}}}{Rp_A}r_1^{\frac{\lambda}{\lambda+1}}r_2^{-\frac{\lambda}{\lambda+1}}V$$

$$\Delta_{2H}Y=Y_3\text{-}Y_{2H}=\frac{1-Q^{\frac{\lambda+1}{\lambda}}r_2}{Rr_2p_B}\varphi^{-\frac{1}{\lambda+1}}V$$

named the revenue effect.

We shall apply now the Slutsky method for our analysis.

At the modify of the price of B, the revenue for the same optimal combination of goods is:

$$V' = p_A X_1 + p'_B Y_1 = p_A \frac{r_1^{\frac{\lambda}{\lambda+1}} V}{Sp_A} + p'_B \frac{\varphi^{-\frac{1}{\lambda+1}} V}{Sp_B} = \frac{S + \varphi^{-\frac{1}{\lambda+1}}(r_2 - 1)}{S} V .$$

therefore:

$$X_{2S} = \frac{r_1^{\frac{\lambda}{\lambda+1}} r_2^{-\frac{\lambda}{\lambda+1}} V'}{Rp_A} = \frac{r_1^{\frac{\lambda}{\lambda+1}} r_2^{-\frac{\lambda}{\lambda+1}} \left(S + \varphi^{-\frac{1}{\lambda+1}}(r_2 - 1) \right) V}{RSp_A}$$

$$Y_{2S} = \frac{\varphi^{-\frac{1}{\lambda+1}} V'}{Rp'_B} = \frac{\varphi^{-\frac{1}{\lambda+1}} \left(S + \varphi^{-\frac{1}{\lambda+1}}(r_2 - 1) \right) V}{RSr_2 p_B} .$$

and the corresponding utility:

$$U_2 = T\left(\alpha X_{2S}^{-\lambda} + \beta Y_{2S}^{-\lambda}\right)^{-\frac{1}{\lambda}} = \frac{TV\beta^{-\frac{1}{\lambda}} \varphi^{-\frac{1}{\lambda}} \left(S + \varphi^{-\frac{1}{\lambda+1}}(r_2 - 1) \right) R^{-\frac{\lambda+1}{\lambda}} r_1}{Sr_2 p_A}$$

The substitution effect after Slusky (which not preserves the utility) gives us a difference:

$$\Delta_{1S}X = X_{2S} - X_1 = \frac{r_2^{-\frac{\lambda}{\lambda+1}} \left(S + \varphi^{-\frac{1}{\lambda+1}}(r_2 - 1) \right) - R}{RSp_A} r_1^{\frac{\lambda}{\lambda+1}} V$$

$$\Delta_{1S}Y = Y_{2S} - Y_1 = \frac{S + \varphi^{-\frac{1}{\lambda+1}}(r_2 - 1) - Rr_2}{RSr_2 p_B} \varphi^{-\frac{1}{\lambda+1}} V$$

and the revenue effect (after Slutsky):

$$\Delta_{2S}X = X_3 - X_{2S} = \frac{-\varphi^{-\frac{1}{\lambda+1}}(r_2 - 1)}{RSp_A} r_1^{\frac{\lambda}{\lambda+1}} r_2^{-\frac{\lambda}{\lambda+1}} V$$

$$\Delta_{2S}Y=Y_3-Y_{2S}=\frac{-\varphi^{-\frac{1}{\lambda+1}}(r_2-1)}{RSr_2p_B}\varphi^{-\frac{1}{\lambda+1}}V$$

We shall define, in what follows, the ratio:

$\alpha_Y=\frac{Y_2-Y_1}{Y_3-Y_1}$ - the share from the total consumption change for Y due to the substitution effect;

$\beta_Y=\frac{Y_3-Y_2}{Y_3-Y_1}$ - the share from the total consumption change for Y due to the revenue effect;

$r_Y=\frac{\beta_Y}{\alpha_Y}=\frac{Y_3-Y_2}{Y_2-Y_1}$ - the ratio between the revenue effect and the substitution effect.

We have obviously: $\alpha_Y+\beta_Y=1$ and $r_Y=\frac{1}{\alpha_Y}-1=\frac{1}{\frac{1}{\beta_Y}-1}$.

In the case of Hicks, we have:

- $\alpha_{YH}=\frac{\Delta_{1H}Y}{\Delta_{1H}Y+\Delta_{2H}Y}=\frac{\left(Q^{\frac{1}{\lambda}}-1\right)Qr_2}{1-Qr_2}$

- $\beta_{YH}=\frac{\Delta_{2H}Y}{\Delta_{1H}Y+\Delta_{2H}Y}=\frac{1-Q^{\frac{\lambda+1}{\lambda}}r_2}{1-Qr_2}$

- $r_{YH}=\frac{\beta_{YH}}{\alpha_{YH}}=\frac{1-Q^{\frac{\lambda+1}{\lambda}}r_2}{\left(Q^{\frac{1}{\lambda}}-1\right)Qr_2}$

In the case of Slutsky, we have:

- $\alpha_{YS}=\dfrac{\Delta_{1S}Y}{\Delta_{1S}Y+\Delta_{2S}Y}=1+\dfrac{\left(Q-r_2^{-\frac{\lambda}{\lambda+1}}\right)(r_2-1)}{(1-Qr_2)\left(1-r_2^{-\frac{\lambda}{\lambda+1}}\right)}$

- $\beta_{YS}=\dfrac{\Delta_{2S}Y}{\Delta_{1S}Y+\Delta_{2S}Y}=\dfrac{-\left(Q-r_2^{-\frac{\lambda}{\lambda+1}}\right)(r_2-1)}{(1-Qr_2)\left(1-r_2^{-\frac{\lambda}{\lambda+1}}\right)}$

- $r_{YS}=\dfrac{\beta_{YS}}{\alpha_{YS}}$

Because $Q=\dfrac{R}{S}=\dfrac{r_1^{\frac{\lambda}{\lambda+1}}r_2^{-\frac{\lambda}{\lambda+1}}+\varphi^{-\frac{1}{\lambda+1}}}{r_1^{\frac{\lambda}{\lambda+1}}+\varphi^{-\frac{1}{\lambda+1}}}$ we have: $1-r_2Q=\dfrac{\left(1-r_2^{\frac{1}{\lambda+1}}\right)r_1^{\frac{\lambda}{\lambda+1}}+(1-r_2)\varphi^{-\frac{1}{\lambda+1}}}{r_1^{\frac{\lambda}{\lambda+1}}+\varphi^{-\frac{1}{\lambda+1}}}$

therefore: if $r_2<1$ then $1-r_2Q>0$ and if $r_2>1$ then $1-r_2Q<0$.

Let analyse now the inequality: $\alpha_{YH}>\alpha_{YS}$. We have:

$$\frac{\left(Q^{\frac{1}{\lambda}}-1\right)Qr_2}{1-Qr_2}>1+\frac{\left(Q-r_2^{-\frac{\lambda}{\lambda+1}}\right)(r_2-1)}{(1-Qr_2)\left(1-r_2^{-\frac{\lambda}{\lambda+1}}\right)}\text{ therefore:}$$

$$\frac{\left(Q^{\frac{1}{\lambda}}Qr_2-1\right)\left(1-r_2^{-\frac{\lambda}{\lambda+1}}\right)-\left(Q-r_2^{-\frac{\lambda}{\lambda+1}}\right)(r_2-1)}{(1-Qr_2)\left(1-r_2^{-\frac{\lambda}{\lambda+1}}\right)}>0$$

Because $(1-r_2Q)\left(1-r_2^{-\frac{\lambda}{\lambda+1}}\right)<0$ we must have:

$$Q^{\frac{\lambda+1}{\lambda}}\left(r_2-r_2^{\frac{1}{\lambda+1}}\right)-Q(r_2-1)+r_2^{\frac{1}{\lambda+1}}-1<0.$$

Let now the function: g(Q)= $Q^{\frac{\lambda+1}{\lambda}}\left(r_2 - r_2^{\frac{1}{\lambda+1}}\right) - Q(r_2-1) + r_2^{\frac{1}{\lambda+1}} - 1$

We have: $g'(Q) = \frac{\lambda+1}{\lambda}Q^{\frac{1}{\lambda}}\left(r_2 - r_2^{\frac{1}{\lambda+1}}\right) - (r_2-1)$ =0⇒

$$Q_{root} = \left(\frac{\lambda}{\lambda+1}\right)^{\lambda} r_2^{-\frac{\lambda}{\lambda+1}}\left(\frac{r_2-1}{r_2^{\frac{\lambda}{\lambda+1}}-1}\right)^{\lambda} > 0.$$

and, with the notation: u= $\frac{r_2^{\frac{\lambda}{\lambda+1}}-1}{r_2-1} \in (0,1)$, we have: $g(Q_{root})=$

$$\frac{(r_2-1)}{r_2^{\frac{\lambda}{\lambda+1}}u^{\lambda}}\left((1-u)u^{\lambda} - \frac{\lambda^{\lambda}}{(\lambda+1)^{\lambda+1}}\right).$$

Lemma 1 $(1-u)u^{\lambda} < \frac{\lambda^{\lambda}}{(\lambda+1)^{\lambda+1}}$ $\forall u \in (0,1)$ $\forall \lambda > 0$

Proof Let note h:(0,1)→**R**, h(u)=(1-u)u^{λ}. Because $h'(u) = u^{\lambda-1}(\lambda - (\lambda+1)u)$=0 has the root: $u_0 = \frac{\lambda}{\lambda+1}$ we obtain that: h has a maximum value in u_0, therefore $(1-u)u^{\lambda} < \frac{\lambda^{\lambda}}{(\lambda+1)^{\lambda+1}}$. **Q.E.D.**

From the upper relation, we obtain now that: $g(Q_{root})>0$ for $r_2<1$ and $g(Q_{root})<0$ for $r_2>1$.

Lemma 2 $(x-1)\left(\frac{\lambda+1}{\lambda}\frac{x - x^{\frac{1}{\lambda+1}}}{x-1} - 1\right) > 0$ $\forall x>0$ $\forall \lambda>0$

Proof Let note h:**R**-{1}→**R**, h(x)= $\frac{\lambda+1}{\lambda}\frac{x - x^{\frac{1}{\lambda+1}}}{x-1} - 1$. We have

$$h'(x) = \frac{\lambda x^{\frac{1}{\lambda+1}} + x^{-\frac{\lambda}{\lambda+1}} - \lambda - 1}{\lambda(x-1)^2}$$

But $k(x)=\lambda x^{\frac{1}{\lambda+1}}+x^{-\frac{\lambda}{\lambda+1}}-\lambda-1$, $k'(x)=\frac{\lambda}{\lambda+1}x^{-\frac{2\lambda+1}{\lambda+1}}(x-1)$ and k(1)=0, implies that: k(x)>0 ∀x>0 therefore: $h'(x)>0$ ∀x>0. The function h being increasing and h(0)=-1, $\lim_{x\to1}h(x)=0$, $\lim_{x\to\infty}h(x)=\frac{1}{\lambda}$ we have that for 0<x<1: h(x)<0 and for x>1: h(x)>0. Multiplied h with (x-1) we shall obtain the conclusion of the lemma. **Q.E.D.**

We have now $g'(1)=\frac{\lambda+1}{\lambda}\left(r_2-r_2^{\frac{1}{\lambda+1}}\right)-(r_2-1)=(r_2-1)\left(\frac{\lambda+1}{\lambda}\frac{r_2-r_2^{\frac{1}{\lambda+1}}}{r_2-1}-1\right)$. From the lemma, we have therefore: $g'(1)>0$. Because $g'(0)=1-r_2$, $\lim_{Q\to\infty}g'(Q)=\left(1-r_2^{-\frac{\lambda}{\lambda+1}}\right)\infty$ we have that:

- if $r_2<1$: $Q_{root}>1$
- if $r_2>1$: $Q_{root}<1$

On the other hand, we have that: $g(0)=r_2^{\frac{1}{\lambda+1}}-1$, g(1)=0, $\lim_{Q\to\infty}g(Q)=\left(1-r_2^{-\frac{\lambda}{\lambda+1}}\right)\infty$.

We obtain then:

- if $r_2<1$: g is an increasing function on $(0,Q_{root})$ and decreasing on (Q_{root},∞) and also has one single root, except 1, in $(Q_{root},\infty)\subset(1,\infty)$.
- if $r_2>1$: g is a decreasing function on $(0,Q_{root})$ and is increasing on (Q_{root},∞) and also has one single root, except 1, in $(0,Q_{root})\subset(0,1)$

Let note $\overline{Q}$ - the single root of g. The upper specified values of g concludes that:

- if $r_2<1$: g<0 for $Q\in(0,1)\cup(\overline{Q},\infty)$ and g>0 for $Q\in(1,\overline{Q})$.
- if $r_2>1$: g>0 for $Q\in(0,\overline{Q})\cup(1,\infty)$ and g<0 for $Q\in(\overline{Q},1)$.

In terms of our indicators, we have that $\alpha_{YH}>\alpha_{YS}$ if $r_2<1$ and $Q\in(0,1)\cup(\overline{Q},\infty)$ or $r_2>1$ and $Q\in(\overline{Q},1)$ where $\overline{Q}$ is the root of the equation:

$$Q^{\frac{\lambda+1}{\lambda}}\left(r_2 - r_2^{\frac{1}{\lambda+1}}\right) - Q(r_2-1) + r_2^{\frac{1}{\lambda+1}} - 1 = 0$$

Also, $\alpha_{YH}<\alpha_{YS}$ if $r_2<1$ and $Q\in(1,\overline{Q})$ or $r_2>1$ and $Q\in(0,\overline{Q})\cup(1,\infty)$.

For the determination now of the real root $\overline{Q}$ of g, we shall apply the Newton method of approximation for functions of one variable. Because the starting point Q_0 for a function $g:[a,b]\to\mathbf{R}$, who maintains the monotony and the concavity is those for which $g(Q_0)g''(Q_0)>0$ and at us, if $r_2<1$: $g''<0$, $r_2>1$: $g''>0$, we must choose Q_0, in the case $r_2<1$ such that $g(Q_0)<0$ and in the case $r_2>1$ such that $g(Q_0)>0$.

On the other hand, if $r_2<1$, we have: $\overline{Q}>1$ and we shall choose the starting point Q_0 sufficiently large and if $r_2>1$, we have: $\overline{Q}<1$ and we shall choose the starting point Q_0 sufficiently small.

We have now, from the Newton's method:

(48) $$Q_{n+1}=Q_n-\frac{g(Q_n)}{g'(Q_n)}=\frac{Q_n^{\frac{\lambda+1}{\lambda}}\left(r_2-r_2^{\frac{1}{\lambda+1}}\right)+\lambda\left(1-r_2^{\frac{1}{\lambda+1}}\right)}{(\lambda+1)Q_n^{\frac{1}{\lambda}}\left(r_2-r_2^{\frac{1}{\lambda+1}}\right)-\lambda(r_2-1)},\ n\geq 0.$$

In the figure 1, we have on the horizontal axis the values of r_2 and on vertical axis the value of $\overline{Q}$ for which $\lambda=2$:

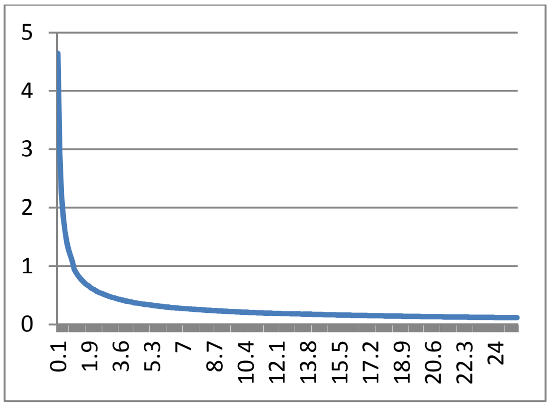

Figure 1. The chart of the roots $\overline{Q}$ for the case $\lambda=2$ in the case of a CES-function

If $\lambda \to 0$ we know that the CES-function becomes Cobb-Douglas.

In the figure 2, we have on the horizontal axis the values of r_2 and on vertical axis the value of $\overline{Q}$ for which $\lambda = \frac{1}{2}$:

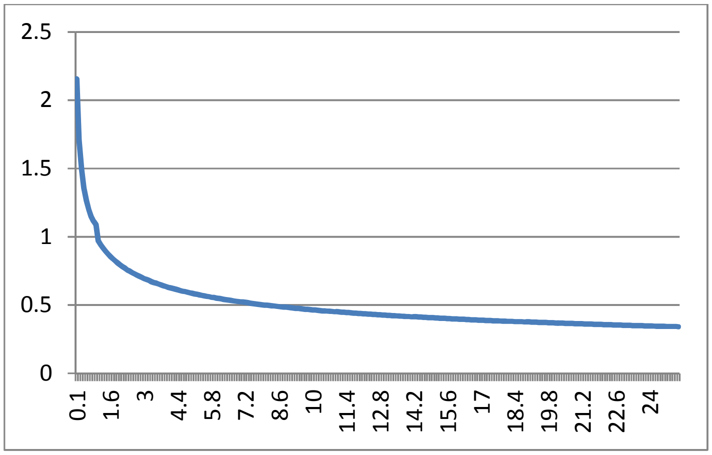

Figure 2. The chart of the roots $\overline{Q}$ for the case $\lambda = \frac{1}{2}$ in the case of a CES-function

If $\lambda \to 0$ we know that the CES-function becomes Cobb-Douglas.

3. Conclusion

Considering the single real root $\overline{Q}$ of the equation: $Q^{\frac{\lambda+1}{\lambda}}\left(r_2 - r_2^{\frac{1}{\lambda+1}}\right) - Q(r_2 - 1) + r_2^{\frac{1}{\lambda+1}} - 1 = 0$ we have that: $\alpha_{YH} > \alpha_{YS}$ if $r_2 < 1$ and $Q \in (0,1) \cup (\overline{Q}, \infty)$ or $r_2 > 1$ and $Q \in (\overline{Q}, 1)$ that is the share from the total consumption change for Y due to the substitution effect is smaller in the case of Slutsky than in the hicksian case. Also, $\beta_{YH} < \beta_{YS}$ that is the share from the total consumption change for Y due to the revenue effect is higher in the case of Slutsky than in the hicksian case.

If $r_2 < 1$ and $Q \in (1, \overline{Q})$ or $r_2 > 1$ and $Q \in (0, \overline{Q}) \cup (1, \infty)$ we have that $\alpha_{YH} < \alpha_{YS}$ that is the share from the total consumption change for Y due to the substitution effect is higher in the case of Slutsky than in the hicksian case and, of course $\beta_{YH} > \beta_{YS}$ that is the share from the total consumption change for Y due to the revenue effect is smaller in the case of Slutsky than in the hicksian case.

4. References

Arrow, K.J., Chenery, H.B., Minhas, B.S. and Solow, R.M. (1961). Capital Labour Substitution and Economic Efficiency. *Review of Econ and Statistics*, 63, pp. 225-250.

Cobb, C.W. and Douglas, P.H. (1928). A Theory of Production. *American Economic Review*, 18, pp. 139–165.

Ioan, C.A. (2008). *Mathematics I*. Bucharest: Sinteze.

Ioan, C.A. (2008). *Mathematics II*. Bucharest: Sinteze.

Mishra, S.K. (2007). *A Brief History of Production Functions.* India: North-Eastern Hill University, Shillong,

Revankar, N.S. (1971). A Class of Variable Elasticity of Substitution Production Functions. *Econometrica/Econometrics*, 39(1), pp. 61-71.